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Combined effect of suspended particles and rotation on thermosolutal convection in a viscoelastic fluid saturating a Darcy-Brinkman porous medium

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Abstract

In this paper, the combined effect of suspended (fine dust) particles and rotation on the onset of thermosolutal convection in an elastico-viscous fluid in a porous medium is studied. For the porous medium, the Brinkman model is employed and Rivlin-Ericksen model is used to characterize viscoelastic fluid. By applying normal mode analysis method, the dispersion relation has been derived and solved analytically. It is observed that the rotation, stable solute gradient, suspended particles, gravity field and viscoelasticity introduce oscillatory modes. For stationary convection, it is observed that the rotation, stable solute gradient have a stabilizing effect and suspended particles have a destabilizing effect on the system whereas Darcy number and medium permeability have stabilizing/destabilizing effects under certain conditions. The effects of rotation, stable solute gradient, suspended particles, Darcy number and medium permeability have also been shown graphically.

Keywords*:* Brinkman porous medium; Rivlin-Ericksen fluid; rotation; suspended particles; thermosolutal convection; viscosity; viscoelasticity

1. Introduction

The problem of thermosolutal convection in porous medium has attracted considerable interest during the last few decades, because it has various applications in geophysics, soil sciences, ground water hydrology, astrophysics, food processing, oceanography, limnology and engineering etc. Many researchers have investigated thermosolutal convection problems by taking different types of fluids. A detailed account of the thermal instability of a Newtonian fluid under varying assumptions of hydrodynamics and hydromagnetics has been given by Chandrasekhar [1]. Chandra [2] observed a contradiction between the theory and experiment for the onset of convection in fluids heated from below. He performed the experiment in an air layer and found that the instability depended on the depth of the layer. A Be′nard-type cellular convection with the fluid descending at a cell centre was observed when the predicted gradients were imposed for layers deeper than 10 mm. A convection which was different in character from that in deeper layers occurred at much lower gradients than predicted if the layer depth was less than 7 mm, and called this motion, "Columnar instability". He added an aerosol to mark the flow pattern.

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Veronis [3] has investigated the problem of thermohaline convection in a layer of fluid heated from below and subjected to a stable salinity gradient. Lapwood [4] has studied the convective flow in a porous medium using linearized stability theory. The Rayleigh instability of a thermal boundary layer in flow through a porous medium has been considered by Wooding [5]. Scanlon and Segel [6] have considered the effect of suspended particles on the onset of Be′nard convection and found that the critical Rayleigh number was reduced solely because the heat capacity of the pure gas was supplemented by the particles. The suspended particles were thus found to destabilize the layer.

Stommel and Fedorov [7] and Linden [8] have remarked that the length scalar characteristic of double-diffusive convecting layers in the ocean may be sufficiently large so that the Earth's rotation might be important in their formation. Moreover, the rotation of the Earth distorts the boundaries of a hexagonal convection cell in a fluid through a porous medium and the distortion plays an important role in the extraction of energy in the geothermal regions. Brakke [9] explained a double-diffusive instability that occurs when a solution of a slowly diffusing protein is layered over a denser solution of more rapidly diffusing sucrose. The scientific importance of the field has also increased because hydrothermal circulation is the dominant heat transfer mechanism in the development of young oceanic crust [10]. Generally, it is accepted that comets consist of a dusty ‘snowball’ of a mixture of frozen gases which in the process of their journey changes from solid to gas and vice-versa. The physical properties of comets, meteorites and inter-planetary dust strongly suggest the importance of porosity in the astrophysical context [11].

In recent years, considerable interest has been evinced in the study of Rivlin-Ericksen viscoelastic fluid having relevance in chemical technology and industry. Rivlin and Ericksen [12] have proposed a theoretical model for such viscoelastic fluid. Srivastava and Singh [13] have studied the unsteady flow of a dusty elastico-viscous Rivlin-Ericksen fluid through channels of different cross-sections in the presence of time-dependent pressure gradient. Sharma and Rana [14] have studied thermal instability of a incompressible Walters' (Model B') elastico-viscous fluid in the presence of variable gravity field and rotation in porous medium and found that rotation has a stabilizing effect on the system. Garg et. al. [15] has studied the rectilinear oscillations of a sphere along its diameter in conducting dusty Rivlin-Ericksen fluid in the presence of magnetic field, whereas the instability of streaming Rivlin-Ericksen fluids in porous medium studied by Sharma and Rana [16].

