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Review Article

Review of the economic-mathematical analysis for the management of irrigation water and other crop production parameters

A. Shabani[1*], A. R. Sepaskhah[2], M. M. Moghimi[1], A. R. Zarei[3]

[1] Department of Water Science and Engineering, College of Agricultural Science, Fasa University, Fasa, I. R. Iran

[2] Irrigation Department, & Drought Research Center, School of Agriculture, Shiraz University, Shiraz, I. R. Iran

[3] Department of Range and Watershed Management (Nature Engineering), College of Agricultural Science, Fasa University, Fasa, I. R. Iran

* Corresponding Author: Shabani@Fasau.ac.ir

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ABSTRACT- Plant growth and yield are influenced by many production parameters such as the amount of irrigation water, fertilization, plant density, etc. Due to the limitation of resources and agricultural inputs, increased production costs, food demand, population growth, and environmental problems, the development of scheduling approaches to use production parameters is necessary. In this study, firstly, crop yield and net benefit responses to variation of production parameters and secondly, economic-mathematical analysis of production parameters such as irrigation water were reviewed. Presented analyses in this study were categorized into four cases including 1) constant or variable crop price, 2) single, two, or multiple variables production functions, 3) limited land area and water or not limited, and 4) variable water price. The economic-mathematical analysis presented about deficit irrigation in this study was extended to determine the optimum value of other production parameters such as fertilizer rate, plant density, seed density, corm planting intensity, etc.

INTRODUCTION

Limitation of resources and agricultural inputs, increase in production costs, increasing food demand, population growth, and environmental problems, have made inevitable the necessity of the development of scheduling approaches in the use of resources and agricultural inputs. Applying optimum values of agricultural inputs and other manageable production parameters can be considered one method for coping with limitations and shortages of these inputs.

Mathematical programming and heuristic methods are two procedures for solving optimization problems (Rodríguez et al., 2018). Understandability, analyzability, and the ability to determine the exact answers are the most important advantages of mathematical solution methods for solving optimization problems. However, in the heuristic methods, researchers may not be able to obtain the exact answer and should be satisfied to obtain acceptable or enough good answers (Burke et al. 2003).

Heuristic methods should be used in cases where problems cannot be stated in mathematical form (Rodríguez et al. 2018), the mathematical form of optimization problems is not solvable, however, an exact method of solving optimization problems is available, but it is not computationally attractive and obtaining acceptable answers is limited by computing time (Zanakis and Evans 1981). Furthermore, heuristics methods are categorized into a group known as the black-box type (Muñoz et al. 2015) that gives a final answer which may be not the correct answer (Kaveh and Talatahari 2010). In some cases, the optimization problems may have more than one answer that one of them is the correct one. The heuristic methods only give one answer that may be not correct or optimal (Yang 2010). Analysis of sensitivity in mathematical methods compared with heuristic methods is much easier and more comprehensible. As reported by Yoo and Kim (2014), for carrying out an analysis of sensitivity in the heuristic methods, the software should be run many times along with various inputs that are time-consuming.

To determine optimum values of production parameters, an assessment of crop yield response to variation of these parameters is necessary. Therefore, the relationship between the yield responses to different production parameters should be determined.

Crop yield Response to Production Parameters

As shown in Fig. 1, plant growth and yield responses to major production parameters such as the amount of irrigation water, fertilization, plant density, etc., are initially increasing and then decreasing over the increase in the level of most of these parameters. The crop yield increases with an increase in the level of production parameters until the level that would maximize yield. Zheng et al. (2019) by assessing 1490 paired data points from 21 counties reported that irrigation water (146 mm) increases maize grain yield by 30.35% compared with non-irrigated systems. Tomato yield first increased by applying 223.0 mm of irrigation water and application of 250 kg N ha–1 and then decreased by 297.0 mm and 350 kg ha–1 (Du et al. 2017).

The effect of production parameters on crop yield increase depends on its availability, environmental conditions, agronomic practice, and crop characteristics (Baeza et al. 2019; Liu et al. 2019; Bronson et al. 2019; Carciochi et al. 2019; Zhihui et al. 2016). Before the point of maximum, the increase in production parameters results in an increase in crop yield. After the maximum point, yield decreases due to the response of plant or soil to an excessive amount of production parameters. Excessive irrigation or over-irrigation causes crop yield reduction due to 1) an increase in nutrient loss from the root zone resulting from nutrient leaching to the sub-soil (Pulido-Bosch et al. 2018; Li et al. 2018; Kapur et al. 2017) or conversion nitrate into gas forms of nitrogen under waterlogging conditions, 2) coming up groundwater level and increase in soil salinity (Haj-Amor et al. 2017; Pulido-Bosch et al. 2018; Cui et al. 2019), 3) reduction in the temperature of the soil due to water evaporation from the soil surface, 4) change in soil hydraulic conductivity due to deformation of soil structure (Zhang et al. 2017), 5) increase in diseases (Chartzoulakis and Bertaki 2015) and 6) deficiency of air for aeration of plants roots and other soil beneficial organisms (Herzog et al. 2016).

Overusing fertilizer can result in more susceptibility of crops to drought and decrease harvest indices. On the other hand, full irrigation can only result in high yields if sufficient N-fertilizer is applied (Geerts and Raes 2009; Woli et al. 2016; Barzegari et al. 2017). Excess fertilizer results in an overabundance of soluble salts in the root zone soil that cause root burn (Hazra 2016). Han et al. (2017) and Azizian and Sepaskhah (2014) reported that continuous excessive fertilization results in an increase in soil acidity and salinity, and a decrease in the diversity of nitrifying microbial communities. Overusing fertilizer can result in crop susceptibility to disease attacks (Dusserre et al. 2017; Hong-xing et al. 2017). Overusing nitrogen fertilizer decreases stalk lodging resistance (Zhang et al. 2016) and high vegetative growth in rice (Leghari et al. 2016).

The relationship between grain yield and plant density is usually described by a quadratic model that constitutes four major regions including relatively rapid growth, slow growth, zero growth, and declining growth (Assefa et al. 2016). Grain yield commonly reaches a maximum at proper plant densities, however, it declines at higher densities due to a decrease in the growth of the lateral branch and pod formation on lower branches (French 2016). As reported by Ren et al. (2017), with an increase in plant density of maize from 30,000 plants ha^{-1} to 135,000 plants ha^{-1}, chlorophyll a and b, net photosynthetic rate, amount of chloroplasts content, the number of grana and yield per plant decreased. Lower-relative growth rate, -relative elongation rate, fewer branch numbers, and smaller shoot diameter resulted from the high plant density (Li et al. 2015). High plant density in maize causes interplant competition to reach more light, nutrients, and water. This may decrease the final yield due to a decrease in the number of ears per plant and kernels per ear (Sangoi 2001). Therefore, optimum plant density that maximizes yield is dependent on the cropping system, environment, cultivar and, planting date (Dong et al. 2010).

