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# THREE-DIMENSIONAL ANALYSIS OF BEARING CAPACITY OF SQUARE FOOTINGS ON CONVEX SLOPES BY FINITE ELEMENT METHOD*

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**Abstract–** Soil slopes are sometimes curved in plan rather than being straight. This curvature, especially when it is convex, decreases the bearing capacity of the slope. Finite element method has been used here in this research to investigate the reducing effect of curvature on bearing capacity of a slope. Failure of soil has been assumed to be according to Mohr-Coulomb criteria; but the flow rule of the soil has been considered unassociated. Both cohesive and frictional soils have been used in these investigations to observe the effect of soil type on the problem. Investigations consist of four types of cohesive soils having different strengths and four types of frictional soils having different friction angles. The finite element model consisted of a square footing adjacent to a slope that is convex in plan. Besides changing the curvature; the slope angle and the footing to slope distance have also been varied to see their effects as well. Based on the results of these investigations it has been concluded that convex curvature generally reduces the bearing capacity and this effect is more pronounced in steeper slopes. While this reduction is limited to 7% in case of cohesive soils it can be as high as 15% in case of frictional soils. Therefore this effect should be considered in design of slopes that are convex in plan.

**Keywords–** Three dimensional analysis, stability, finite element method, curved slopes, convex

## 1. INTRODUCTION

Foundations are sometimes constructed on soil slopes. The bearing capacity of the soil under such foundations decreases due to the presence of such a slope adjacent to the foundation. This problem has drawn the attention of researchers in the field of geotechnical engineering for many years. When there is no distance between the footing and slope edge, the common practice is to introduce ground factors $g_i$'s in different terms of the bearing capacity equation as [1-3]:

$$q_{ult} = cN_c g_c + qN_q g_q + \tfrac{1}{2}\gamma B N_\gamma g_\gamma \tag{1}$$

If there is distance between the footing and slope edge, however, the approach is to modify the bearing capacity factors $N_i$'s, rather than using the ground factors in the equation [4]. The general bearing capacity equation is for the plane strain condition which resembles long footings along which the loading condition, geometry or material does not change. In practice, the footings are usually limited in size and this situation makes the problem inherently three-dimensional, so that the plane strain solution is no longer valid. Common practice here is to introduce some shape factors in the general equation in order to compensate for the three dimensional behavior. There is a debate here about how to account for shape effects when there is a slope nearby. Because of deviation of failure mechanism from a symmetric one, it is usually recommended to forget about those shape the bearing capacity that usually increase factors (i.e.,

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$s_c$ and $s_q$), but to consider the one that is a reduction factor i.e., $s_\gamma$. All of these are because the problem of footing of limited size adjacent to a slope is essentially three dimensional; and we still want to use simple equations developed for two dimensional problems and at the same time be on the safe side in our calculations. Analytical methods used for tackling these kinds of problems are few [5, 6]. This is because such problems are too complicated to be analyzed by simple analytical methods and this is where numerical methods should come into play. Numerical analysis of three dimensional bearing capacity of foundations on straight slopes has been performed in the past. These have been based on limit equilibrium method (LEM) [5], limit analysis method (LAM) [6-8], finite element method (FEM), discrete element method (DEM) [9, 10] and finite difference method (FDM) [11, 12].

The problem would be further complicated when the slope is not straight in plan. The resistance against bearing capacity failure toward the slope is further reduced when the slope is convex in plan. The curvature of the slope in plan causes the failure mechanism due to bearing capacity to open from the convex side down the slope, rather simpler. The first approach to study the influence of curvature of a slope on its bearing capacity was made by considering axially symmetric conditions. The numerical analysis was made using the rigorous method of stress characteristics [13]. The three-dimensional counterpart problem was numerically investigated later using the upper bound limit analysis and the three-dimensional zero extension line methods [14, 15]. In this paper; the problem of bearing capacity of a square footing on top of a convex slope has been studied using the finite element method. Analyses have been performed using the finite element code ABAQUS [16]. Both cohesive and frictional soils have been considered for this study. The curvature of the slope in plan has been varied to observe its influence on the bearing capacity. At the same time, the distance to slope edge, slope angle and soil strength have also been varied to see their influence on the problem.

## 2. THE FINITE ELEMENT MODEL

Investigations were made using the finite element code ABAQUS. Modeling includes a square footing of width $B$ at the surface of a concave slope as shown in Fig. 1. The radius of curvature of the slope in plan is $R$, and the slope angle is $\beta$. The distance between the footing and slope edge is $a$. The slope height $H$ is taken large enough to avoid any base interference. All dimensions of the problem are normalized to the footing width $B$. Therefore, the normalized radius of curvature and footing edge to slope crest distance would be represented by the ratios $R/B$ and $a/B$, respectively.

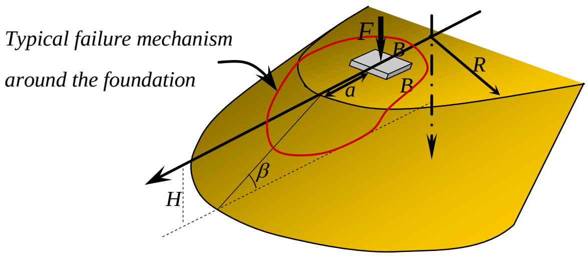


Fig. 1. Schematic diagram of a convex slope supporting a square foundation

### *a) Soil model*

The soil is modeled as an isotropic elastic-perfectly plastic material. Yielding of soil is assumed to be according to Mohr-Coulomb failure criteria. A smooth plastic potential function suggested by Menétrey and Willam (1995) available in ABAQUS was used to model the deformation behavior of soil [16]. This function appears as a hyperbola in meridian planes, i.e., planes passing through the opposite edges of Mohr-Coulomb yield surface (See Fig. 2a). It appears as a piecewise elliptic curve in the deviatoric planes passing all corners of Mohr-Coulomb yield locus (See Fig. 2b). In this way, the singularity in derivatives occurring in tension and compression paths at the meridian planes are avoided when Mohr-Coulomb yield condition is used together with normality flow rule. But the flow rule is non-associative because, the assumed plastic potential function does not match with the Mohr-Coulomb yield locus everywhere. Deviation of the plastic potential from the yield function in this model is characterized by eccentricity parameters $\varepsilon$ and $e$ in the meridian and deviatoric planes, respectively (See Figs. 2a and 2b). For further details refer to ABAQUS benchmark manual [16].