The investigation in porous media has been started with the simple Darcy model and gradually is extended to Darcy-Brinkman model. A good account of convection problems in a porous medium is given by Vafai and Hadim [17], Ingham and Pop [18] and Nield and Bejan [19]. Kuznetsov and Nield [20] studied thermal instability in a porous medium layer saturated by a nanofluid in a Darcy-Brinkman porous medium.

Recently, Attia [21] studied hiemenz flow through a porous medium of a non-Newtonian Rivlin-Ericksen fluid with heat transfer whereas the onset of convection in Rivlin-Ericksen fluid in a Darcy-Brinkman porous medium was studied by Rana [22]. The Bénard problem (the onset of convection in a horizontal layer uniformly heated from below) for an incompressible Rivlin-Ericksen rotating fluid permeated with suspended particles and variable gravity field in porous medium was studied by Rana and Kumar [23]. The corresponding problem for a compressible Rivlin-Ericksen rotating fluid permeated with suspended dust particles in porous medium has been studied by Rana [24]. In the present paper, the study is extended to thermosolutal convection in Rivlin-Ericksen rotating fluid permeated with suspended particles in a Darcy-Brinkman porous medium. This necessitates the inclusion of two additional parameters, namely stable solute gradient and Darcy number.

2. Mathematical Model and Perturbation Equations

Here, we consider an infinite, horizontal, incompressible Rivlin-Ericksen elastico-viscous fluid layer of depth d, bounded by the planes z = 0 and z = d in an isotropic and homogeneous medium of porosity ε and permeability k_1, which is acted upon by gravity g(0, 0, -g) as shown in Fig. 1. This layer is heated and soluted from below such that a uniform temperature gradient $\beta\left(=\left|\frac{dT}{dz}\right|\right)$ and a uniform solute gradient $\beta'\left(=\left|\frac{dC}{dz}\right|\right)$ are maintained. The character of equilibrium of this initial static state is determined by supposing that the system is slightly disturbed and then following its further evolution.

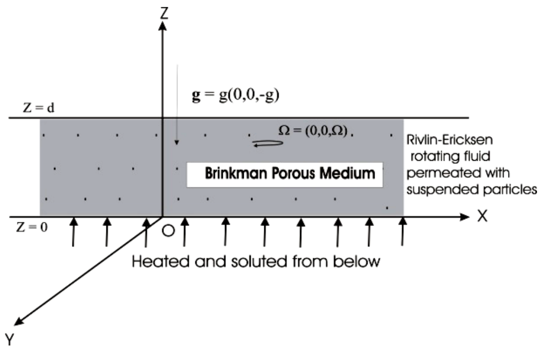

Fig. 1. Schematic Sketch of physical situation

Let ρ, υ, υ', p, ε, T, α, α', and $\boldsymbol{v}(0, 0, 0)$, denote respectively, the density, kinematic viscosity, kinematic viscoelasticity, pressure, medium porosity, temperature, thermal coefficient of expansion, an analogous solvent coefficient of expansion and velocity of the fluid.

The equations expressing the conservation of momentum, mass, temperature and concentration for Rivlin-Ericksen elastico-viscous fluid (Chandrasekhar [1], Sharma and Rana [14], Kuznetsov and Nield [20], Rana [22]) in a Brinkman porous medium are

$$\frac{\rho}{\varepsilon}\left[\frac{\partial v}{\partial t}+\frac{1}{\varepsilon}(\boldsymbol{v}.\nabla)\boldsymbol{v}\right]=-\nabla p+\boldsymbol{g}\rho-\frac{1}{k_1}\left(\mu+\mu'\frac{\partial}{\partial t}\right)\boldsymbol{v}+\tilde{\mu}\nabla^2\boldsymbol{v}+\frac{K'N}{\varepsilon}(\boldsymbol{v_d}-\boldsymbol{v})+\frac{2\rho}{\varepsilon}(\boldsymbol{v}\times\Omega), \tag{1}$$

$$\nabla.v=0, \tag{2}$$

$$E\frac{\partial T}{\partial t}+(\boldsymbol{v}.\nabla)T+\frac{mNC_{pt}}{\rho C_f}\left[\varepsilon\frac{\partial}{\partial t}+v_d.\nabla\right]T=\kappa\,\nabla^2\mathrm{T}, \tag{3}$$

$$E'\frac{\partial C}{\partial t}+(\boldsymbol{q}.\nabla)C+\frac{mNC_{pt}'}{\rho C_f}\left[\in\frac{\partial}{\partial t}+\boldsymbol{q_d}.\nabla\right]T=\kappa'\,\nabla^2\mathrm{C}, \tag{4}$$