Crop Yield Under Loss/No Loss Production Parameters

In the field, whole applied production parameters cannot be used by the plant to produce marketable yield. As shown in Fig. 2, the amount of yield under conditions of loss occurrences in the applied production parameters is different from those values under no loss conditions. In the case of irrigation water, losses are deep percolation, runoff, and whole non-transpiration water like evaporation. In the case of fertilization, losses are nutrient leaching and conversion of nutrients into gas forms and entering the atmosphere and the whole amount of fertilizer which is not taken up by the plant and goes out of plant reach, and in the case of seeding, losses included not germinated seeds. In other words, here, the loss is the amount of production parameters that does not enter the plant growth process and does not affect the yield production. It should be noted that the shape of the yield production curve under no loss conditions is linear or nonlinear for some production parameters, e.g., for irrigation water, the relationship between net applied water (equal to transpiration) and yield is linear while for fertilization, this relationship is nonlinear. In fact, for the nonlinear relationship under no loss conditions, the amount of applied production parameters can be divided into two parts: First, the part of applied production parameters that results in an increase in yield, and second, the part that results in a decrease in yield. For more explanation, the applied nitrogen fertilizer can be divided into three parts, first, one part of the applied nitrogen fertilizer cannot be taken up by the plant. This part is named losses. Second, the part of nitrogen fertilizer that is taken up by the plant increases the yield. and third, the part of nitrogen fertilizer that is taken up by the plant decreases the yield due to the negative effect on plant growth. In reality, losses are inevitable and crop production occurs along with losses. Therefore, losses should be considered in economic analyses.

Losses amount, as shown in Fig. 2, is different at low and high levels of the applied production parameters. The higher the level of applied production parameters, the higher losses occur. A high level of irrigation water increases deep percolation, runoff, and evaporation in cases of short irrigation intervals (English 1990). Fertilizer leaching increases with increasing the applied fertilizer due to the fact that plants cannot take up the whole fertilizer. Increasing losses means low efficiency of the farming system, increase in cost and, decrease in benefit.

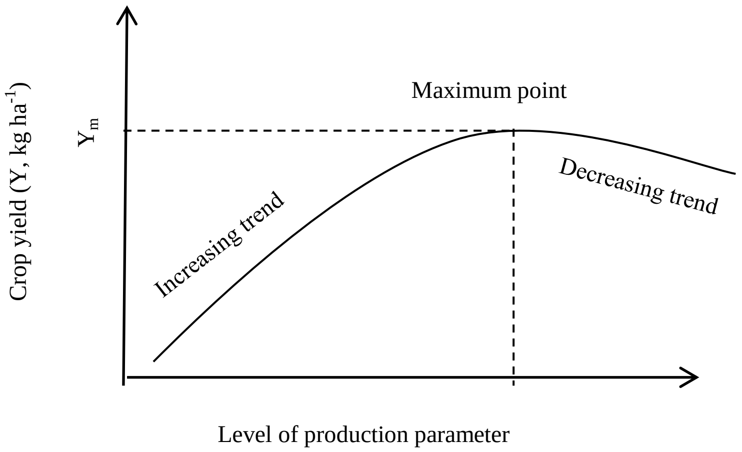

Fig. 1. Crop yield response to increase in level of production parameters

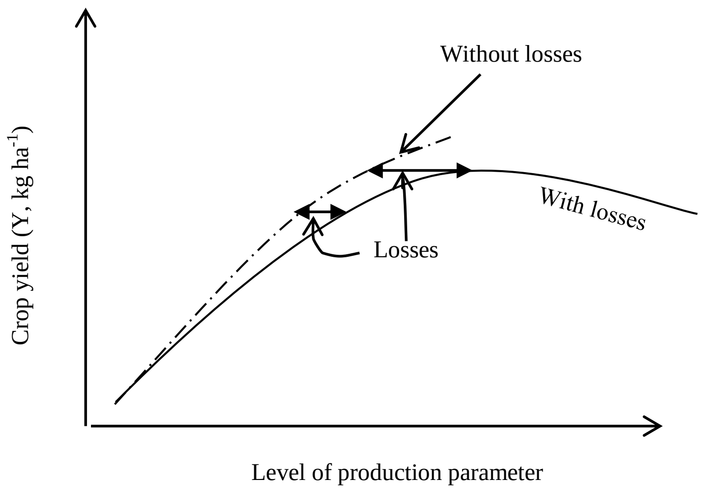

Fig. 2. Crop production curve under loss/no loss production parameters

Production Function

Single Variable

There are numerous studies that show a quadratic relationship between crop yield and production parameters. For example, it has been reported that there are quadratic relationships between nitrogen application rate and forage mass of tropical grasses (Johnson et al. 2001), between corn grain yield and nitrogen (N) application rate (Schlegel and Havlin 2017), between phosphorus fertilizer rate and forage yield (Berg et al. 2005), between soybean seed yield, seed protein and seed mineral contents with plant density (Rahman et al. 2011), between rice panicles and grain yield and rice seeding rates (Chauhan et al. 2011) and especially between marketable yield and applied irrigation water (wheat grain) (Kang et al. 2002), cabbage, spinach, rape, carrot, tomato and, onion (Imtiyaz et al. 2000), cotton for drip, furrow, and sprinkler irrigation methods (Cetin and Bilgel 2002), sugarbeet (Tarkalson et al. 2018), and etc.). Therefore, a binomial equation (Eq. 1) that has a maximum point is used for describing the mathematical relationship between crop yield and production parameters as follows (Fig. 1):

$$Y(x) = ax^2 + bx + c \qquad \text{Eq. (1)}$$

where Y is the marketable yield (kg ha^{-1}) as the dependent variable, x is the independent variable including applied irrigation water, fertilizer rate, plant density, seeding rate, and other production parameters (applied amount per hectare), and a, b and c are the constant coefficients.

It should be noted that due to single variable production functions being empirical, it can be used for analysis in cases where one production parameter is growth limited factor and all climatic and production parameters affecting crop yield should be mentioned.

Two or Multiple Variables

It is clear that crop yield is not a function of a single parameter. Several parameters can affect crop yield. The effect of each parameter on crop yield is not independent of the effects of other parameters. In other words, one parameter can decrease or increase the effects of other parameters on crop yield and they may interact with each other. Schlegel & Havlin (2017) based on 19-year field experimental data demonstrated that the relationship between maize grain yield and nitrogen rate is related to phosphorus application rate under full irrigation conditions (Fig. 3). So that the N rates required for maximum grain yield at 20 and 40 kg P ha^{-1} were 187 and 195 kg N ha^{-1}, respectively. In other words, crop yield variation depends on both fertilizer rates.