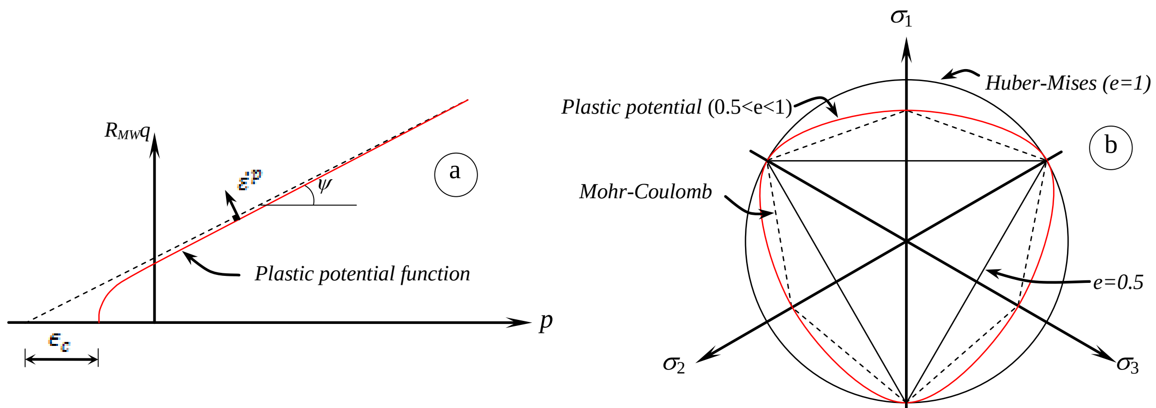


Fig. 2. Deviation of the plastic potential from the *M-C* yield function in a) meridian and b) deviatoric planes

***b) Domain size and finite element discretization***

The assumed problem has symmetry with respect to a vertical radial plane from the center of curvature that bisects the footing (See Figs. 1 and 3). Therefore, only half of the domain has been used for modeling and analysis, taking advantage of the symmetry (Fig. 3).

In distributing the elements, care has been taken to use smaller elements around the corners and edges of the footing. This is vital to capture the high displacement and stress gradients created around these regions. The size of elements has been increased with distance from the footing.

The boundaries of the finite element model should not be so close to affect the solution. Extensive preliminary investigations were made to find the required size of the finite element mesh in each direction for different cases. The required distances from the edges of footing were found to be not more than 3.5*B*. The depth was taken 4*B* from the base of the footing. These dimensions were found to guarantee that the zones of observed displacement fields and plastic shearing are well within the boundaries of the model.

Maintaining the proportion of the element sizes, the finite element meshes were made finer to that extent beyond which substantial improvement in the answer was not observed. It was concluded that the elements of 0.1*B* in size adjacent to footing were suitable enough to obtain answers of sufficient accuracy.

***c) The boundary conditions***

Loading is applied by specifying uniform vertical downward displacement for nodes under footing. These nodes are not allowed to displace laterally. In this way, the footing-soil interface is assumed to be rough. At vertical planes comprising the boundaries of the model including the plane of symmetry, the displacement of nodes normal to these planes is restricted. Nodes at the ground surface are free to move in any direction but those at the bottom plane are locked from any movement.

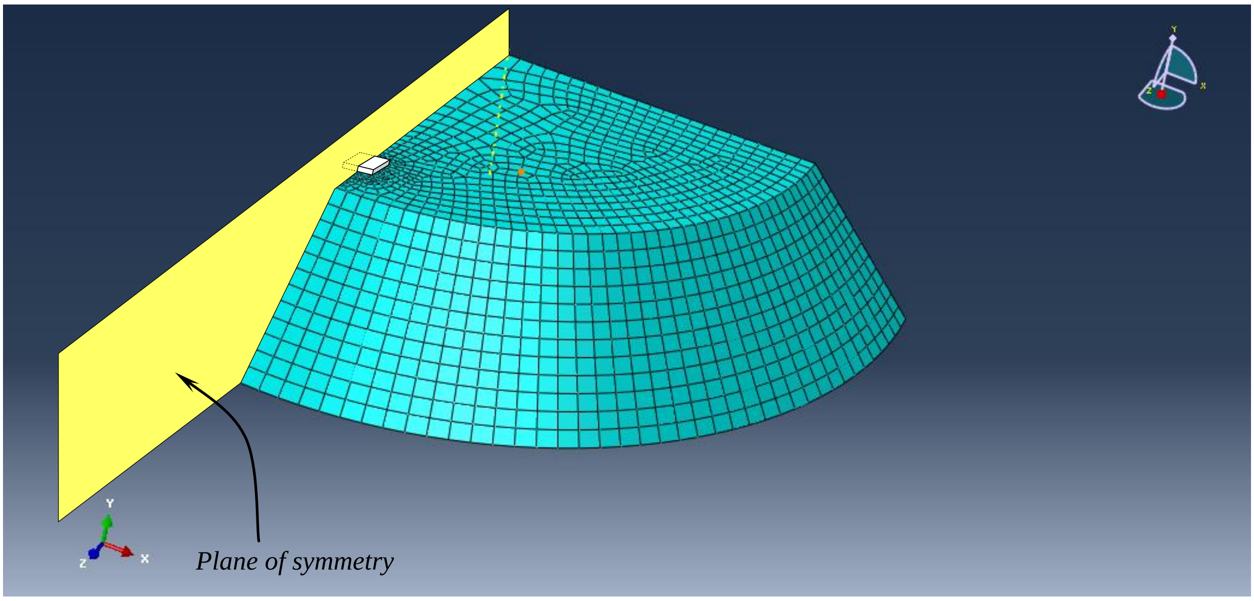


Fig. 3. Typical finite element mesh for half of the domain of the problem

***d) Material properties***

Both cohesive and frictional soils were used for the investigations. Four types of cohesive soil with a small friction angle of 5 degrees have been used with cohesions in the range of 50 to 400 kPa. Four types of frictional soil having a small cohesion of 5kPa have been used. The friction angle of these soils has been in the range of $20^o$ to $45^o$. The properties of these eight soils are tabulated in Table 1. Values of elastic modulus and Poisson's ratio assumed for all soils have been 20Mpa and 0.3 respectively. It must be mentioned that the value of bearing capacity is not affected by the elastic modulus assumed for the soil. The effect of reduction in elastic modulus *E* is just to extend the overall load-settlement curve of the footing to a larger range of settlement. The bearing capacity which is obtained where the load-settlement curve reached a plateau is not affected by what is assumed for *E*.