where $v_d(\bar{x},t)$ and $N(\bar{x},t)$ denote the velocity and number density of the particles respectively, $K' = 6\pi\eta\rho\upsilon$, where η is particle radius, is the Stokes drag coefficient, $v_d = (l,r,s)$ and $\bar{x} = (x,y,z)$. Here

$$E = \varepsilon + (1-\varepsilon)\left(\frac{\rho_s C_s}{\rho_0 C_f}\right)$$

which is constant, κ is the thermal diffusivity and $\tilde{\mu}$ is effective viscosity of porous medium; ρ_s, C_s; ρ_0, C_f denote the density and heat capacity of solid (porous) matrix and fluid respectively and E' is a constant analogous to E but corresponding to solute rather than heat; κ, κ' are the thermal diffusivity and solute diffusivity respectively.

The equation of state is

$$\rho = \rho_0[1-\alpha(T-T_0)+\alpha'(C-C_0)], \tag{5}$$

where the suffix zero refers to values at the reference level z = 0.

If mN is the mass of particles per unit volume, then the equations of motion and continuity for the particles are

$$mN\left[\frac{\partial \boldsymbol{v_d}}{\partial t} + \frac{1}{\varepsilon}(v_d.\nabla)\boldsymbol{v_d}\right] = K'N(\boldsymbol{v}-\boldsymbol{v_d}), \tag{6}$$

$$\varepsilon\frac{\partial N}{\partial t} + \nabla.(N\boldsymbol{v_d}) = 0, \tag{7}$$

The presence of particles adds an extra force term proportional to the velocity difference between particles and fluid and appears in the equation of motion (1). Since the force exerted by the fluid on the particles is equal and opposite to that exerted by the particles on the fluid, there must be an extra force term, equal in magnitude but opposite in sign, in the equations of motion for the particles (6). The buoyancy force on the particles is neglected. Interparticles reactions are not considered either since we assume that the distance between the particles is quite large compared with their diameters. These assumptions have been used in writing the equations of motion (6) for the particles. The initial state of the system is taken to be quiescent layer (no settling) with a uniform particle distribution number N_0. The initial state is

$$\boldsymbol{v} = (0,0,0), \boldsymbol{v_d} = (0,0,0), \quad T = -\beta z + T_0,\ C = -\beta' z + C_0,$$
$$\rho = \rho_0(1+\alpha\beta z - \alpha'\beta' z),\ N_0 = \text{constant} \tag{8}$$

is an exact solution to the governing equations.

Let $\boldsymbol{v}$(u,v,w), θ, $\gamma, \delta p$ and $\delta\rho$ denote, respectively, the perturbations in fluid velocity $\boldsymbol{v}$(0,0,0), temperature T, solute concentration C, pressure p and density ρ.

Therefore, the change in density $\delta\rho$ caused by perturbation θ and γ in temperature is given by

$$\delta\rho = -\rho_0(\alpha\theta - \alpha'\gamma). \tag{9}$$

The linearized perturbation equations governing the motion of fluid are

$$\frac{1}{\varepsilon}\frac{\partial \boldsymbol{v}}{\partial t} = -\frac{1}{\rho_m}\nabla\delta p - g(\alpha\theta - \alpha'\gamma) - \frac{1}{k_1}\left(\upsilon + \upsilon'\frac{\partial}{\partial t}\right)\boldsymbol{v} + \frac{\tilde{\mu}}{\rho_0}\nabla^2\boldsymbol{v} + \frac{K'N_0}{\rho_0\in}(\boldsymbol{v_d}-\boldsymbol{v}) + \frac{2}{\varepsilon}(\boldsymbol{v}\times\Omega), \tag{10}$$

$$\nabla.\boldsymbol{v} = 0, \tag{11}$$

$$(E+b\varepsilon)\frac{\partial\theta}{\partial t} = \beta(w+bs) + \kappa\nabla^2\theta, \tag{12}$$

$$(E'+b'\in)\frac{\partial\gamma}{\partial t} = \beta'(w+b's) + \kappa'\nabla^2\gamma, \tag{13}$$

where $\upsilon = \frac{\mu}{\rho_0}$, $\upsilon' = \frac{\mu'}{\rho_0}$ and w stand for kinematic viscocity, kinematic viscoelasticity, thermal diffusivity and vertical fluid velocity, respectively, $b = \frac{mN_0C_{pt}}{\rho_0 C_f}$, $b' = \frac{mNC_{pt'}}{\rho_0 C_{f'}}$ and w, s are the vertical fluid and particles velocity.