Therefore, as shown by Zand-Parsa and Sepaskhah (2001), crop yield can function as two or more variables as follows (Eq. 2) (Fig. 4):

$$Y(x, y) = ax^2y^2 + bx^2y + cxy^2 + dx^2 + ex + fy^2 + gy + hxy + i \qquad \text{Eq. (2)}$$

where Y is the crop yield, x, and y are the production parameters, and a, b, c, d, e, f, g, h and i are the constant coefficients. Some of the constant coefficients in Eq. (2) may not be significant, therefore, their values should be considered zero.

Concept of Economic Model

As many researchers concluded studies about deficit irrigation (English 1990; Sepaskhah et al. 2006; Banda et al. 2019), there is a curvilinear relationship between gross revenue and the level of production parameters (Fig. 5).

There are three important levels of applied production parameters. First: the level of the production parameter that maximizes yield and gross revenue (is shown as X_m in Fig. 5). The marginal yield is zero at this level of the applied production parameter. Profit (net revenue = gross revenue – cost, the distance between gross revenue and cost lines in Fig. 5) is not maximum at that point. Second: when the land is limiting (shown as X_l in Fig. 5), the goal would be the use of the amount of production parameters that maximize the net revenue per unit of land. The value of X_l is lower than X_m. At the X_l, the marginal cost is equal to the marginal gross revenue. For values of higher and lower than X_l, the profit per unit of land decreases. Under the land limited condition, farmers put all available land under cultivation and will be able to save production parameters by choosing Xl instead of Xm. If the land is not limiting, by selecting the level of production parameter lower than X_m, the saved production parameters at that level compared to X_m can be applied to cultivate the extra land area. Therefore, the total gross revenue obtained from the sum of original and extra cultivated lands will be increased. Third: the land is not limiting, but the amount of the applied production parameters are limiting. The amount of applied production parameters can be limited due to limitation of resources (e.g., water shortage) or limitation and the high price of agricultural inputs (due to high demand and low supply) and governmental rules for restricting the amounts of fertilizers applied by each farmer due to environmental impacts such as soil and water pollution. Therefore, when the production parameter is limited, the profit per unit of land is equal to the profit from full irrigation at one point which is shown as X_x in Fig. 5. Total gross revenue is maximum at point X_x (Sepaskah et al. 2006). For lower than X_x, total gross revenue is decreased due to decreasing the gross revenue from the original land area. However, the extra land area is higher than original land area when the amount of production parameter is higher than X_x. Under production parameter limiting conditions, the area of cultivated land is calculated as X_m/X_x, and the area of cultivated land increases as (X_m/X_x -1). The profit per unit production parameter is maximum under this condition.

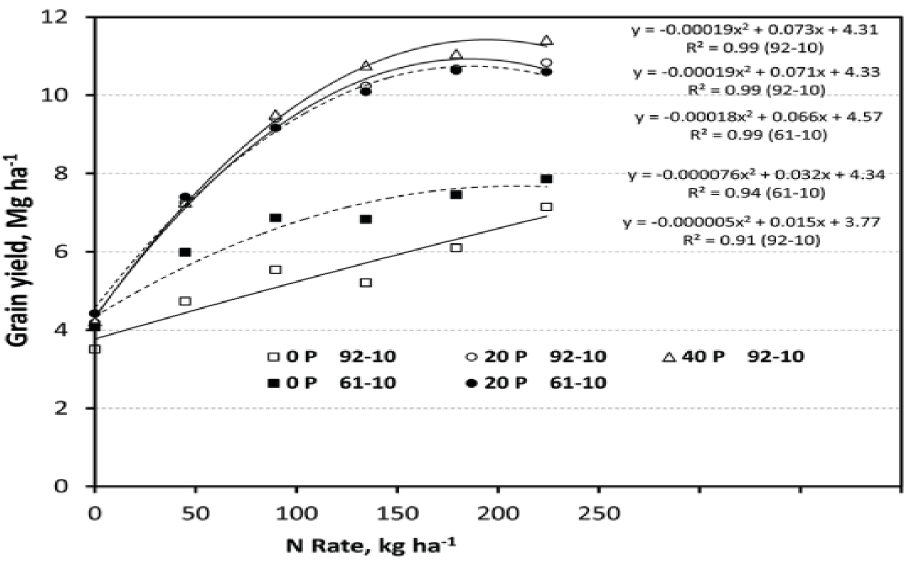

Fig. 3. Maize grain yield response to nitrogen (N) and phosphorus (P) rate for full irrigation from 1992-2010 and 1961-2010 (Retrieved from Schlegel & Havlin. 2017)

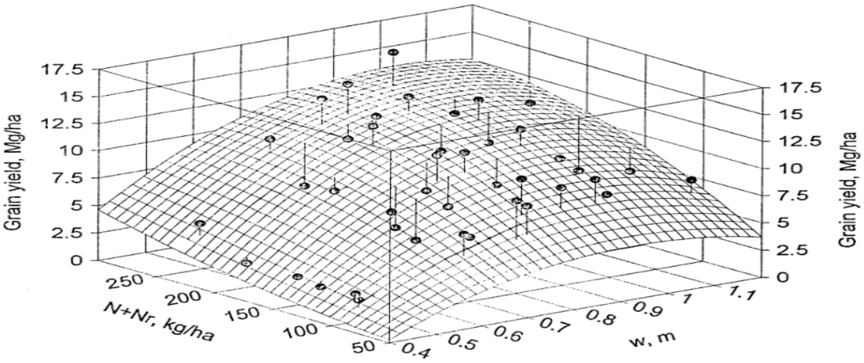

Fig. 4. Relationships between maize grain yield and the sum of applied water (w) and applied and residual nitrogen ($N+N_r$), (Retrieved from Zand-Parsa and Sepaskhah 2001)

