

Research Article

Evaluation of a Micromechanism-Inspired Ductile Fracture Criterion for Formability and Fracture Prediction in Single Point Incremental Forming of AA1050 Aluminum Sheet

Seyed Amin Teymouri Sendesi¹ , Mohammad Mehdi Kasaei^{1, 2} , Majid Majidi^{1*}  and Seyed Jalal Hashemi³ 

¹ Department of Mechanical Engineering, Qa. C., Islamic Azad University, Qazvin, Iran

² Institute of Science and Innovation in Mechanical and Industrial Engineering (INEGI), Porto, Portugal

³ Department of Mechanical Engineering, Technical and Vocational University (TVU), Tehran, Iran

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ABSTRACT

This study presents an experimental and numerical investigation of ductile fracture prediction in the single-point incremental forming (SPIF) of AA1050 aluminum sheets. The mechanical properties of the material were determined using uniaxial tensile tests, and the combined Swift-Voce hardening law was employed to define the plastic behavior of the material at strains beyond the onset of necking. The micromechanism-inspired ductile fracture criterion DF2014 was calibrated using uniaxial, plane-strain, in-plane shear, and notched tensile tests through the curve fitting toolbox in MATLAB and implemented in ABAQUS/Explicit 2022 through a VUSDFLD user subroutine, incorporating both linear and nonlinear damage accumulation rules. SPIF simulations were performed for a straight groove, a truncated cone, and a truncated pyramid, both with variable wall angles, and validated against experimental measurements of in-plane principal strains and thickness variations. The results demonstrate that incorporating the nonlinear damage accumulation rule, which accounts for the non-proportional loading history and evolving stress state during SPIF, substantially improves fracture prediction compared with the linear rule. The resulting predicted fracture initiation depths for the truncated cone and truncated pyramid agreed well with the experimental results, with errors of 6.1% and 4.6%, respectively. On the other hand, the variable cut-off value for stress triaxiality in the DF2014 criterion and its ability to predict fracture under stress states with negative triaxiality did not have a significant effect on the results due to the predominance of tensile stress states during the forming of the truncated cone and truncated pyramid.

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1. Introduction

Incremental sheet forming (ISF) is an advanced sheet forming process that produces complex and asymmetric geometries through localized, incremental deformation

using a forming tool. Its flexibility, low tooling requirements, and suitability for rapid prototyping and customized production make it attractive compared with conventional forming methods [1].

* Corresponding author

E-mail address: majid.majidi@iau.ac.ir (M. Majidi)

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The influence of various process parameters in incremental forming has been investigated by many researchers, with particular attention to their effects on formability, surface roughness, dimensional accuracy, and forming forces. Gheysarian and Honarpisheh [2] investigated the effects of tool diameter, step size, tool path, and sheet arrangement on the formability and forming forces of Al/Cu bimetal sheets in SPIF. The results showed that smaller tool diameters and step sizes improved formability, while the spiral tool path increased formability by about 16%. Statistical analysis confirmed the reliability of the developed models and identified the significant process parameters. Cusanno et al. [3] investigated the effects of feed rate, spindle speed, and step down on fracture depth, forming load, and surface roughness in SPIF. The results showed that higher spindle speed increased forming depth, while lower step down improved surface quality. They also identified process parameter combinations that provided a suitable balance among these performance indicators. Kashi et al. [4] investigated the effects of tool rotational speed, vertical step size, annealing condition, and forming direction on the formability of AA3105-H14 sheets. The results showed that vertical step size and annealing condition were the most influential parameters. Increasing spindle speed and annealing also improved formability and reduced localized thinning. Farhadi et al. [5] investigated the thickness distribution in a three-layer aluminum polyethylene sheet formed by TPIF. The upper aluminum layer showed greater thinning due to direct tool contact, while the PE core reduced deformation in the lower layer. Higher feed rates and step depths increased thinning, whereas higher spindle speeds reduced localized thinning.

Fracture prediction in forming processes has always attracted considerable attention from researchers due to its potential to reduce costs and facilitate proper process design. In incremental forming, many studies have also investigated the forming limits of various alloys and the capability of ductile fracture criteria to predict fracture. Ghane Shalmani et al. [6] developed a three-dimensional M-K model to construct a generalized forming limit diagram (GFLD) considering normal and through-