3. The Dispersion Relation

Following the normal mode analyses, we assume that the perturbation quantities have x, y and t dependence of the form

$$[w,s,\zeta,\theta,\gamma] = [W(z),S(z),Z(z),\Theta(z),\Gamma(z)]exp(ilx + imy + nt), \tag{14}$$

where l and m are the wave numbers in the x and y directions, $k = (l^2+m^2)^{1/2}$ is the resultant wave number and n is the frequency of the harmonic disturbance, which in general is a complex constant.

Using equation (14), equations (10)-(13) in non-dimensional form, become

$$\left[1+\left(\frac{P_l}{\varepsilon}+\frac{MP_l}{\varepsilon(1+\tau_1\sigma)}+F\right)\sigma - D_A(D^2-a^2)\right](D^2-a^2)W + \frac{2\Omega d^3 P_l}{\in\upsilon}DZ + \frac{ga^2d^2P_l\alpha}{\upsilon}\Theta = 0, \tag{15}$$

$$\left[1+\left(\frac{P_l}{\varepsilon}+\frac{MP_l}{\varepsilon(1+\tau_1\sigma)}+F\right)\sigma - D_A(D^2-a^2)\right]Z = \left(\frac{2\Omega d}{\in\upsilon}\right)P_l DW, \tag{16}$$

$$[D^2-a^2-E_1P_r\sigma]\Theta = -\frac{\beta d^2}{\kappa}\left(\frac{B+\tau_1\sigma}{1+\tau_1\sigma}\right)W, \tag{17}$$

$$[D^2-a^2-E_1'S_c\sigma]\Gamma = -\left(\frac{\beta' d^2}{\kappa'}\right)\left(\frac{B'+\tau_1\sigma}{1+\tau_1\sigma}\right)W, \tag{18}$$

where we have put

$a = kd, \sigma = \frac{nd^2}{\upsilon}$, $F = \frac{\upsilon'}{d^2}$, $E_1 = E + b\varepsilon$, $E_1' = E' + b'\varepsilon$, $\tau = \frac{m}{K'}$, $\tau_1 = \frac{\tau\upsilon}{d^2}$, $M = \frac{mN_0}{\rho_0}$, B=b+1 and $P_l = \frac{k_1}{d^2}$ is the dimensionless medium permeability, $P_r = \frac{\upsilon}{\kappa}$, is the thermal Prandtl number, $S_c = \frac{\upsilon}{\kappa'}$, is the

Schmidt number and $D_A = \frac{\tilde{\mu} k_1}{\mu d^2}$, is the Darcy number modified by the viscosity ratio and $D^* = d\frac{d}{dz}$ and the superscript * is suppressed.

Eliminating Θ, Γ and Z between equations (15)-(18), we obtain

$$\left[1 + \left(\frac{P_l}{\varepsilon} + \frac{MP_l}{\varepsilon(1+\tau_1\sigma)} + F\right)\sigma - D_A(D^2 - a^2)\right](D^2 - a^2)(D^2 - a^2 - E_1 P_r \sigma)(D^2 - a^2 - E_1{}' S_c \sigma)W - Ra^2 P_l \left(\frac{G-1}{G}\right)\left(\frac{B+\tau_1\sigma}{1+\tau_1\sigma}\right)(D^2 - a^2 - E_1{}' S_c \sigma)W + Sa^2\left(\frac{B'+\tau_1\sigma}{1+\tau_1\sigma}\right)(D^2 - a^2 - E_1 P_r \sigma)W + \frac{T_A P_l{}^2 (D^2-a^2-E_1 P_r\sigma)(D^2-a^2-E_1{}' S_c\sigma)}{1+\left(\frac{P_l}{\varepsilon}+\frac{MP_l}{\varepsilon(1+\tau_1\sigma)}+F\right)\sigma - D_A(D^2-a^2)} D^2 W = 0, \quad (19)$$

where $R = \frac{g\alpha\beta d^4}{\upsilon\kappa}$ is the thermal Rayleigh number, $S = \frac{g_0 \alpha' \beta' d^4}{\upsilon\kappa'}$ is the analogous solute Rayleigh numberand $T_A = \left(\frac{2\Omega d^2}{\varepsilon\upsilon}\right)^2$ is the modified Taylor number.