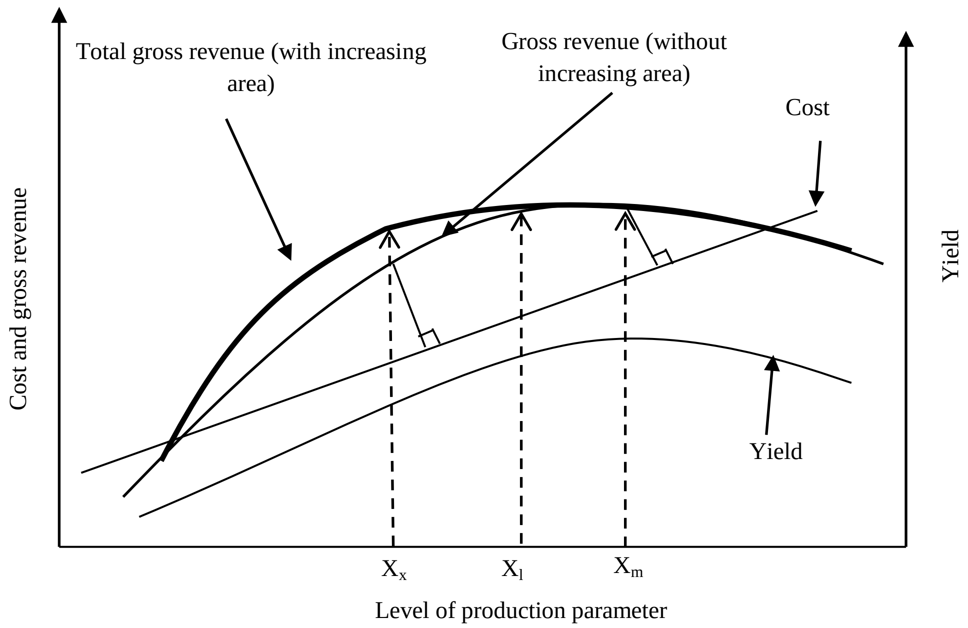

Fig. 5. Revenue, cost and yield functions

The Mathematical Formulation of Deficit Production Parameters

Farm net income is determined as follows:

$$I_f(\mathrm{X}) = A(X).i_l(X) \quad \text{Eq. (3)}$$

$$i_l(\mathrm{X}) = [P_c Y(X) - C(X)] \quad \text{Eq. (4)}$$

where A (X) is the cultivated area (ha), Y(X) is the crop yield (kg ha^{-1}) that is expressed as a function of production parameters, C(X) is the production costs per unit area (money ha^{-1}) that is expressed as a function of production parameters, Pc is the crop price in money kg^{-1}, i_l(X) is the net income per unit area (money ha^{-1}), and I_f(X) is the total net income (money) from all cultivated area. The goal is to maximize the total net income. There are many states for Eq. (4). Four situations based on a review of the literature are considered including 1) crop price can be constant or variable, 2) production function may be a single variable or two or multiple variables, 3) land area and production parameters can be limited or not limited, 4) the agricultural inputs price may be varied in some cases. For example, water price is variable for different levels of irrigation water salinities.

Further details about the mentioned states will be explained in the following sections. Since the major economic-mathematical analysis by investigators is focused on deficit irrigation, the economic-mathematical analyses of deficit irrigation based on the abovementioned four states are presented in this study. It is obvious that these analyses can be used for other manageable production parameters, as the production functions have non-linear behaviors related to their variations.

Single Variable Production Functions with Constant Crop Price

When only one production parameter limits crop growth and yield (i.e., the yield is as a function of one production parameter), the goal is determining the amount of that parameter which results in maximum net income. For example, based on the analysis presented by English (1990), when the yield is a function of applied water (W), the optimum applied water per unit area (m^3 ha^{-1}) that maximize net income can be determined by setting the derivative of Eq. (3) considering that W becomes zero as follows:

$$\frac{\partial I_f(W)}{\partial W} = A(W).\frac{\partial i_l(W)}{\partial W} + i_l(W)\frac{\partial A(W)}{\partial W} = 0 \quad \text{Eq. (5)}$$

where A(w) is the irrigated area (ha), W is the applied water (m^3 ha^{-1}), i_l(W) is the net income per unit area (money ha^{-1}), and I_f(W) is the total net income from all irrigated area (money ha^{-1}).

As mentioned above, in the case of limiting land, A(W) is a constant value and excess water cannot be applied to cultivate the extra land area. Therefore, ∂A(W)/∂W in Eq. (5) is zero. Therefore, the optimum irrigation water can be determined by setting the derivative of Eq. (4) considering that W becomes zero as follows:

$$\frac{\partial i_l(W)}{\partial W} = \frac{\partial [P_c Y(W) - C(W)]}{\partial W} = 0 \qquad \text{Eq. (6)}$$

Therefore, under land limited conditions, the optimum irrigation water amount can be determined by solving:

$$P_c \frac{\partial Y(W)}{\partial W} - \frac{\partial C(W)}{\partial W} = 0 \qquad (7)$$

When the water is limited but the land area is not limited, the land area is a function of water as follows:

$$A(W) = \frac{W_T}{W}. \qquad (8)$$

where W_T is the total available water supply (m^3). The derivative of A(W) is as follows:

$$\frac{\partial A}{\partial W} = -\frac{W_T}{W^2} \qquad (9)$$

By substituting Eqs. (8) and (9) in Eq. (5) and simplifying, the following equation is achieved:

$$\frac{W_T}{W}\left[P_c \frac{\partial Y(W)}{\partial W} - \frac{\partial C(W)}{\partial W}\right] + [P_c Y(W) - C(W)]\left[-\frac{W_T}{W^2}\right] = 0 \qquad (10)$$

By solving Eq. (10) for W, the optimal values of irrigation water can be obtained under water-limited conditions. The resulting equations are presented by Banda et al. (2019), Amiri et al. (2016), Zhang and Oweis (1999), and English and Raja (1996).

Based on an economic analysis of deficit irrigation for wheat in Oregon, cotton in California, and maize in Zimbabwe by English and Raja (1996), deficit irrigation of 16 and 39% for wheat, 15 and 28% for cotton, and 15 and 59% for maize resulted in a profit increase of 8.3 and 49, 13.2 and 44.1, 3.8 and 68% under land- and water-limiting conditions compared to full irrigation condition (W_m), respectively.

Two-Variable Production Functions with Constant Crop Price

Eleven years after the analysis presented by English (1990), Zand-Parsa and Sepaskhah (2001) applied the above-mentioned analysis to a two-variable production function. In their study, optimum values of irrigation water and N for maize were determined.

When the crop yield and cost functions are as a function of applied irrigation water and N, the Eqs. (1) and (2) changes as follows:

$$I_f(W, N+Nr) = A(W).\, i_l(W, N+Nr) \qquad \text{Eq. (11)}$$

$$i_l(W, N+Nr) = [P_c * Y(W, N+Nr) - C(W, N)] \qquad \text{Eq. (12)}$$

where N is the nitrogen (N) application rate ($kgha^{-1}$), and Nr is the soil residual N ($kgha^{-1}$).