Table 1. Soil types and their properties

| **Soil No.** | **Soil Type Description** | $c$ (kN/m$^2$) | $\phi^o$ | **Density** (kN/m$^3$) | ***E*** (Mpa) | $\nu$ |
|---|---|---|---|---|---|---|
| 1 | Cohesive | 50 | 5 | 19 | 20 | 0.3 |
| 2 | | 100 | | | | |
| 3 | | 200 | | | | |
| 4 | | 400 | | | | |
| 5 | Frictional | 5 | 20 | 20 | | |
| 6 | | | 30 | | | |
| 7 | | | 37 | | | |
| 8 | | | 45 | | | |

## 3. PRELIMINARY VERIFICATIONS

As a preliminary verification of modeling the bearing capacity in ABAQUS, it was decided to compare the results of finite element analysis in the absence of any slope, with those obtained using the formulas suggested by famous researchers in the past. These include Terzaghi, Meyerhof, Hansen and Vesic. A 5m by 5m rough square footing is considered on the surface of each of 8 soils of Table 1. Only one-fourth of the problem has to be modeled because, in this case, the problem has two planes of symmetry. The finite element mesh for this ¼ of the problem is shown in Fig. 4. A quarter of the footing is located at the corner that is closest to the view. The ultimate bearing capacity in finite element analysis is found when the load-settlement curve levels off. Comparison of the results is made in the form of Table 2. It is concluded that the results of finite element analysis are more or less within the range of values obtained from well known bearing capacity calculations.

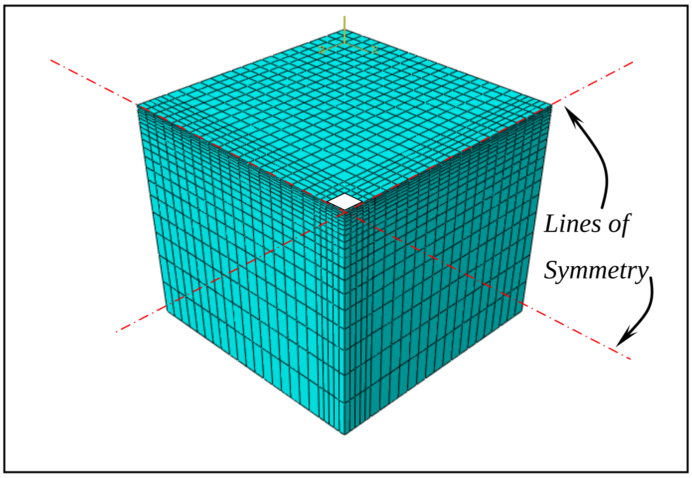


Fig. 4. The finite element mesh for a quarter of the bearing capacity problem

Table 2. Comparison of FEM results with values obtained from well-known bearing capacity calculations

| **Soil No.& parameters** | **Soil Type** | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | **Cohesive** | | | | **Frictional** | | | |
| | **1** $c$=50 kPa $\phi$=5$^o$ | **2** $c$=100kPa $\phi$=5$^o$ | **3** $c$=200kPa $\phi$=5$^o$ | **4** $c$=400kPa $\phi$=5$^o$ | **5** $c$=5 kPa $\phi$=20$^o$ | **6** $c$=5 kPa $\phi$=30$^o$ | **7** $c$=5 kPa $\phi$=37$^o$ | **8** $c$=5 kPa $\phi$=45$^o$ |
| **Terzaghi** | 495.6 | 972.5 | 1926.2 | 3833.7 | 301.4 | 923.3 | 2433.1 | 9528.6 |
| **Meyerhof** | 405.4 | 807.1 | 1610.6 | 3217.5 | 277.3 | 1259.5 | 4237.0 | 22244 |
| **Hansen** | 404.9 | 807.8 | 1613.4 | 3224.7 | 194.6 | 697.8 | 1914.2 | 7368.1 |
| **Vesic** | 415.6 | 818.5 | 1624.1 | 3235.4 | 267.8 | 914.8 | 2478.5 | 9496.2 |
| **This Study (FEM)** | 442 | 813 | 1577 | 3056 | 426 | 1212 | 3135 | 11313 |

Two typical examples have been used to compare the result of finite element analysis with what is obtained by formulas suggested by Hansen and Vesic when there is a slope nearby.

In the first example, a slope of 60$^o$ in a clay soil of 200 kPa shear strength with a unit weight of 19 kN/m$^3$ at $R/B$=60 has been considered. A 5m by 5m square footing is placed 0.5 m from the edge of the slope. Because the footing is not exactly at the edge of the slope, the calculation is made using the method suggested by Bowles [4] using $N'_c$ instead of using $g'_c$. The shape factor $s_c$ for Hansen and Vesic in this case would be 1.2 and 1.195, respectively. Interpolating between the values of $N'_c$ given for different ratios of $a/B$, we get a value of $N'_c$=3.823 for $a/B$ of 0.1. The resulting ultimate bearing capacity for Hansen and Vesic would be 918 and 914 kPa, respectively. The corresponding value obtained from finite element analysis was 1140 kPa. Some of the difference could probably be due to the small friction angle assumed for the cohesive material in the finite element analysis.