Here we assume that the temperature at the boundaries is kept fixed, the fluid layer is confined between two boundaries and adjoining medium is electrically non-conducting. The boundary conditions appropriate to the problem are [1]:

$$W = D^2W = D^4W = \Theta = 0 \text{ at } z=0 \text{ and } 1. \quad (20)$$

The case of two free boundaries, though slightly artificial is the most appropriate for stellar atmospheres. Using the boundary conditions (20), we can show that all the even order derivatives of W must vanish for z = 0 and z = 1 and hence the proper solution of W characterizing the lowest mode is

$$W = W_0 \sin \pi z; \; W_0 \text{ is a constant.} \quad (21)$$

Substituting equation (21) in (19), we get

$$R_1 x P = \left[1 + \left(\frac{P}{\varepsilon} + \frac{MP}{\varepsilon(1+\tau_1 i\sigma_1)} + \pi^2 F\right) i\sigma_1 + D_{A_1}(1+x)\right](1+x)(1+x+E_1 P_r i\sigma_1)\left(\frac{1+\tau_1\pi^2 i\sigma_1}{B+\tau_1\pi^2 i\sigma_1}\right) + \frac{S_1 x(1+x+E_1 P_r i\sigma_1)}{(1+x+E_1{}' S_c\sigma)}\left(\frac{B'+\tau_1\pi^2 i\sigma_1}{B+\tau_1\pi^2 i\sigma_1}\right) + \frac{T_{A_1} P^2 (1+x+E_1 P_r i\sigma_1)}{1+\left(\frac{P}{\varepsilon}+\frac{MP}{\varepsilon(1+\tau_1 i\sigma_1)}+\pi^2 F\right) i\sigma_1 + D_{A_1}(1+x)}\left(\frac{1+\tau_1\pi^2 i\sigma_1}{B+\tau_1\pi^2 i\sigma_1}\right), \quad (22)$$

where we have put

$$R_1 = \frac{R}{\pi^4}, \; S_1 = \frac{S}{\pi^4}, \; T_{A_1} = \frac{T_A}{\pi^4}, \; D_{A_1} = \frac{D_A}{\pi^2} x = \frac{a^2}{\pi^2}, \; i\sigma_1 = \frac{\sigma}{\pi^2}, \; P = \pi^2 P_l.$$

Equation (22) is required dispersion relation accounting for the onset of thermosolutalconvection in Rivlin-Ericksen rotating viscoelastic fluid permeated with suspended particles in a Darcy-Brinkman porous medium.

4. Stability of the System and Oscillatory Modes

Here, we examine the possibility of oscillatory modes, if any, in Rivlin-Ericksen viscoelastic fluid due to the presence of rotation, suspended particles, viscoelasticity, medium permeability and gravity field. Multiply equation (15) by W^* the complex conjugate of W, integrating over the range of z and making use of equations (16)-(18) with the help of boundary conditions (20), we obtain

$$\left[1 + \left(\frac{P_l}{\varepsilon} + \frac{MP_l}{\varepsilon(1+\tau_1\sigma)} + F\right)\sigma\right] I_1 - D_A(I_2 + d^2 I_5) - \frac{g a^2 \alpha\kappa P_l}{\upsilon\beta}\left(\frac{1+\tau_1\sigma^*}{B+\tau_1\sigma^*}\right)(I_3 + E P_r \sigma^* I_4) + d^2\left[1 + \left(\frac{P_l}{\varepsilon} + \frac{MP_l}{\varepsilon(1+\tau_1\sigma)} + F\right)\sigma^*\right] I_6 + \frac{\alpha' a^2 \lambda g_0 \kappa'}{\upsilon\beta'}\left(\frac{1+\tau_1\sigma^*}{B'+\tau_1\sigma^*}\right)(I_7 + E_1{}' S_c \sigma^* I_8) = 0, \quad (23)$$

where

$$I_1 = \int_0^1 (|DW|^2 + a^2|W|^2)\, dz,$$
$$I_2 = \int_0^1 (|DW|^4 + 2a^2|DW|^2 + a^4|W|^2)\, dz,$$
$$I_3 = \int_0^1 (|D\Theta|^2 + a^2|\Theta|^2)\, dz,$$
$$I_4 = \int_0^1 |\Theta|^2 dz,$$
$$I_5 = \int_0^1 (|DZ|^2 + a^2|Z|^2)\, dz,$$
$$I_6 = \int_0^1 |Z|^2 dz,$$
$$I_7 = \int_0^1 (|D\Gamma|^2 + a^2|\Gamma|^2)\, dz,$$
$$I_8 = \int_0^1 |\Gamma|^2 dz.$$

