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On the Petrovsky inverse problem with memory term and nonlinear boundary feedback

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Abstract

In this paper we consider a Petrovsky viscoelastic inverse source problem with memory term in the boundary condition. We obtain sufficient conditions on relaxation function and initial data for which the solutions of problem are asymptotically stable when the integral overdetermination tends to zero as time goes to infinity.

Keywords: Inverse problem; asymptotic stability; boundary feedback

1. Introduction

We study the asymptotic behavior of solutions for inverse problem of determining a pair of functions $\{u(x,t), f(t)\}$ that satisfy

$$u_{tt} + \Delta^2 u - \int_0^t g(t-\tau)\Delta^2 u(\tau)\, d\tau - a_1 \Delta u + a_2 u_t = f(t)\omega(x),\ \ x \in \Omega, t > 0 \quad (1)$$

$$\begin{cases} u(x,t) = 0, & x \in \Gamma_0\,, t > 0 \\ \Delta u(x,t) = \int_0^t g(t-\tau)\Delta u(\tau)\, d\tau - a_3 |\nabla u|^p \nabla u, & x \in \Gamma_1\,, t > 0 \end{cases} \quad (2)$$

$$u(x,0) = u_0(x), u_t(x,0) = u_1(x), \qquad x \in \Omega \quad (3)$$

$$\int_\Omega u(x,t)\omega(x)\, dx = \phi(t), \qquad t > 0 \quad (4)$$

where Ω is a bounded domain of $\mathbb{R}^n (n \geq 1)$ with smooth boundary $\Gamma_0 \cup \Gamma_1 = \partial\Omega$ such that $meas(\Gamma_1) > 0$. Here a_1, a_2 and a_3 are positive numbers. Moreover, $\phi(t)$ and $\omega(x)$ are functions that satisfy specific conditions that will be enunciated later.

Such problems occur in many mathematical models of applied sciences. Applications include recovery of inclusions from anomalies of their gravitational fields, reconstruction of the interior of the human body from exterior electrical, ultrasonic and magnetic measurements, recovery of interior structural parameters of detail of machines and of the underground from similar data and locating flying or navigated objects from their acoustic or electromagnetic fields. In contrast with the extensive literature on global behavior of solutions for direct problems in partial differential equations, we know few results about inverse problems. For example, we consider (1)-(4) in the absence of the viscoelastic term and with homogeneous boundary condition this problem reduces to the following inverse problem,

$$\begin{gathered} u_{tt} - \Delta u + a_1 u_t + a_2 |u|^p u + b(x,t,u,\nabla u) = f(t)\omega(x), \qquad x \in \Omega > 0 \\ u(x,t) = 0, \quad x \in \Gamma, \qquad t > 0 \\ u(x,0) = u_0(x),\ \ u_t(x,0) = u_1(x), x \in \Omega, \\ \int_\Omega u(x,t)\omega(x) dx = \phi(t), \qquad t > 0 \end{gathered} \quad (5)$$

This problem has been studied by Eden and Kalantarov (2006). They proved the global behavior of solutions. Also Guvenilir and Kalantarov (2006) established the global nonexistence of solutions to an inverse problem for differential operator equation. Existence and unique solvability of parabolic and hyperbolic inverse source problems has been studied by Gozukizil and Yaman (2007), (2008). They proved these results by using the contraction mapping theorem.

Recently, Tahamtani and Shahrouzi (2013) studied asymptotic behavior of solutions for the following inverse problem:

$$u_{tt} + \Delta^2 u - \alpha_1 \Delta u + \alpha_2 u_t + \alpha_3 |u|^p u + b(x,t,u,\nabla u,\Delta u) = f(t)\omega(x), \quad x \in \Omega, t > 0$$

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$$u(x,0)=0,\ \ \Delta u=-c_0\partial_\nu u(x,t),\quad x\in\partial\Omega, t>0$$
$$u(x,0)=u_0(x),\qquad u_t(x,0)=u_1(x),\qquad x\in\Omega$$
$$\int_\Omega u(x,t)\omega(x)dx=\phi(t),\quad t>0$$

and showed that the solutions of this problem under some appropriate conditions are stable if α_1, α_2 are large enough, $\alpha_3\geq 0$ and $\phi(t)$ tends to zero as time goes to infinity and establish a blow-up result, if $\alpha_3<0$ and $\phi(t)=k$ are a constant. For more information about inverse problems we refer the readers to Belov and Shipina (1988); Bui (2002); Gbur (2001); Shahrouzi and Tahamtai (2012); Shidfar et al. (2010).