In the second example, a slope of 30$^o$ in a frictional soil having 30$^o$ friction angle and 20 kN/m$^3$ unit weight is considered under the same footing. The footing is located very close to the edge of the slope and the ratio $R/B$ is taken 100. It is believed that curvature effect is minimum at this large value of $R/B$. For both Hansen and Vesic shape factors for cohesion and $\gamma$-terms in this case are similar, being 1.61 and 0.6, respectively. However their suggestion for ground factor is different. Formulae suggested by Hansen give $g_c$=0.796 and $g_\gamma$=0.182, whereas those suggested by Vesic give $g_c$=1 and $g_\gamma$=0.179. The resulting bearing capacities according to the suggestions of Hansen and Vesic would be 275.5 and 363 kPa, respectively.

Corresponding value obtained from the finite element analysis has been 720. Part of the difference here could be due to the assumption of smooth soil-footing interface in conventional bearing capacity calculations.

## 4. RESULTS OF FINITE ELEMENT ANALYZES

In all cases the bearing capacity increases with increase in the radius of curvature in plan. Typical result obtained from the finite element analysis looks like what is shown in Fig. 5. The three-dimensional bearing capacity $q$ increases with $R$ as shown. The bearing capacity when the radius is large is called $q_{(L)}$. The bearing capacity can be normalized to $c$ or $\gamma B$ in cases of cohesive and frictional soils, respectively; but in order to see the effect of curvature, the results will be presented in the non-dimensional form of Fig. 6. The radius of curvature $R$ is normalized to $B$. The bearing capacity for any curvature is normalized to that at the same condition when $R/B$ is very large. Therefore the results are presented in the form of $q/q_{(L)}$ vs. $R/B$ -graphs to show the effect of curvature. The maximum value of $R/B$ used for the analyses was 60 because a significant change in the bearing capacity with further increase in this ratio was never observed.
The distance from the edge of footing to the edge of the slope $a$, is a factor that influences the results. The non-dimensional ratio of $a/B$ is used to see the influence of this factor on the results. Analyses were made for values of $a/B$ from 1.0 to 0.1 to see the effect of proximity of the footing to the slope edge.

### *a) Convex slopes in cohesive material*

Four levels of strength have been considered for the cohesive soil in the analyses. These include 50, 100, 200, and 400 kPa cohesions as tabulated in Table 1. The strength is expressed in the form of non-dimensional stability number $c/\gamma B$. The levels of stability numbers considered for cohesive soils would then be 0.5, 1, 2 and 4.

**1. Effect of proximity to slope, a/B:** The variation of non-dimensional bearing capacity $q/c$ with distance of footing from the slope edge $a/B$, has been compared for two typical stability numbers 1 and 2 for a $30^o$-slope at different curvatures, in Fig. 7. As expected, the bearing capacity increases with increase in distance of footing from the slope edge, so that the aggravating effect of slope fades after a distance of about footing width. Comparison of the results for slopes of different curvatures indicates lower bearing capacity with increase in curvature.

The influence of proximity of slope in reducing the bearing capacity has been shown in a different way in Fig. 8. Curves for lower values of $a/B$ fall below those for higher values of this ratio. Also, comparison of these figures with each other indicates the reduction of bearing capacity with increase in the slope angle as expected.

**2. Effect of curvature:** Effect of curvature is better recognized when we consider the variation in $q/q_{(L)}$ with $R/B$. Results of analyses for $30^o$, $60^o$ and $90^o$-slope angles of these soils are given in Fig. 9. As shown, the results indicate increase in bearing capacity with increase in the radius of curvature. Therefore, the curvature of slope in plan decreases the bearing capacity of convex slopes of cohesive soils. Results also indicate the radius beyond which the effect of curvature diminishes depends on the soil strength, slope angle and the distance between the footing edge and slope. The radius of influence increases with increase in slope angle and the distance from the slope but decreases with increase in soil strength. In general, the curvature of slope of cohesive soil was not seen to have any significant effect on reducing the bearing capacity when the ratio $R/B$ exceeds 20.

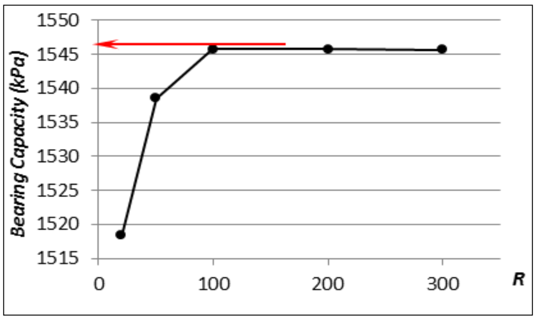


Fig. 5. Bearing capacity as a function of radius of curvature of the slope

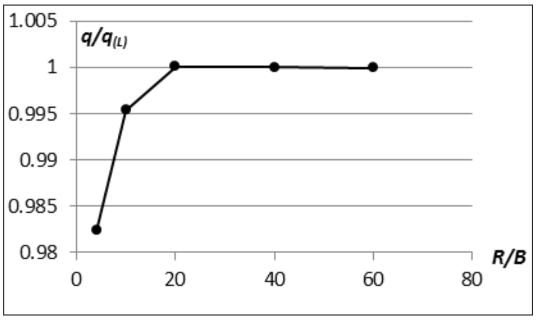


Fig. 6. A typical non-dimensional graph showing the influence of curvature on bearing capacity

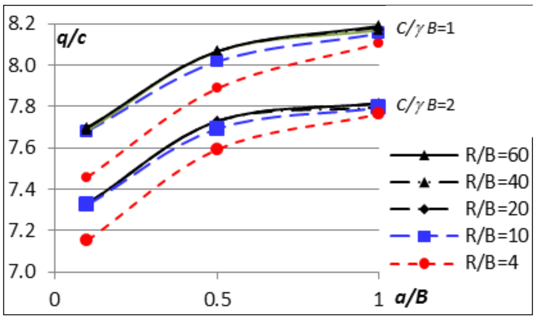


Fig. 7. Influence of slope proximity on the bearing capacity of cohesive soils for $\beta = 30^o$.