The integral part I_1-$I_8$are all positive definite. Putting $\sigma = i\sigma_i$ in equation (23), where σ_i is real and equating the imaginary parts, we obtain

$$\sigma_i\left[\left(\frac{P_l}{\varepsilon} + \frac{MP_l}{\varepsilon(1+\tau_1{}^2\sigma_i{}^2)} + F\right)(I_1 - d^2 I_6) + \frac{g a^2 \alpha\kappa P_l}{\upsilon\beta}\left\{\left(\frac{\tau_1(B-1)}{B^2+\tau_1{}^2\sigma_i{}^2}\right) I_3 + \left(\frac{\tau_1(B-1)}{B^2+\tau_1{}^2\sigma_i{}^2}\right) E p_1 I_4\right\} - \frac{\alpha' a^2 \lambda g_0 \kappa'}{\upsilon\beta'}\left\{\left(\frac{\tau_1(B'-1)}{B'^2+\tau_1{}^2\sigma_i{}^2}\right) I_7 + \frac{B'+\tau_1{}^2\sigma_i{}^2}{B'^2+\tau_1{}^2\sigma_i{}^2} E_1{}' S_c I_8\right\}\right] = 0, \quad (24)$$

Equation (24) implies that $\sigma_i = 0$ or $\sigma_i \neq 0$, which means that modes may be non-oscillatory or oscillatory. The oscillatory modes introduced are due to the presence of viscoelasticity, suspended particles and rotation which were non-existent in their absence.

5. The Stationary Convection

For stationary convection, putting $\sigma = 0$ in equations (22), we obtain

$$R_1 = \frac{1+x}{xB}\left[\frac{1+x}{P} + \frac{(1+x)^2 D_{A_1}}{P} + \frac{T_{A_1} P}{1+(1+x)D_{A_1}}\right] + \frac{S_1 B'}{B}. \quad (25)$$

Equation (25) expresses the modified Rayleigh number R_1 as a function of the dimensionless wave number x and the parameters T_{A_1}, S_1, B, B',D_{A_1}, P and Rivlin-Ericksen viscoelastic fluid behave like an ordinary Newtonian fluid since elastico-viscous parameter F vanishes with σ.

To study the effects of stable solute gradient, rotation, Darcy number, suspended particles and medium permeability, we examine the behavior of $\frac{\partial R_1}{\partial S_1}$, $\frac{\partial R_1}{\partial T_{A_1}}$, $\frac{\partial R_1}{\partial D_{A_1}}$, $\frac{\partial R_1}{\partial B}$ and $\frac{\partial R_1}{\partial P}$ analytically and numerically.

From equation (25), we get

$$\frac{\partial R_1}{\partial S_1} = \frac{B'}{B}, \tag{26}$$

which is positive, implying thereby the stable solute gradient has a stabilizing effect. Also in Fig. 2, R_1 increases with the increase in S_1 which clearly verifies the result numerically.

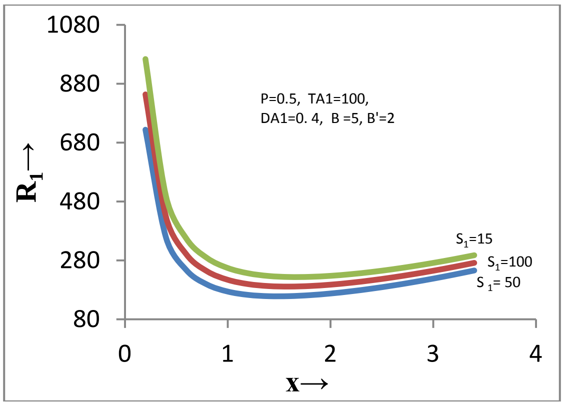

Fig. 2. Variation of Rayleigh number R_1 with dimensionless wave number x for different values of Stable solute gradient S_1

From equation (25), we obtain

$$\frac{\partial R_1}{\partial T_{A_1}} = \frac{1+x}{xB}\left(\frac{P}{1+(1+x)D_{A_1}}\right), \tag{27}$$

which is positive, implying thereby the stabilizing effect of rotation on the thermosolutal convection in Rivlin-Ericksen rotating fluid permeated with suspended particles in a Brinkman porous medium, which is in agreement with the result derived by Rana [24] and Sharma and Rana [14, 16]. Also, in Fig. 3, R_1 increases with the increase in T_{A_1} which clearly verifies the result numerically.