In the analysis presented by Zand-Parsa and Sepaskhah (2001), it was assumed that A can only be depended on applied water under water-limited conditions. However, due to environmental considerations and governmental laws, it is possible that A be dependent on N too. Land area owned by the farmers is the common criterion of the fertilizer allocation to each farmer. If the allocated fertilizer is lower than the optimum value of fertilizer, the farmers will be forced to use the allocated amount. In case when the allocated fertilizer is higher than the optimum value of fertilizer, the farmers choose the optimum values to apply and they can sell extra fertilizer or use it for cultivating the plant that has the optimum value of fertilizer higher than the allocated fertilizer. Here, similar to the analysis presented by Zand-Parsa and Sepaskhah (2001), the irrigated area is determined by Eq. (8) and is independent of the fertilizer rate. When the land is limited, the net income per unit area should be maximized for specific irrigation water and nitrogen application. Therefore, to determine the optimum value of water and nitrogen, the partial derivative of I_f(W, N+Nr) in Eq. (11) with respect to W and N should be set to zero as follows:

$$\frac{\partial I_f(W, N+Nr)}{\partial W} = A.\frac{\partial i_l(W, N+Nr)}{\partial W} + i_l(W, N+Nr)\frac{\partial A}{\partial W} = 0 \qquad \text{Eq. (13a)}$$

$$\frac{\partial I_f(W, N+Nr)}{\partial N} = A.\frac{\partial i_l(W, N+Nr)}{\partial n} + i_l(W, N+Nr)\frac{\partial A}{\partial N} = 0 \qquad \text{Eq. (13b)}$$

As mentioned above, under the limited-land conditions, ∂A(W)/∂W and ∂A(W)/∂N in Eqs. (13a) and (13b) are zero. After substituting Eq. (12) and its derivative form in Eqs. (13a) and (13b) and simplifying, the optimum levels of W and N under land limiting conditions can be obtained as the following form:

$$P_c * \frac{\partial Y(W, N+Nr)}{\partial W} - \frac{\partial C(W, N)}{\partial W} = 0 \qquad \text{Eq. (14a)}$$

$$P_c * \frac{\partial Y(W, N+Nr)}{\partial N} - \frac{\partial C(W, N)}{\partial N} = 0 \qquad \text{Eq. (14b)}$$

When the W and N are limiting and not limiting, respectively, ∂A(W)/∂W equals Eq. (9), and ∂A(W)/∂N is zero in Eqs. (13a) and (13b). After substituting Eq. (12) and its derivative form in Eqs. (13a) and (13b) and simplifying, the optimum W and N under water limiting conditions can be obtained as the following form:

$$W_w\left[P_c * \frac{\partial Y(W, N+Nr)}{\partial W} - \frac{\partial C(W, N)}{\partial W}\right] = [P_c * Y(W, N+Nr) - C(W, N)] \qquad \text{Eq. (15a)}$$

$$p_c * \frac{\partial Y(W, N+Nr)}{\partial N} - \frac{\partial c(W, N)}{\partial N} = 0 \qquad \text{Eq. (15b)}$$

A nonlinear system of equations should be applied to solving Eqs. (14a), (14b), (15a,) and (15b) for determining W and N under land and water-limited conditions.

For maize farming in Bajgah Agricultural Experiment Station at Shiraz University in Iran, Zand-Parsa and Sepaskhah (2001) reported that optimum irrigation water depth and applied nitrogen were 0.99 m and 212 kg Nha^{-1} and 0.735 m and 206 kg Nha^{-1} under land- and water-limiting conditions, respectively. Applying the above amount of irrigation water and nitrogen fertilizer under water-limiting conditions caused a net income increase of 18% compared to land-limiting conditions. They concluded that the values of applied water and nitrogen fertilizer under land limiting are equal to their maximum values when the price of water and nitrogen is low.

The above analysis was used by Sepaskhah et al. (2008) for determining optimum levels of irrigation water and saffron corm planting intensity by replacing saffron corm planting intensity with $N+N_r$ in the aforementioned equations.

Single Variable Production Functions with Variable Crop Price

In the two aforementioned cases, analyses were done based on constant crop prices. In some crops (e.g., sugar beet and sugarcane), the sale price of crops is not constant in Iran, Germany, Poland, and some other countries (Shabani and Sepaskhah 2019; Artyszak et al. 2019; Loel et al. 2014; Hoffmann 2010). Crop price variability in these cases is different from the crop price variation caused by the supply and demand system or government pricing. In this pricing system, crop price depends on the produced sugar concentration. For instance, some countries such as Iran and Germany, considers a base crop price for the unit weight of sugar beetroot with a sugar concentration of 16% (P_{c16}). Then the sugar concentration of sugar beet is determined by the sugar refining factory. Afterward, the sugar beet price is determined based on sugar concentration. Sugar beet price increases by increasing the sugar concentration (Shabani 2019; Shabani et al. 2018). Therefore, any parameters (e.g., irrigation and fertilizer application) that affect the sugar concentration cause increasing or decreasing the crop price.

It has been reported that some factors increase the concentration of sugar in sugar beet. For example, water stress is leading to sucrose accumulation and increases in sugar concentration in sugar beet (Mahmoud et al. 2018). Also, applying calcium promotes the sugar concentration of sugar beet under drought-stress conditions (Hosseini et al. 2019). Similarly, osmotic stress increases the sucrose content in sugar beetroot (Wu et al., 2016). It has also been reported that at a low range of N application, the sugar concentration is increased with increasing N, but at a high range of N application rate, the sugar beet shows reverse behavior (Sadeghi-Shoae et al. 2015 and Chatterjee et al. 2018).

According to the analysis presented by Shabani (2019), in cases when the crop price, yield, and cost functions are a function of applied water, Eq. (4) should be modified as follows:

$$i_l(\mathrm{W}) = [P_c(W)Y(W) - C(W)] \qquad \text{Eq. (16)}$$

Therefore, for land limited conditions, the optimum irrigation water amount can be determined by solving

$$\frac{\partial i_l(W)}{\partial W} = P_c(W)\frac{\partial Y(W)}{\partial W} + \frac{\partial P_c(W)}{\partial W}\mathrm{Y(W)} - \frac{\partial C(W)}{\partial W} = 0 \qquad (17)$$

For water limiting conditions, by substituting Eqs. (8), (16), and (17) in Eq. (5) the following equation is obtained:

$$\frac{W_T}{W}\left[P_c(W)\frac{\partial Y(W)}{\partial W} + \frac{\partial P_c(W)}{\partial W}\mathrm{Y(W)} - \frac{\partial C(W)}{\partial W}\right] + [P_c(W)Y(W) - C(W)]\left[-\frac{W_T}{W^2}\right] = 0 \qquad \text{Eq. 18)}$$

By simplifying Eq. (18), Eq. 19 is obtained:

$$W\left[P_c(W)\frac{\partial Y(W)}{\partial W} + \frac{\partial P_c(W)}{\partial W}\mathrm{Y(W)} - \frac{\partial C(W)}{\partial W}\right] = [P_c(W)Y(W) - C(W)] \qquad \text{Eq. (19)}$$

Optimal applied water can be calculated by solving Eq. (19) for W when water and crop price are limited and variable, respectively. The resulting equations are presented by Shabani et al. (2018). In their study, sugar beet price was a function of the sum of irrigation water and seasonal rainfall, and optimum applied water was determined under land and water-limiting conditions. Obtained results by these investigators showed that applying optimum water under land- (98.5 cm water depth) and water-limited conditions (56.6 cm water depth) caused a 1.2 and 12% increase in net income per unit area and per unit water compared to maximum yield conditions, respectively.