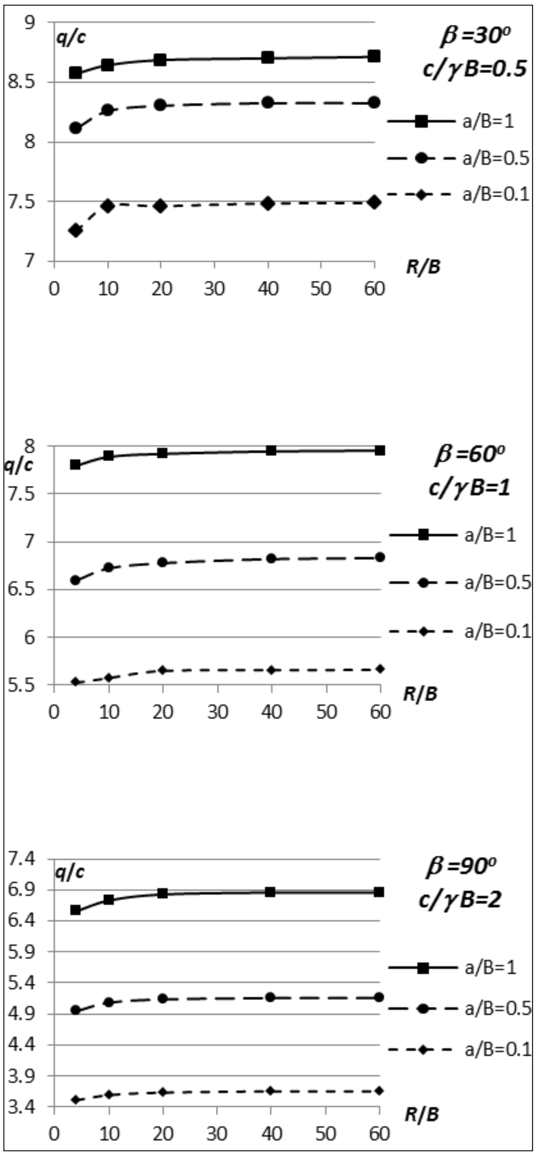


Fig. 8. Influence of slope proximity on the bearing capacity of cohesive soils for different slope angles.

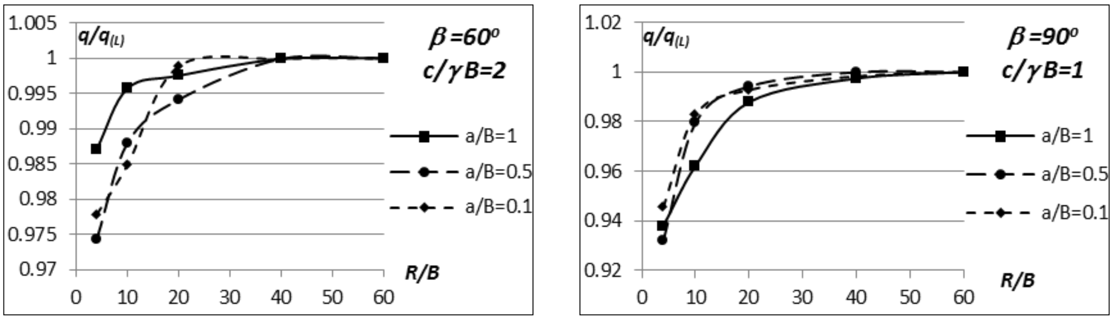


Fig. 9. Effect of curvature on bearing capacity of cohesive slopes

The results also indicate more reduction in bearing capacity with curvature when the slope is steeper. The reduction in bearing capacity with increase in curvature was limited to 7%. This was obtained when a vertical cut in a soil having the least strength had a relatively low radius of curvature (Fig. 9).

**3. Effect of soil strength c/γB:** Influence of soil strength on the curvature effects is shown for $a/B$=0.5 and 1 for slope of 30°, 60° and 90° in Fig. 10. Usually, curves for lower strength soils are located below those for higher strength. Therefore, the aggravating effect of plan curvature is more pronounced in case of soils having lower strengths. Comparison of the results for $a/B$ of 0.5 and 1 in the figure indicates more reduction in $q/q_{(L)}$ for $a/B$=0.5. This indicates increase in the effect of curvature as the footing approaches the edge of the slope.

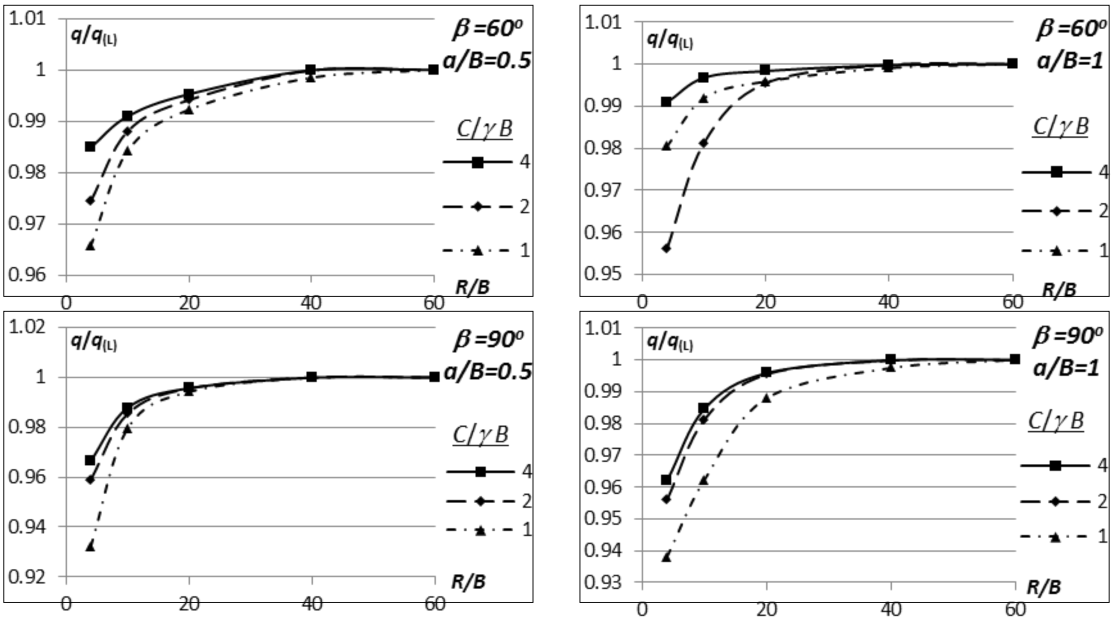


Fig. 10. Curvature effects on bearing capacity of cohesive soil slopes having different strengths

**4. Influence of slope angle, β:** In Fig. 11, the ratio $q/q_{(L)}$ is drawn vs. $R/B$ for different slope angles to show the influence of soil slope on curvature effects. These results indicate the influence of curvature in reducing the bearing capacity is higher for steeper slopes. The maximum reduction in bearing capacity obtained in the analysis was 7% which belonged to the slope angle of 90° when the stability number $c/\gamma B$ was 1, $a/B$ was 0.5 and $R/B$ was 4.