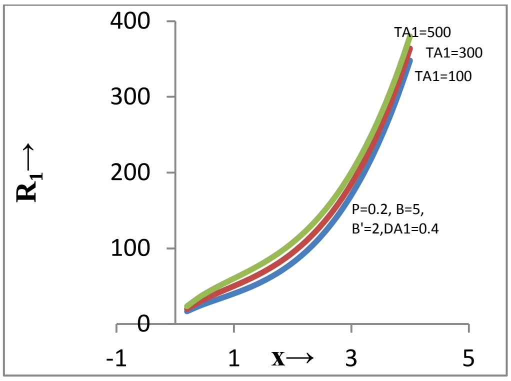

Fig. 3. Variation of Rayleigh number R_1 with dimensionless wave number x for different values of Taylor number T_{A_1}

From equation (35), we get

$$\frac{\partial R_1}{\partial D_{A_1}} = \frac{(1+x)^2}{xB}\left[\frac{1+x}{P} - \frac{T_{A_1}P}{\{1+(1+x)D_{A_1}\}^2}\right], \tag{28}$$

From equation (28), we found that the modified Darcy number has a stabilizing effect if

$$\frac{1+x}{P} > \frac{T_{A_1}P}{\{1+(1+x)D_{A_1}\}^2},$$

and destabilizing effect if

$$\frac{1+x}{P} < \frac{T_{A_1}P}{\{1+(1+x)D_{A_1}\}^2},$$

on the thermosolutal convection in Rivlin-Ericksen viscoelastic rotating fluid is permeated with suspended particles in a Brinkman porous medium. Also, in Fig. 4, R_1 increases/decreases with the increase in D_{A_1}. Thus Darcy number has stabilizing/destabilizing effects, which clearly verify the result numerically. However, in the absence of rotation the modified Darcy number has a stabilizing effect which is an agreement with the result derived by Kuznetsov and Nield [20] and Rana [22].

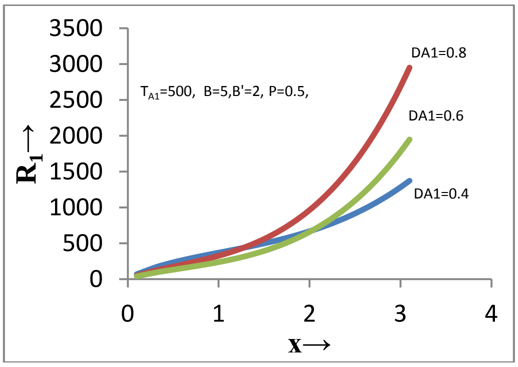

Fig. 4. Variation of Rayleigh number R_1 with dimensionless wave number x for different values of Darcy number D_{A_1}

From equation (25), we get

$$\frac{\partial R_1}{\partial B} = -\frac{1+x}{xB^2}\left[\frac{1+x}{P} + \frac{(1+x)^2 D_{A_1}}{P} + \frac{T_{A_1}P}{1+(1+x)D_{A_1}}\right] - \frac{S_1 B'}{B^2}, \quad (29)$$

which is negative. Hence, suspended particles have a destabilizing effect on the thermosolutal convection in Rivlin-Ericksen viscoelastic fluid in a Brinkman porous medium. This destabilizing effect is in agreement with the earlier work of Scanlon and Segel [6], Rana and Kumar [23] and Rana [24]. Also, in Fig. 5, R_1 decreases with the increase in suspended particles parameter B. Hence, suspended particles have a destabilizing effect, which clearly verifies the result numerically.

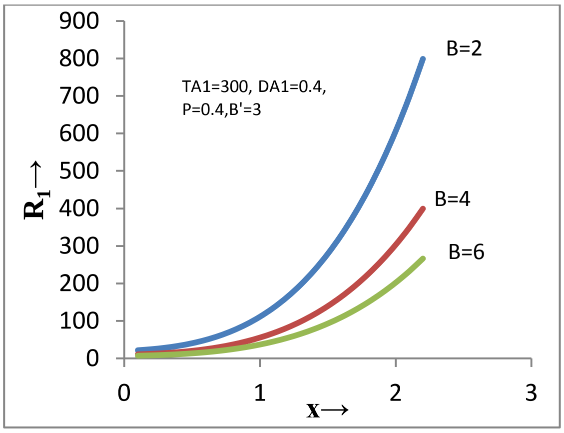

Fig. 5. Variation of Rayleigh number R_1 with dimensionless wave number x for different values of suspended particles parameter B

It is evident from equation (25) that

$$\frac{\partial R_1}{\partial P} = -\frac{1+x}{xB}\left[\frac{1+x}{P^2} + \frac{(1+x)^2 D_{A_1}}{P^2} - \frac{T_{A_1}}{1+(1+x)D_{A_1}}\right], \quad (30)$$

From equation (30), we found that the medium permeability has a stabilizing effect if

$$\frac{1+x}{P^2} + \frac{(1+x)^2 D_{A_1}}{P^2} < \frac{T_{A_1}}{1+(1+x)D_{A_1}},$$

and destabilizing effect if

$$\frac{1+x}{P^2} + \frac{(1+x)^2 D_{A_1}}{P^2} > \frac{T_{A_1}}{1+(1+x)D_{A_1}},$$

on the thermosolutal convection in Rivlin-Ericksen viscoelastic fluid permeated with suspended particles in a Brinkman porous medium. This destabilizing effect is in agreement with the earlier work of Scanlon and Segel [6], Sharma and Rana [14, 16], Rana [22, 24]. Also in Fig. 6, R_1 increase/decrease with the increase in medium permeability parameter P. Hence, medium permeability has stabilizing/destabilizing effects, which clearly verify the result numerically.