In direct problems, it is worth mentioning some papers in connection with existence and blow up of solutions for viscoelastic equations. Cavalcanti et al. (2002) were the first to study exponential decay for solutions of

$$u_{tt}-\Delta u+\int_0^t g(t-\tau)\Delta u(\tau)\,d\tau+a(x)u_t = u|u|^{p-2}. \qquad (6)$$

This work was later improved by Cavalcanti and Oquendo (2003) and Berrimi and Messaoudi (2006) using different methods. Messaoudi (2002) and (2003) showed that, concerning nonexistence, Todorova and Georgiev's results (1994) can be extended to (6) using the concavity method with a modification in energy functional due to the different nature of the problem.

Recently, Zarai et al. (2013) considered elastic membrane equation with memory term and nonlinear boundary damping. They proved a general decay and blow up result for some solutions of the following problem:

$$u_{tt}-M(t)\Delta u+\int_0^t h(t-s)\Delta u(s)\,ds = 0 \quad in\ \Omega\times(0,\infty)$$
$$u(x,0)=u_0(x)\ \ and\ \ u_t(x,0)=u_1(x)\quad in\ \bar{\Omega}$$
$$u=0 \qquad on\ \Gamma_1\times(0,\infty)$$
$$M(t)\frac{\partial u}{\partial \nu}-\int_0^t h(t-s)\frac{\partial u}{\partial \nu}(s)\,ds+\alpha|u_t|^{m-2}u_t = |u|^{p-2}u\quad on\ \Gamma_0\times(0,\infty)$$

Motivated by the aforementioned works, we take a_1, a_2 in the appropriately domain, then prove that solutions of (1)-(4) are asymptotically stable when $\phi(t)$ tends to zero as time goes to infinity. Our approaches are based on the Lyapunov function and perturbed energy method.

2. Preliminaries and main results

In this section, we present some materials needed in the proof of our main results. We shall assume that the functions $\omega(x), \phi(t)$ and the functions appearing in the data satisfy the following conditions\newline

$$(A1)\quad u_0\in H_0^1(\Omega)\cap L^{p+2}(\Omega),\quad \int_\Omega u_0(x)\omega(x)dx=\phi(0),$$
$$(A2)\quad \omega\in H^4(\Omega)\cap H_0^3(\Omega)\cap L^{p+2}(\Omega),\quad \int_\Omega \omega^2(x)dx=1,$$
$$(A3)\quad g(t)\geq 0\ ,g'(t)\leq 0,\quad 1-\int_0^\infty g(t)dt=l>0.$$

Throughout this paper all the functions considered are real-valued. We denote by $\|.\|_q$ the L^q-norm over Ω. In particular, the L^2-norm is denoted $\|.\|$ in Ω and $\|.\|_{\Gamma_i}$ in Γ_i. Also $(.,.)$ denotes the usual L^2-inner product. We use familiar function spaces $H_0^1(\Omega), H^4(\Omega)$.

We recall the Poincare inequality

$$\|u\|^2\leq B_2\|\nabla u\|^2, \qquad (7)$$

where B_2 is the optimal constant.

Also, the Young's inequality is sometimes used,

$$ab\leq \beta a^q+C(\beta,q)b^{q'}, \qquad (8)$$

where $a,b\geq 0,\ \beta>0,\ C(\beta,q)=\frac{1}{q'(\beta q)^{-\frac{q'}{q}}}$ are constants and $\frac{1}{q}+\frac{1}{q'}=1$.

Adapting the idea of Prilepko et.al (2000), the key observation is that the problem (1)-(4) is equivalent to the following direct problem,

$$u_{tt}+\Delta^2 u-\int_0^t g(t-\tau)\Delta^2 u(\tau)\,d\tau-a_1\Delta u + a_2 u_t = f(t)\omega(x),\quad x\in\Omega, t>0 \qquad (9)$$

$$\begin{cases} u(x,t)=0, & x\in\Gamma_0, t>0\\ \Delta u(x,t)=\int_0^t g(t-\tau)\Delta u(\tau)\,d\tau-a_3|\nabla u|^p\nabla u, & x\in\Gamma_1, t>0\end{cases} \qquad (10)$$

$$u(x,0)=u_0(x),\qquad u_t(x,0)=u_1(x), x\in\Omega \qquad (11)$$

in which the unknown function $f(t)$ is replaced by

$$f(t)=\phi''(t)+a_2\phi'(t)+(\Delta u,\Delta\omega)+a_1(\nabla u,\nabla\omega) -\int_0^t g(t-\tau)(\Delta u(\tau),\Delta\omega)\,d\tau, \qquad (12)$$