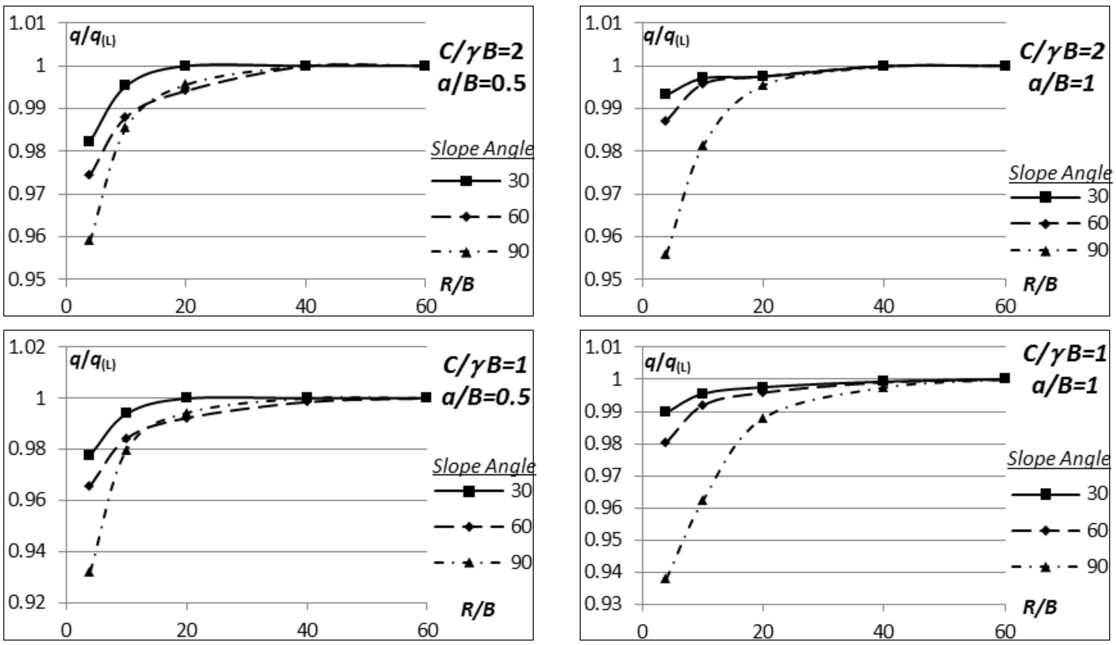


Fig. 11. Curvature effects on bearing capacity of cohesive soil slopes having different slope angles

### *b) Convex slopes in frictional material*

The frictional materials with friction angles 30, 37 and 45-degrees have been considered for the analyses; all with a small cohesion equal to 5 kPa. The slope angles were varied in each case but did not exceed the friction angle of the soil.

**1. Effect of proximity to slope, a/B:** The variation of non-dimensional bearing capacity ratio $q/\gamma B$ with distance of footing from the slope edge $a/B$, for $30^o$-slope in frictional soils having friction angles $37^o$ and $45^o$, has been compared in Fig. 12. The results are presented at different curvatures. As expected, the bearing capacity increases with increase in distance of footing from the slope edge. The figure also indicates that there is not much difference between the curves of different curvatures. The bearing capacity in this case is more influenced by proximity to the slope rather than the curvature.

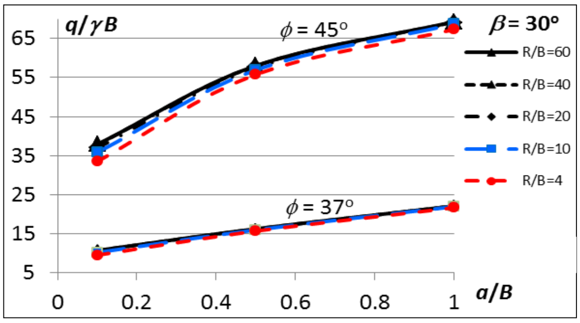


Fig. 12. Influence of slope proximity on the bearing capacity of frictional soils

Figure 13 shows the effect of proximity of foundation to slope in a different way. Obviously, curves for lower values of $a/B$ fall below those for higher values of this ratio.

**2. Effect of curvature:** In order to see the effect of curvature on the bearing capacity of slopes of frictional soils, this data is drawn in terms of $q/q_{(L)}$ vs. $R/B$-graphs in Fig. 14. The figure indicates increase in bearing capacity with increase in the radius of curvature. There seems to be no significant increase in bearing capacity when the radius exceeds 15 to 20 times footing dimension. The results also indicate the reduction in bearing capacity can be as high as 17%.