Two-Variable Production Functions with Variable Crop Price

When the crop yield, cost, and crop price are as a function of applied water and N, the Eqs. (3) and (4) should be modified as follows (Shabani and Sepaskhah 2019):

$$I_f(W, N+Nr) = A.\, i_l(W, N+Nr) \qquad \text{Eq. (20)}$$

$$i_l(W, N+Nr) = [P_c(W, N+Nr) * Y(W, N+Nr) - C(W, N)] \qquad \text{Eq. (21)}$$

When the land is limiting, the net income per unit area should be maximized. Therefore, in case the crop price is variable, the Eqs. (14a) and (14b) should be modified as follows:

$$P_c(W, N+Nr) * \frac{\partial Y(W,N+Nr)}{\partial W} + \frac{\partial P_c(W,N+Nr)}{\partial W} * \mathrm{Y}(W, N+Nr) - \frac{\partial C(W,N)}{\partial W} = 0 \qquad \text{Eq. (22a)}$$

$$P_c(W, N+Nr) * \frac{\partial Y(W,N+Nr)}{\partial N} + \frac{\partial P_c(W,N+Nr)}{\partial N} * \mathrm{Y}(W, N+Nr) - \frac{\partial C(W,N)}{\partial N} = 0 \qquad \text{Eq. (22b)}$$

Therefore, in the conditions of limited-land and variable crop prices, optimum irrigation water and nitrogen amounts can be determined by solving the nonlinear system of Eqs. (22a), (22b).

It is assumed that the irrigated area is a function of applied water and it is not dependent on the N application. Therefore, the value of A can be determined by Eq. (8). When the water is limiting and crop price is variable, the net income per unit applied water should be maximized. Therefore, in these cases Eqs. (15a) and (15b) should be modified as follows:

$$W\left[P_c(W, N+Nr) * \frac{\partial Y(W,N+Nr)}{\partial W} + \frac{\partial P_c(W,N+Nr)}{\partial W} * \mathrm{Y}(W, N+Nr) - \frac{\partial C(W,N)}{\partial W}\right] = [P_c(W, N+Nr) * Y(W, N+Nr) - C(W, N)] \qquad \text{Eq. (23a)}$$

$$P_c(W, N+Nr) * \frac{\partial Y(W,N+Nr)}{\partial N} + \frac{\partial P_c(W,N+Nr)}{\partial N} * \mathrm{Y}(W, N+Nr) - \frac{\partial C(W,N)}{\partial N} = 0 \qquad \text{Eq. (23b)}$$

By ssolving the nonlinear system of Eqs. (23a) and (23b) for W and N, the optimum amounts of W and N under limited water and variable crop price conditions are determined. The resulting equations are presented by Shabani and Sepaskhah (2018). In their study, sugar beet price was dependent on the sum of irrigation water and seasonal rainfall, and nitrogen fertilizer and optimum applied water and nitrogen fertilizer were determined under land and water limiting conditions. Optimum water, nitrogen, and total net income at different rainfall depths under conditions of maximum yield, limited-land and water limiting obtained by Shabani and Sepaskhah (2018) are shown in Fig. 6. An increase in rainfall occurrence resulted in a decrease in applied water (lower distance from the center) and an increase in the amount of total net income (more distance from the center). The differences between total net income under water limiting and maximum yield conditions were higher than those values between land limiting and maximum yield conditions (more distance between green and red lines with respect to the blue line, Fig. 6). The optimum applied water was lower and nitrogen fertilizer was higher under water-limiting conditions compared to land-limiting and maximum yield conditions. Total net income was higher under water-limiting conditions compared to land-limiting and maximum yield conditions.

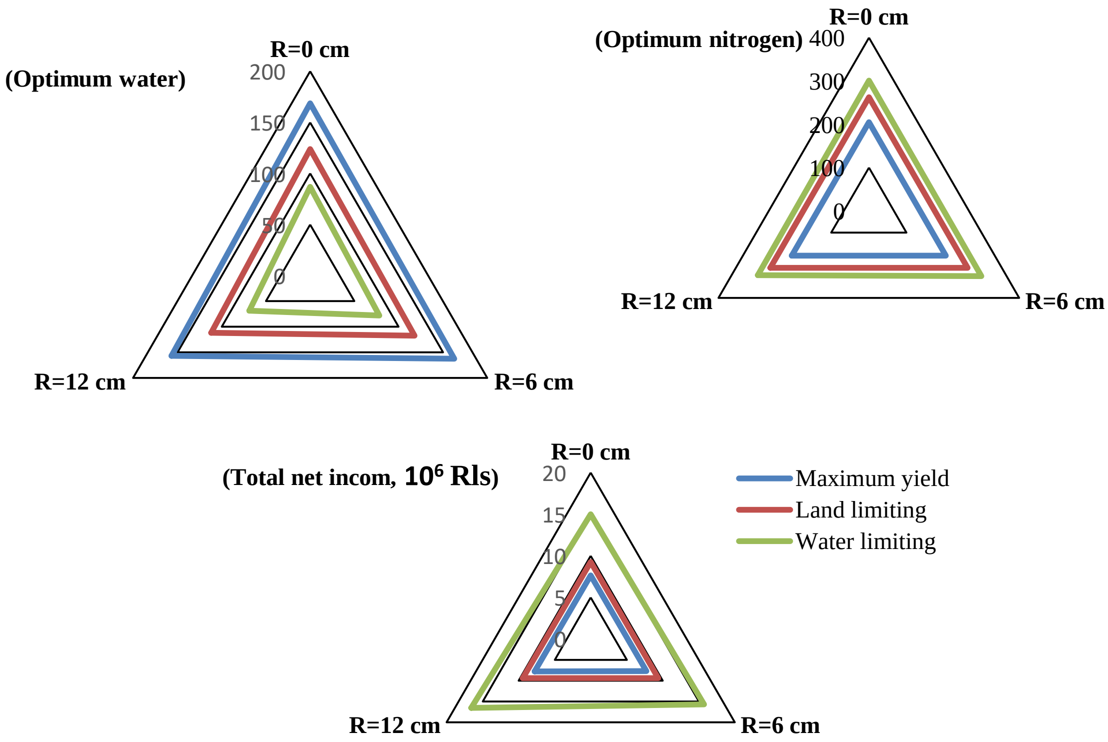

Fig. 6. Optimum water (cm), Optimum nitrogen (kgha^{-1}), and total net income (Iranian Rials (Rls)) at different rainfall depths (R) under conditions of maximum yield, land limiting, and water limiting