So the energy associated with problems (9)-(11) is given by

$$E(t) = \frac{1}{2}\left(\|u_t\|^2 + \left(1 - \int_0^t g(s)ds\right)\|\Delta u\|^2 + (g * \Delta u)(t) + a_1\|\nabla u\|^2 \right) + \frac{a_3}{p+2}\|\nabla u\|_{p+2,\Gamma_1}^{p+2} \tag{13}$$

where $(g * v)(t) = \int_0^t g(t-\tau)\|v(t) - v(\tau)\|^2 \, d\tau$.

Now, we are in a position to state our asymptotic stability result:

Theorem 2.1. Let the conditions $(A1) - (A3)$ be satisfied and suppose that ϕ, ϕ' and ϕ'' are continuous functions defined on $[0, \infty)$, such that ϕ'' is a bounded function and ϕ, ϕ' tend to zero as t goes to infinity. Also, there exist positive constants M, N such that

$$\frac{2N + M\delta}{2M} \leq a_2 \leq \frac{2a_1}{\delta B_2^2},$$
$$\int_0^\infty g(s)ds \leq \frac{2\xi}{1 + 2\xi}$$

and

$$N \geq \max\{\frac{2\delta M + 1 - l}{l}, \frac{\delta l M - l + 1}{2\xi l + l - 1}, \frac{\delta M}{p+2}\}$$

where δ, ξ are positive numbers such that ξ is sufficiently small. Then the solutions of problem (1)-(4) are asymptotically stable and

$$\lim_{t\to\infty} E(t) = 0.$$

3. Asymptotic stability

As mentioned earlier, the proof of asymptotic stability result, Theorem 2.1 is given in this section. In order to carry the proof we need the following Lemmas.

Lemma 3.1. Under the conditions of Theorem 2.1, the energy functional $E(t)$, defined by (13), satisfies

$$\frac{d}{dt}E(t) \leq -a_2\|u_t\|^2 + f(t)\phi'(t). \tag{14}$$

Proof: Multiplying equation (9) by u_t, performing an integration by parts and using (A3) and (13) our conclusion follows.

Lemma 3.2. Under the conditions of Theorem 2.1, the function $f(t)$, defined by (12), satisfies for some $M.N > 0$

$$|M\phi'(t) + N\phi(t)|f(t) \leq \frac{\delta M}{2}\|\Delta u\|^2 + \frac{1-l}{2l}(g * \nabla u)(t) + \frac{\delta M a_1}{2}\|\nabla u\|^2 + H(t), \tag{15}$$

where $\delta > 0$ and

$$H(t) = |M\phi'(t) + N\phi(t)||\phi''(t)) + a_2\phi'(t)| + \frac{a_1|M\phi'(t) + N\phi(t)|^2}{2\delta M}\|\nabla\omega\|^2 + \left(\frac{1+\delta M}{2\delta M}\right)|M\phi'(t) + N\phi(t)|^2\|\Delta\omega\|^2. \tag{16}$$

Proof: We have from definition of $f(t)$

$$\begin{aligned}|M\phi'(t) + N\phi(t)|f(t) &= |M\phi'(t) + N\phi(t)|\big(\phi''(t) + a_2\phi'(t)\big) \\ &+ |M\phi'(t) + N\phi(t)|\int_0^t g(t-\tau)(\Delta u(\tau), \Delta\omega)\, d\tau \\ &+ |M\phi'(t) + N\phi(t)|(\Delta \mathrm{u}, \Delta\omega) \\ &+ a_1|M\phi'(t) + N\phi(t)|(\nabla \mathrm{u}, \nabla\omega).\end{aligned} \tag{17}$$