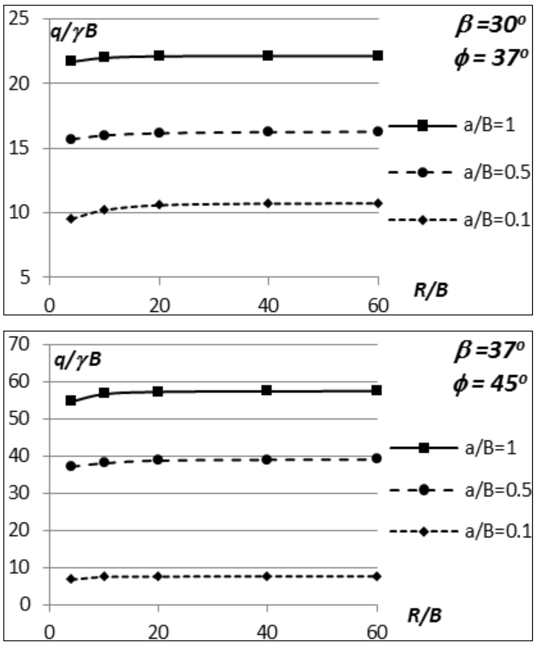


Fig. 13. Influence of slope proximity on the bearing capacity of frictional soils

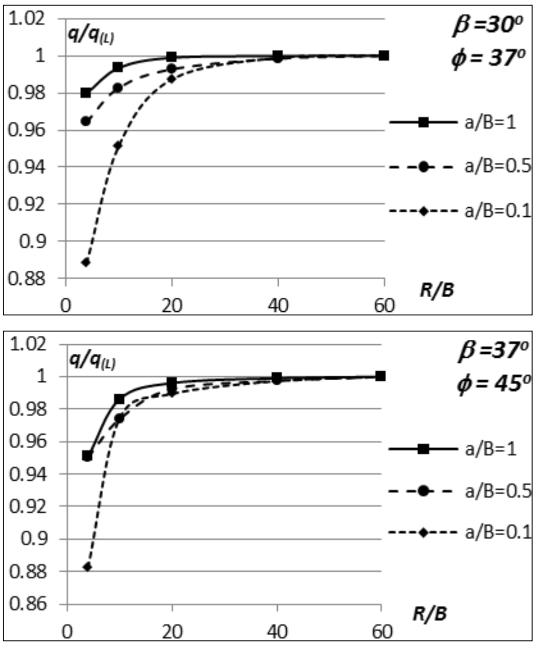


Fig. 14. Effect of curvature on bearing capacity of frictional soil slopes

**3. Effect of soil strength parameter, $\phi$:** This is only shown for $a/B$=0.1 and 0.5 because, as was shown above, the curvature does not have a significant effect at higher value of $a/B$. Figure 15 shows the curvature effects for soils of different strengths. The general trend is so that the curves for higher friction angles are located a little below those for lower friction angles. This indicates a slight increase in curvature effects with increase in soil strength parameter, $\phi$. The greater effect of curvature at low values of $a/B$ mentioned before is also clear in this figure.

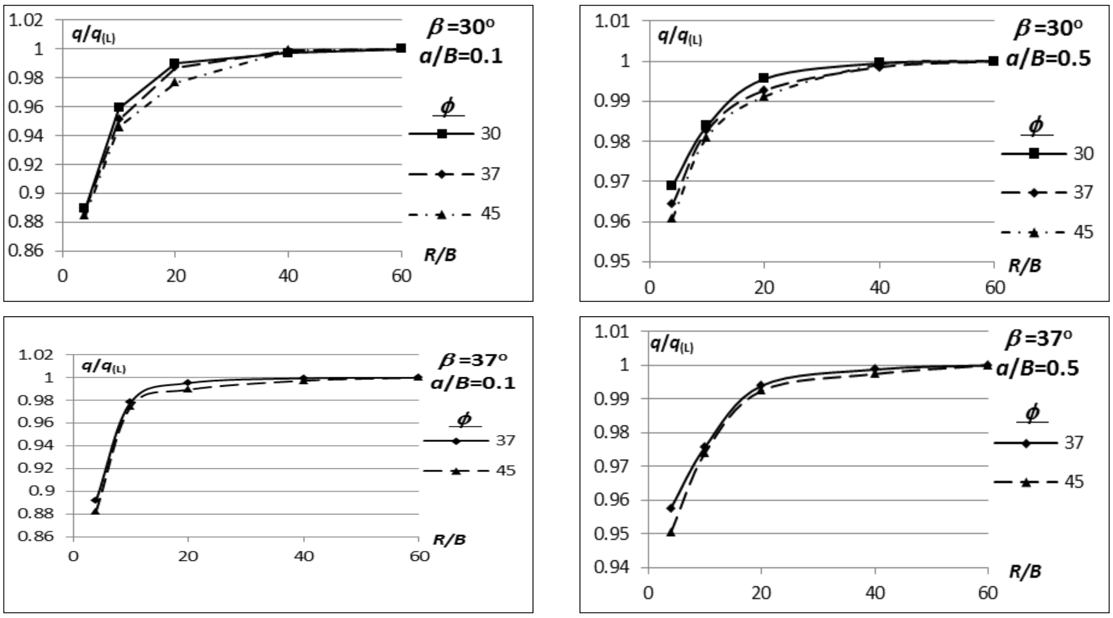


Fig. 15. Curvature effects on bearing capacity of frictional soil slopes having different strengths

**4. Influence of slope angle, $\beta$:** Figure 16 shows the curvature effects for different slope angles. There seems to be no considerable effect of slope angle when $a/B$ is 0.1. For the case of $a/B$=0.5, curves for steeper slopes fall below those for flatter ones. We may conclude that increase in the slope angle increases the effect of curvature in reducing the bearing capacity in this case. This is consistent with what we found in the case of cohesive soils.