Two-variable Production Function with Variable Crop and Water Price

Due to water resource shortage, saline water usage is unavoidable. The cost function includes two parts. First, constant costs involving land rent and preparation, seeding, weeding, pesticides and herbicides, harvest, and variable costs including costs of water, fertilizer, and other manageable production parameters that increase (or decrease) as use increases (or decreases). Irrigation water price is not constant when water is saline and depends on the amount of salinity, due to the fact that water price decreases with increasing water salinity. Shamshiri et al. (2019) determined the optimal water consumption of sugar beet in conditions of the irrigation water salinity and the dependency of the yield price on its quality. In their study, sugar beet production function was obtained based on the amount and salinity of irrigation water. Therefore, similar to the earlier analysis, the production function is a two-variable function, and crop and water prices are variables. Hence, Eqs. (20) and (21) should be modified as follows:

$$I_f(W, EC_{iw}) = A.\, i_l(W, EC_{iw}) \quad \text{Eq. (24)}$$

$$i_l(W, EC_{iw}) = [P_c(W, EC_{iw}) * Y(W, EC_{iw}) - C(W, EC_{iw})] \quad \text{Eq. (25)}$$

where EC_{iw} is the irrigation water salinity and W is the applied water. There is no data available about the variation of irrigation water prices based on water salinity levels. Therefore, Shamshiri et al. (2019) determined the cost function as follows:

$$C(W, EC_{iw}) = C_0 + W[P_{W0}(1 - b(EC_{iw} - EC_{iwth})] \quad \text{Eq. (26)}$$

where C_0 is the constant cost, P_{wo} is the price of freshwater, EC_{iw} is the irrigation water salinity (dS m^{-1}), EC_{iwth} is the threshold irrigation water salinity for yield reduction of the crop (dS m^{-1}) and b is the yield reduction per unit irrigation water salinity. They considered two assumptions: first, for the salinity of water lower than the threshold, the water price did not reduce and it was equal to non-saline water. Second, for water salinity higher than the threshold, water price reduction per unit of water salinity was equal to yield reduction per unit of irrigation water salinity. Although these assumptions should be revised based on real data. Eq. (26) can be suggested for pricing the saline water. Anyhow, under irrigation saline water, either Eq. (26) or another equation is used to determine the price of saline water, the water price is variable. Therefore, in Eqs. (22a), (22b), (23a) and (23b), N+Nr must be replaced with EC_{iw} as follows:

$$P_c(W, EC_{iw}) * \frac{\partial Y(W, EC_{iw})}{\partial W} + \frac{\partial P_c(W, EC_{iw})}{\partial W} * Y(W, EC_{iw}) - \frac{\partial C(W, EC_{iw})}{\partial W} = 0 \quad \text{Eq. (27a)}$$

$$P_c(W, EC_{iw}) * \frac{\partial Y(W, EC_{iw})}{\partial N} + \frac{\partial P_c(W, EC_{iw})}{\partial N} * Y(W, EC_{iw}) - \frac{\partial C(W, EC_{iw})}{\partial N} = 0 \quad \text{Eq. (27b)}$$

$W\left[P_c(W,EC_{iw}) * \frac{\partial Y(W,EC_{iw})}{\partial W} + \frac{\partial P_c(W,EC_{iw})}{\partial W} * \mathrm{Y}(W,EC_{iw}) - \frac{\partial C(W,EC_{iw})}{\partial W}\right] = [P_c(W,EC_{iw}) * Y(W,EC_{iw}) - C(W,EC_{iw})]$ Eq. (28a)

$P_c(W,EC_{iw}) * \frac{\partial Y(W,EC_{iw})}{\partial N} + \frac{\partial P_c(W,EC_{iw})}{\partial N} * \mathrm{Y}(W,EC_{iw}) - \frac{\partial C(W,EC_{iw})}{\partial N} = 0$ Eq. (28b)

Therefore, optimum irrigation water at specific water salinity can be determined by solving the nonlinear system of Eqs. (27a) and (27b) for land limited conditions and Eqs. (28a) and (28b) for water-limited conditions.

Based on the study by Shamshiri et al. (2019), for the salinity of 0 dS m^{-1} and based on current prices of water and sugar beet crop, the optimum amounts of water were 1.87, 1.77, and 1.52 m to obtain the maximum yield, maximum profit under limited-land and maximum profit under limited-water conditions, respectively. The amount of water saving under water-limiting conditions was 18.7 %, and the cultivation area increased by 20% compared to maximum yield conditions.

Considering Rainfall in Calculations of Optimum Amount of Water

There are two strategies for considering rainfall in the calculation of the optimum amount of water. First, the predicting the amount of seasonal rainfall before the start of the growing season. Second, determining optimum applied water based on occurrence probability analysis for a given rainfall (Sepaskhah et al. 2008). In the second strategy, farmers accept some degree of risk in exchange for potential economic gain (English 1981). Sepaskhah and Akbari (2005) considered the rainfall in deficit irrigation analysis and determined the optimum amount of irrigation water. For considering rainfall in analyses, yield functions [Eqs. (1) and (2)] are modified as follows:

$Y(W+R) = a'(W+R)^2 + b'(W+R) + C'$ Eq. (29)

$Y(W+R,y) = a'(W+R)^2y^2 + b'(W+R)^2y + c'(W+R)y^2 + d'(W+R)^2 + e'(W+R) + f'y^2 + g'y + h'(W+R)y + i'$ Eq. (30)

Where, a´, b´ and, c´ are the constants of the production function when the independent variable is the sum of seasonal applied water (W) and rainfall (R). Also, rainfall can only be considered for determining planting area as follows (Sepaskhah et al. 2006; Sepaskhah et al. 2008):

$A(W) = \frac{W_T+R}{W}$ Eq. (31)

However, applying the rainfall in the calculation of planting area can increase the degree of risk for farmers due to increasing planting area along with increasing the other costs related to planting and losses resulting from no occurrence of predicted rainfall. Therefore, in some studies, the planting area is assumed to be independent of rainfall (Shabani et al. 2018; Shabani and Sepaskhah 2019).

As stated by Sepaskhah and Akbari (2005), Sepaskhah et al. (2006), Sepaskhah et al. (2008), and Shabani et al. (2018), by increasing in rainfall, the value of optimum applied water under land- and water-limited conditions decreased linearly and nonlinearly, respectively (Fig. 7). Based on mathematical analysis, the nonlinearity of the relationship between rainfall and the value of optimum applied water under water-limited conditions resulted from using W, as a variable for the irrigated area in Eq. (8) that is behind the brackets in Eqs. (10), (15a), (19), (23a) and (28a). Therefore, choosing the water-limited conditions strategy results in applying a higher ratio of rainfall to applied water that maximizes total net return.