By using the Young's inequality (8), the last three terms in the right hand side of (17) can be estimated as follows, for any $\delta > 0$, taking $a = \|\Delta u\|$, $b = |M\phi'(t) + N\phi(t)|\|\Delta\omega\|$, $q = q' = 2$ and $\beta = \frac{\delta M}{2}$, we deduce that

$$|M\phi'(t) + N\phi(t)||(\Delta \mathrm{u}, \Delta\omega)| \leq \frac{\delta M}{2}\|\Delta \mathrm{u}\|^2 + \frac{|M\phi'(t) + N\phi(t)|^2}{2\delta M}\|\Delta\omega\|^2, \tag{18}$$

and

$$a_1|M\phi'(t) + N\phi(t)||\nabla \mathrm{u}, \nabla\omega| \leq \frac{\delta M a_1}{2}\|\nabla u\|^2 + \frac{a_1|M\phi'(t) + N\phi(t)|^2}{2\delta M}\|\nabla\omega\|^2. \tag{19}$$

We now estimate the integral term in the right-hand side of (19) as follows:

$$\begin{aligned}|M\phi'(t) + N\phi(t)|\left|\int_0^t g(t-\tau)(\Delta u(\tau), \Delta\omega)\, d\tau\right| &\leq \frac{|M\phi'(t) + N\phi(t)|^2}{2}\|\Delta\omega\|^2 + \frac{1}{2}\int_\Omega \left(\int_0^t g(t - \tau)|\Delta u(\tau)|\, d\tau\right)^2 dx \\ &\leq \frac{|M\phi'(t) + N\phi(t)|^2}{2}\|\Delta\omega\|^2 + \frac{1}{2}\int_\Omega \left(\int_0^t g(t-\tau)(|\Delta u(\tau) - \Delta u(t)| + |\Delta u(t)|)\, d\tau\right)^2 dx\end{aligned}$$

Using Schwartz and Young's inequality, and $\int_0^t g(s)\,ds \le \int_0^\infty g(s)\,ds = 1-l$, we obtain

$$\int_\Omega \left(\int_0^t g(t-\tau)(|\Delta u(\tau)-\Delta u(t)| + |\Delta u(t)|)\,d\tau\right)^2 dx \le \int_\Omega \left(\int_0^t g(t-\tau)|\Delta u(\tau) - \Delta u(t)|\,d\tau\right)^2$$

$$+\int_\Omega \left(\int_0^t g(t-\tau)|\Delta u(t)|\,d\tau\right)^2 dx + 2\int_\Omega \left(\int_0^t g(t-\tau)|\Delta u(\tau)-\Delta u(t)|\,d\tau\right)\left(\int_0^t g(t-\tau)|\Delta u(t)|\,d\tau\right)dx$$

$$\le \frac{1}{1-l}\int_\Omega \left(\int_0^t g(t-\tau)|\Delta u(t)|\,d\tau\right)^2 dx + \frac{1}{l}\int_\Omega \left(\int_0^t g(t-\tau)|\Delta u(\tau)-\Delta u(t)|\,d\tau\right)^2 dx$$

$$\le \frac{1}{l}\int_\Omega \int_0^t g(s)ds\int_0^t g(t-\tau)|\Delta u(\tau)-\Delta u(t)|^2\,d\tau dx + \frac{1}{1-l}\int_\Omega |\Delta u(t)|^2\left(\int_0^t g(\tau)\,d\tau\right)^2 dx$$

$$\le (1-l)\|\Delta u\|^2 + \frac{1-l}{l}(g*\Delta u)(t). \qquad (20)$$

Therefore we have

$$|M\phi'(t)+N\phi(t)|\left|\int_0^t g(t-\tau)(\Delta u(\tau),\Delta\omega)\,d\tau\right| \le \frac{|M\phi'(t)+N\phi(t)|^2}{2}\|\Delta\omega\|^2 + \frac{(1-l)}{2}\|\Delta u\|^2 + \frac{1-l}{2l}(g*\Delta u)(t). \qquad (21)$$

Applying estimations (18), (19) and (21) in (17) yields the desired result.

Proof of Theorem 2.1. Inspired by the idea in Tahamtani and Shahrouzi (2013), we define

$$L(t) = ME(t) + N\psi(t), \qquad (22)$$

where

$$\psi(t) = \int_\Omega uu_t dx + \frac{a_2}{2}\|u\|^2 - \xi\int_0^t\int_0^s g(s-\tau)\|\Delta u(\tau)-\Delta u(s)\|^2 d\tau ds, \qquad (23)$$

for some $\xi > 0$ that will be explained later.

It is clear that $L(t)$ and $E(t)$ are equivalent in the sense of the following Lemma.