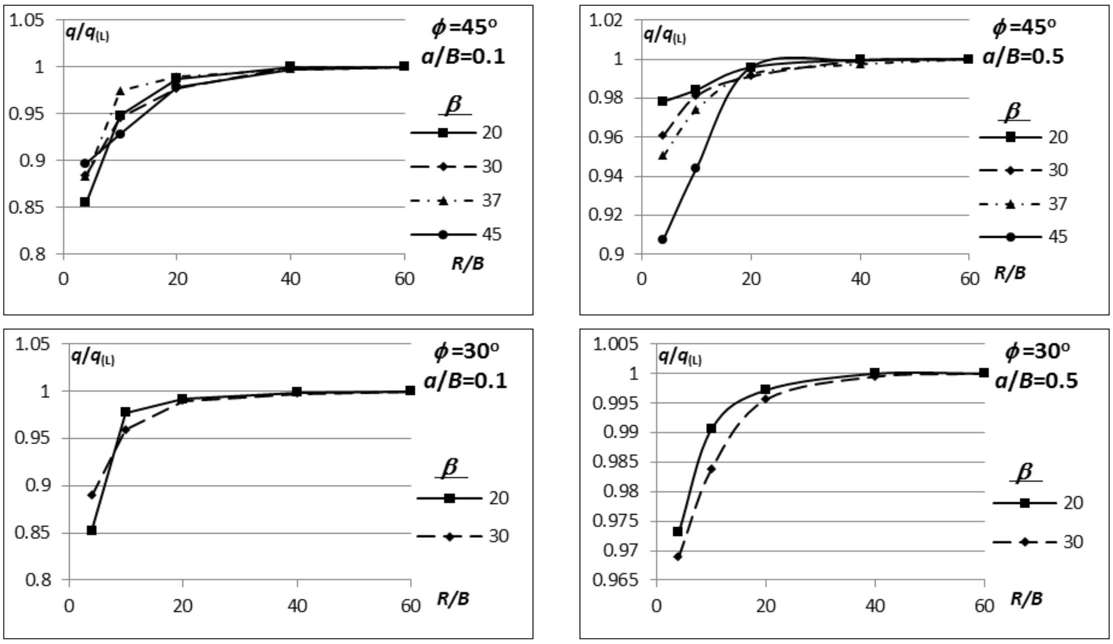


Fig. 16. Curvature effects on bearing capacity of frictional soil slopes having different slope angles

## 5. COMPARISON WITH THE RESULTS OF LIMIT ANALYSIS METHOD

Previous works on the subject differ in the way their results are presented and this makes the possibility of comparison very limited. The only comparison possible is with the results of Farzaneh et al. [16], who used the upper bound method of limit analysis to investigate almost the same problem in part of their work. In their work, $H$ is the height of slope failure mechanism, and is used to define a non-dimensional stability number $\lambda_{\phi\chi} = \gamma H.\tan\phi/\chi$. In the bearing capacity part of their work, Farzaneh et al. have presented their results only for $\lambda_{\phi c}$ of 0 and 1. The height $H$ that is used in the finite element analysis (Fig. 1), has nothing to do with the failure height. The parameter $H$ has been used here to define the thickness of the domain of finite element model. It has been taken large enough to eliminate the effect of bottom boundary of the model on the answer at the worst case. Due to the difference in definition of $H$ in these two studies only comparison has been made here, for $\lambda_{\phi c}$=0, where the effect of $H$ is eliminated. This has been done for the slope angles 90$^o$ and 60$^o$, when $c/\gamma B$ is 4. It should be noted that there is still a very slight friction angle (5$^o$) accompanied by the cohesive material assumed in the finite element analysis. Apart from this, Farzaneh et al. have presented their results in terms of non-dimensional ratio $q_{3D}/q_{2D}$, where $q_{2D}$ is the ultimate bearing capacity for two dimensional plane strain problem [16]. Here, in order to see the effect of curvature, their data has been converted. The ratio $q_{3D}/q_{2D}$ given for any $a/B$ has been divided by the value of this ratio at the end of their graphs relevant to large values of $R/B$, where we believe curvature has no significant effect. The result of this conversion is again drawn vs. $R/B$ in Figs. 16 and 17. Results of finite element analysis are also depicted in the same figures for comparison. Note that the horizontal axis is logarithmic to be consistent with the way Farzaneh et al. have presented their results. Both analyses indicate the effect of curvature becomes important only when the ratio $R/B$ is less than about 10. The results seem more or less similar for 60$^o$-slope quantitatively. For 90$^o$-slope, however, the limit analysis method predicts lower effects with curvature compared to the finite element method.

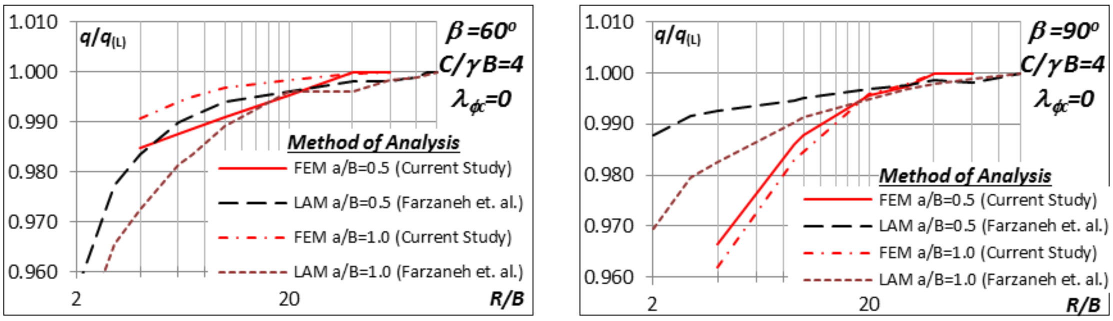


Fig. 17. Comparison of results of finite element analysis and upper bound limit analysis method

## 6. CONCLUSION

In this research work, the effect of convexity of a soil slope in plan on its bearing capacity was investigated using the finite element method. The obtained results generally indicated a reduction in the bearing capacity due to this type of curvature that should not be overlooked in design.

In cohesive soils the aggravating effect of convexity of slope in plan is more significant in steeper slopes of low strength. The maximum reduction in bearing capacity was found to be limited to 7%. This was found when the slope was the steepest, i.e. $\beta$=90$^o$; the soil strength was relatively low ($c/\gamma B$=1), the footing was relatively close to the slope ($a/B$=0.5), and the radius of curvature was 4$B$, i.e., $R/B$=4. The effect of curvature on reducing the bearing capacity would be less if the slope angle is decreased or the soil strength is increased. Trends of reduction in bearing capacity with curvature found by the finite element method were similar to those found by the upper bound method of limit analysis.

The reduction in bearing capacity due to convexity of a slope in frictional soils can be as large as 15%. Effect of curvature was not found to be much influenced by the strength parameter of the soil in this case. However, the steeper slopes of frictional soils were found to be more influenced by curvature as in the case of cohesive soils. The effect of curvature diminished as the radius of curvature exceeded about 10 to 20 times footing dimension.

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