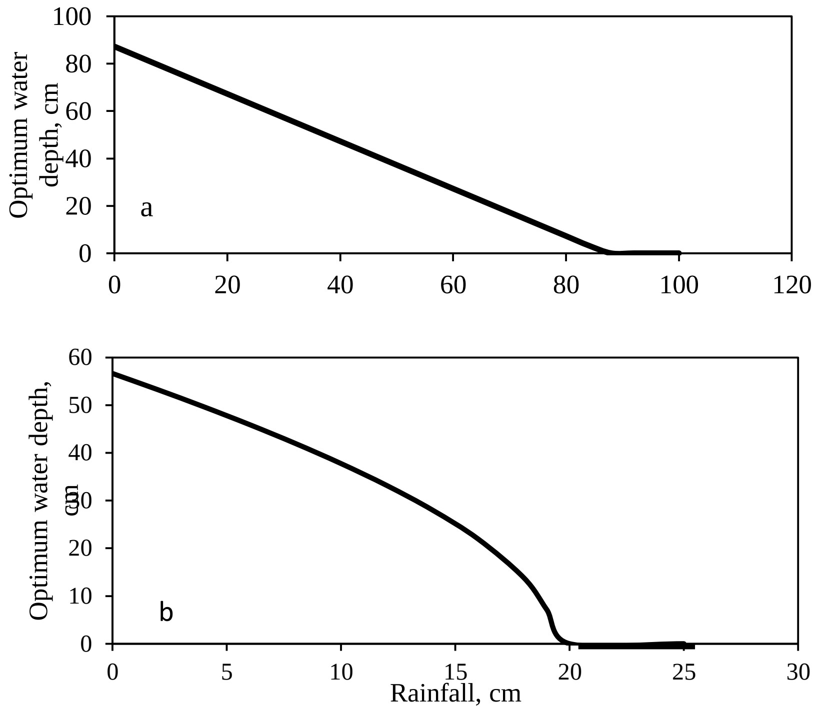

Fig. 7. Relationship between the optimum water depth of sugar beet under limiting land (a) and limiting water (b) conditions with rainfall depth (Retrieved from Shabani et al. 2018).

Sensitivity Analysis

Based on reported results by Shabani and Sepaskhah (2019), by increasing the rainfall, the applied water depth decreased under both water and land-limited conditions. But, the effect of rainfall on optimum applied water under water-limited conditions was higher than its effect on optimum applied water under land limited conditions so the relation between rainfall and optimum applied water was second power under water-limited conditions, while their relationship was linear under land limited conditions. The amount of rainfall only affects the level of optimum water level and not the level of nitrogen, seeding rate, and other agronomic parameters. It should be noted that this is true in the conditions that over-rainfall does not lead to nitrogen leaching or plant growth reduction.

CONCLUSIONS

Determining the optimum value of production parameters is very important due to the limitation of resources and agricultural inputs. A review of literature indicated that the optimum water depth decreases with an increase in water price. Increasing fertilizer cost causes a decrease and an increase in the optimum amount of fertilizer and applied water depth, respectively. The optimum level of water increases with an increase in crop price because maximum net income is achieved at higher applied water depth by increasing crop price. Also, maximum net income decreases by increasing fertilizer costs. This occurs at a lower fertilizer level. Soil residual nitrogen content does not affect the value of applied water depth and the level of applied nitrogen decreases by increasing the soil residual nitrogen content.

Presented concepts and analyses in this study can be extended to determine the optimum value of all manageable production parameters in agriculture and help to select the best strategies for increasing net benefit per area and unit of water and declining the loss of resources to prevent environmental pollution. There are two shortcomings in the presented analyses. First, yield and crop price functions, applied in the presented analyses, are basically empirical, therefore, those should be modified before being used in other climates and cultivars. However, this shortcoming is also true for other yield functions and with less important for crop models. Second, the analysis presents a single value as optimum for the whole growing season. For example, farmers or farm managers can determine one optimum amount of irrigation water for deficit irrigation under water- or land-limited conditions. However, there are some problems with what is the irrigation interval. When is the time of the irrigation event? Is it applied in sensitive or tolerant growth stages to water stress? For the optimum value of fertilizer, when the fertilizer is applied? Is it divided into parts during the growing season? Therefore, all of the conditions governing the yield and cost functions to be obtained should be stated and presented for similar cases. Simplicity, understandability, analyzability, and ease of assessment of price variations on optimum values of production parameters and performing sensitivity analysis are advantages of these analyses.

DISCLOSURE STATEMENT

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مروری بر تحلیل اقتصادی - ریاضی برای مدیریت آب آبیاری و سایر پارامترهای تولیدمحصول

علی شعبانی[۱*]، علیرضا سپاسخواه[۲]، محمد مهدی مقیمی[۱]، عبدالرسول زارعی[۳]

[۱] بخش علوم و مهندسی آب، دانشکده کشاورزی، دانشگاه فسا، فسا، ج. ا. ایران

[۲] بخش مهندسی آب و مرکز مطالعات خشکسالی، دانشکده کشاورزی، دانشگاه شیراز، شیراز، ج.ا. ایران

[۳] بخش مهندسی مرتع و آبخیزداری (مهندسی طبیعت)، دانشکده کشاورزی، دانشگاه فسا، فسا، ج. ا. ایران

*نویسنده مسئول

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قیمت محصول

محدودیت زمین

تابع تولید

محدودیت آب

چکیده رشد و عملکرد گیاه تحت تأثیر بسیاری از پارامترهای تولید مانند میزان آب آبیاری، کوددهی، تراکم بوته و غیره است. با توجه به محدودیت منابع و نهاده های کشاورزی، افزایش هزینه های تولید، تقاضای غذا، رشد جمعیت و مشکلات زیست محیطی، توسعه رویکردهای زمانبندی برای استفاده از پارامترهای تولید ضروری است. در این مطالعه ابتدا عملکرد محصول و پاسخ سود خالص به تغییرات پارامترهای تولید مورد بحث قرار می گیرد، دوم تحلیل اقتصادی-ریاضی پارامتر های تولید مانند آب آبیاری مورد بررسی قرار گرفت. تحلیل های ارائه شده در این مطالعه در چهار مورد شامل ۱) قیمت محصول، ثابت یا متغیر، ۲) تابع های تولید تک، دو یا چند متغیره، ۳) سطح زمین و آب محدود یا نامحدود و ۴) قیمت آب متغییر طبقه بندی شدند. تجزیه و تحلیل اقتصادی-ریاضی ارائه شده در مورد کم آبیاری در این مطالعه برای تعیین مقدار بهینه سایر پارامترهای تولید مانند میزان کود، تراکم بوته، تراکم بذر، شدت کاشت بنه و غیره گسترش یافت.