Lemma 3.3. Under the conditions of Theorem 2.1, there exist two positive constants α_1 *and* α_2 such that

$$\alpha_1 L(t) \le E(t) \le \alpha_2 L(t).$$

We differentiate (22) and use equation (14) to obtain

$$L'(t) \le (N - Ma_2)\|u_t\|^2 - N\xi(g*\Delta u)(t) + Na_2(u_t,u) + N(u_{tt},u) + M\phi'(t)f(t). \qquad (24)$$

It follows from (1) and boundary conditions (2)

$$(u_{tt},u) = -\|\Delta u\|^2 - a_3\|\nabla u\|_{p+2,\Gamma_1}^{p+2} - a_1\|\nabla u\|^2 + \int_\Omega \Delta u\int_0^t g(t-\tau)\Delta u(\tau)d\tau dx - a_2(u_t,u) + \phi(t)f(t). \qquad (25)$$

Utilizing (15) into (25), we get

$$L'(t) \le -(Ma_2 - N)\|u_t\|^2 - N\|\Delta u\|^2 - Na_3\|\nabla u\|_{p+2,\Gamma_1}^{p+2} - N\xi(g*\Delta u)(t) + N\int_\Omega \Delta u\int_0^t g(t-\tau)\Delta u(\tau)d\tau dx - Na_1\|\nabla u\|^2 + |M\phi'(t) - N\phi(t)|f(t). \qquad (26)$$

Consequently, from definition of $\delta L(t)$ and inequality (7), we deduce

$$L'(t) + \delta L(t) \le -\left(Ma_2 - N - \frac{\delta M}{2}\right)\|u_t\|^2 - \left(N - \frac{\delta M}{2}\right)\|\Delta u\|^2 - \left(Na_1 - \frac{\delta M a_1}{2} - \frac{\delta N B_2^2 a_2}{2}\right)\|\nabla u\|^2 - \left(N\xi - \frac{\delta M}{2}\right)(g*\Delta u)(t) - a_3\left(N - \frac{\delta M}{p+2}\right)\|\nabla u\|_{p+2,\Gamma_1}^{p+2} + N\int_\Omega \Delta u\int_0^t g(t-\tau)\Delta u(\tau)d\tau dx + |M\phi'(t) - N\phi(t)|f(t). \qquad (27)$$

Similar to (21), we obtain

$$\int_\Omega \Delta u \int_0^t g(t-\tau)\Delta u(\tau)d\tau dx \le \left(1-\frac{l}{2}\right)\|\Delta u\|^2 + \frac{1-l}{2l}\,(\mathrm{g} * \Delta\mathrm{u})(\mathrm{t}). \qquad (28)$$

By virtue of (28) and Lemma 3.2, (27) becomes

$$L'(t)+\delta L(t) \le -\left(Ma_2 - N - \frac{\delta M}{2}\right)\|u_t\|^2 - \left(\frac{Nl}{2} - \delta M - \frac{1-l}{2}\right)\|\Delta u\|^2 - \left(Na_1 - \delta M a_1 - \frac{\delta N B_2^2 a_2}{2}\right)\|\nabla u\|^2 - \left(N\xi - \frac{\delta M}{2} - (N+1)\frac{1-l}{2l}\right)(g*\Delta u)(t) - a_3\left(N - \frac{\delta M}{p+2}\right)\|\nabla u\|_{p+2,\Gamma_1}^{p+2} + H(t), \qquad (29)$$

where $H(t)$ satisfies (16).

At this point if we choose $\delta > 0$ it is sufficiently small and

$$\frac{2N+M\delta}{2M} \le a_2 \le \frac{2a_1}{\delta B_2^2}, \quad \int_0^\infty g(s)ds \le \frac{2\xi}{1+2\xi},$$

also

$$N \ge \max\{\frac{2\delta M + 1 - l}{l}, \frac{\delta l M - l + 1}{2\xi l + l - 1}, \frac{\delta M}{p+2}\}$$

then we easily derive

$$L'(t) + \delta L(t) \le H(t)$$

thanks to the assumptions on $\phi(t), \phi'(t)$ and $\phi''(t)$. Indeed $\phi(t), \phi'(t)$ tends to zero as t goes to infinity and $\phi''(t)$ is a bounded function, so the right-hand side of last inequality tends to zero. This implies that from Lemma 3.3

$$\lim_{t\to+\infty} E(t) = 0.$$

Therefore, solutions of (1)-(4) are asymptotically stable and the proof of Theorem 2.1 is completed.

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