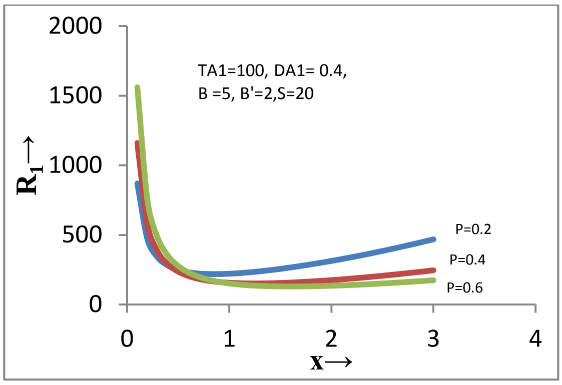

Fig. 6. Variation of Rayleigh number $R_1$with dimensionless wave number x for different values of medium permeability P

6. Conclusion

The combined effect of suspended particles and rotation on thermosolutal convection in Rivlin-Ericksen viscoelastic fluid in a Brinkman porous medium has been investigated. The dispersion relation, including the effects of suspended particles, Darcy number, medium permeability and viscoelasticity on the thermosolutal convection in Rivlin-Ericksen fluid in porous medium is derived. From the analysis, the main conclusions are as follows:

(i)For the case of stationary convection, Rivlin-Ericksen viscoelastic fluid behaves like an ordinary Newtonian fluid.

(ii)For the case of stationary convection, the expressions for $\frac{\partial R_1}{\partial S_1}$, $\frac{\partial R_1}{\partial T_{A_1}}$, $\frac{\partial R_1}{\partial D_{A_1}}$, $\frac{\partial R_1}{\partial B}$ and $\frac{\partial R_1}{\partial P}$ are examined analytically and it has been found that the stable solute gradient, rotation have stabilizing effect and suspended particles have a destabilizing effect on the system whereas Darcy number has stabilizing / destabilizing effect on the system if

$$(1+x)\{1+(1+x)D_{A_1}\}^2 > T_{A_1}P^{2/}(1+x)\{1+(1+x)D_{A_1}\}^2 < T_{A_1}P^2$$

effect and medium permeability has a stabilizing / destabilizing effect on the system if

$$(1+x)\{1+(1+x)D_{A_1}\}^2 < T_{A_1}P^2/(1+x)\{1+(1+x)D_{A_1}\}^2 > T_{A_1}P^2.$$

(iii)The effects of stable solute gradient, rotation, suspended particles, Darcy number and medium permeability on thermal instability of Rivlin-Ericksen viscoelastic fluid permeated with suspended particles in a Brinkman porous medium have also been shown graphically in Figs. 2-6 respectively.

(iv)The oscillatory modes are introduced due to presence of rotation, stable solute gradient, viscoelasticity, suspended particles, and gravity field, which were non-existent in their absence.

Nomenclature

P_l	Dimensionless medium permeability
g	Gravitational acceleration
G	Gravitational acceleration vector
m	Mass of suspended particle
D_A	Modified Darcy number
p	Pressure
K'	Stokes drag coefficient
N	Suspended particles number density
p_1	Thermal Prandtl number
v	Velocity of fluid
v_d	Velocity of suspended particles
k	Wave number of disturbance

Greek Symbols

β	Adverse temperature gradient
$\tilde{\mu}$	Effective viscosity of the porous medium
$\boldsymbol{\rho}$	Fluid density
μ	Fluid viscosity
μ'	Fluid viscoelasticity
υ	Kinematic viscosity
υ'	Kinematic viscoelasticity
ε	Medium porosity
δ	Perturbation in respective physical quantity
θ	Perturbation in temperature
γ	perturbation in solute concentration
η	Radius of suspended particles
κ'	Solute diffusivity
β'	Solute gradient
α'	Solvent coefficient of expansion
β	Temperature gradient
κ	Thermal diffusivity
α	Thermal coefficient of expansion
ζ	z-component of vorticity

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