thickness shear stresses. Their results showed that out-of-plane stresses significantly affect deformation in SPIF and that the isotropic GFLD provides a reliable tool for fracture prediction. Alsharif [7] investigated damage initiation and crack propagation in DP1000 steel using scanning electron microscopy (SEM) bending tests combined with digital image correlation (DIC) and FE modeling. The results showed good agreement between experimental and numerical responses and provided insight into crack initiation, propagation, and microscale damage mechanisms. Mirnia and Shamsari [8] applied the phenomenological modified Mohr-Coulomb (MMC3) model with nonlinear damage accumulation to simulate fracture in SPIF, demonstrating agreement with experimental data. Wang et al. [9] evaluated six ductile damage models for failure prediction in two-point incremental forming (TPIF) and found that the Ayada model provided the most accurate predictions of fracture depth and location. They also showed that double-sided contact affects damage accumulation by reducing stress triaxiality variation through the sheet thickness. Bharti et al. [10] used ISF to produce three parts with varied geometries and evaluated failure predictions using uncoupled phenomenological damage models: Bao-Wierzbicki (BW), Hosford-Coulomb (HC), and modified Mohr-Coulomb (MMC). They concluded that the HC model provided the most accurate prediction of fracture for the investigated loading conditions. Kumar et al. [11] implemented a shear-modified Lemaitre model to predict fracture during incremental forming of truncated cones with different wall angles, showing accurate predictions of both fracture depth and cone wall angle. Alhalaybeh et al. [12] investigated the fracture behavior of a magnesium alloy under SPIF and found that the fracture models effectively captured fracture behavior under multiaxial loading and provided accurate predictions of fracture strain. Qin et al. [13] investigated the ductile fracture of AA1050 in ISF by incorporating temperature effects into the GTN damage model. They developed a temperature-dependent void volume fraction (VVF) function and validated the extended model using FE simulations and experiments. The results showed improved predictions of fracture depth,

location, and formability under varying temperatures. Teymouri et al. [14] evaluated the DF2016 and MMC3 fracture criteria for predicting crack initiation in AA1050 during SPIF. Both criteria showed good agreement with experimental results, while DF2016 provided higher predictive accuracy due to its better representation of stress triaxiality and the Lode parameter. Panich and Amree [15] compared the Lou-Huh and MMC fracture criteria for AA2024-T3 sheets using experimental and numerical approaches. The MMC criterion provided more accurate predictions of fracture strain, while the Lou-Huh model better predicted drawing depth, fracture shape, and location. They also found that fracture loci (FLs) were more effective than FFLC and FFLSC in predicting fracture initiation. Habbachi et al. [16] investigated fracture in Al5754 during SPIF using the Johnson-Cook damage model combined with material anisotropy and Swift-Voce hardening. The model achieved a fracture prediction error of approximately 7.5%, with fracture mainly governed by void nucleation, growth, and coalescence.

The present work specifically examines the predictive capability of the DF2014 ductile fracture criterion for aluminum sheets, with a focus on how linear and nonlinear damage accumulation rules affect its performance under the incremental, non-proportional loading paths typical of SPIF. The criterion parameters are calibrated through a series of tensile tests covering distinct stress states and are then integrated into ABAQUS/Explicit using a VUSDFLD subroutine to simulate fracture initiation in AA1050 sheets. The accuracy of the simulations is evaluated by comparing the predicted and experimental strain distributions and thickness variations. The capability of the DF2014 criterion is assessed based on the predicted and measured fracture depths and crack initiation locations. The effect of the variable cut-off value for stress triaxiality of this criterion on fracture prediction in the SPIF process was also investigated. This approach provides a systematic evaluation of the DF2014 criterion combined with linear and nonlinear damage accumulation rules for predicting

fracture initiation under the complex and continuously evolving stress states characteristic of SPIF.

2. Ductile Fracture Criterion

Ductile fracture in metallic materials is a progressive failure process governed by the nucleation, growth, and coalescence of microscopic voids induced by plastic deformation. During metal forming, microscopic defects are first generated around inclusions, second-phase particles, or grain boundaries. As plastic deformation proceeds, these voids enlarge under the action of hydrostatic tensile stress and eventually coalesce, leading to the initiation of a macroscopic crack. Therefore, the onset of ductile fracture is strongly dependent on the prevailing stress state rather than on the accumulated plastic strain alone. Among the stress-state variables affecting ductile fracture, stress triaxiality has been recognized as one of the most influential parameters because it governs the growth of microscopic voids through the hydrostatic stress component. Stress triaxiality is defined as the ratio of the mean (hydrostatic) stress to the von Mises equivalent stress:

$$\eta = \frac{\sigma_m}{\bar{\sigma}} = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3\bar{\sigma}} \quad (1)$$

Where σ_1 , σ_2 , and σ_3 denote the principal stresses, while the von Mises equivalent stress is expressed as:

$$\bar{\sigma} = \sqrt{\frac{1}{2}[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]} \quad (2)$$

An increase in stress triaxiality promotes void growth by increasing the hydrostatic tensile stress, thereby reducing the fracture strain. Conversely, under compressive stress states, void growth is significantly suppressed because the hydrostatic stress becomes negative. Consequently, fracture cannot be accurately characterized solely by equivalent plastic strain, particularly under complex loading conditions encountered in metal forming processes. Although stress triaxiality plays a dominant role in ductile fracture, extensive experimental observations have demonstrated that materials subjected to identical stress triaxialities may exhibit significantly different fracture strains. This

discrepancy originates from differences in the deviatoric stress state, which cannot be represented by stress triaxiality alone. Therefore, an additional stress-state parameter is required to distinguish between different loading conditions sharing the same hydrostatic stress level. To overcome this limitation, the Lode parameter was introduced to characterize the influence of the intermediate principal stress on the fracture process. The Lode parameter is defined as:

$$L = \frac{2\sigma_2 - \sigma_1 - \sigma_3}{\sigma_1 - \sigma_3} \quad (3)$$

The Lode parameter governs the distribution of the deviatoric stresses and is closely related to the contribution of maximum shear stress to void coalescence. Consequently, incorporating the Lode parameter enables fracture criteria to distinguish between stress states having identical stress triaxialities but different shear characteristics. Based on these considerations, Lou et al. [17] proposed the DF2014 ductile fracture criterion, a micromechanism-inspired model motivated by the mechanisms of void nucleation, growth, and shear coalescence. The criterion incorporates both stress triaxiality and the Lode parameter into the fracture locus and is expressed as follows:

$$\left(\frac{2}{\sqrt{L^2+3}}\right)^{C_1} \left(\left\langle \frac{f(\eta, L, C)}{f(\frac{1}{3}, -1, C)} \right\rangle\right)^{C_2} \bar{\varepsilon}_f^p = C_3 \quad (4)$$

Where $\bar{\varepsilon}_f^p$ is the equivalent plastic strain to fracture, C_1 , C_2 , C_3 , and C are material constants determined experimentally. C_1 governs shear coalescence of voids, C_2 controls void growth, C_3 corresponds to the fracture strain under uniaxial tension, and C models the height of the cut-off value for stress triaxiality. A distinctive feature of the DF2014 criterion is the introduction of a cut-off value for stress triaxiality that is expressed as a function of the Lode parameter, thereby defining a variable cut-off stress triaxiality.

$$f(\eta, L, C) = \frac{\sigma_1}{\bar{\sigma}_{VM}} + C = \eta + \frac{3-L}{3\sqrt{L^2+3}} + C \quad (5)$$

Unlike conventional fracture criteria that assume a constant cut-off stress triaxiality, the DF2014 formulation allows this threshold to vary continuously with the Lode parameter. Consequently, the criterion can represent the influence of both the hydrostatic and deviatoric stress components over a much wider range of loading conditions. The parameter C represents the offset controlling the position of the cut-off stress triaxiality and reflects the material sensitivity to compressive stress states. Varying this parameter changes the fracture locus in the negative stress-triaxiality region and therefore affects the predicted fracture strain under loading conditions involving compressive stress components. This feature significantly extends the applicability of the DF2014 criterion compared with conventional stress-state-dependent fracture models employing a fixed cut-off value.

In practical metal forming operations such as single point incremental forming, the stress state continuously changes throughout the deformation process because of localized deformation and the repeated contact between the forming tool and the sheet. Consequently, the loading path is highly non-proportional, and fracture cannot be accurately predicted using a proportional-loading formulation. For this reason, the DF2014 criterion is employed in an incremental integral form:

$$\int_0^{\bar{\varepsilon}_f^p} \left(\frac{2}{\sqrt{L^2+3}}\right)^{C_1} \left(\left\langle \frac{f(\eta, L, C)}{f(\frac{1}{3}, -1, C)} \right\rangle\right)^{C_2} d\bar{\varepsilon}^p = C_3 \quad (6)$$

The integral formulation continuously updates the accumulated damage according to the instantaneous stress state experienced by each material point during deformation. Consequently, it provides a more realistic description of ductile fracture under non-proportional loading paths than formulations based on constant loading conditions, making it particularly suitable for numerical simulations of incremental sheet forming processes.

3. Experimentation

3.1. Material characterization

This study evaluates the predictive performance of the DF2014 ductile fracture criterion for fracture initiation

during the SPIF process. The investigated material is a 1-mm-thick AA1050 aluminum alloy sheet, which is widely used in industries such as food and pharmaceutical packaging. Its chemical composition is listed in Table 1.

To determine the mechanical properties of the sheet material, uniaxial tensile tests were conducted in accordance with ASTM-E8 standards. The results of these tests are summarized in Table 2. This material exhibits low elongation, making plastic deformation challenging in conventional metal forming processes. The uniaxial tensile test, however, provides an accurate stress-strain curve only up to the onset of diffuse necking. Beyond this stage, stress and strain distributions become non-uniform, requiring the extrapolation of pre-necking curves to predict post-necking behavior [18, 19].

A combination of the Swift and Voce hardening laws was used to extrapolate the true stress-strain curve to high strain values, as follows [20]:

$$\sigma = \lambda \left[K(\varepsilon_0 + \varepsilon_p)^n \right] + (1 - \lambda) [A + B(1 - e^{-C\varepsilon_p})] \quad (7)$$

In this hybrid model, K is the strength coefficient, n is the hardening exponent of the Swift law, and A , B , and C are the parameters of the Voce law. ε_0 represents the pre-strain, and λ is the weighting factor. The parameters of the Swift and Voce laws, except for the weighting factor λ , were determined by fitting the corresponding constitutive equations to the experimental true stress-plastic strain data up to the onset of necking, as identified from the uniaxial tensile test (Table 3). The fitting procedure was performed using the curve fitting toolbox in MATLAB. Subsequently, the weighting factor λ was

determined by fitting the combined Swift-Voce model, using the previously obtained parameters, to the experimental true stress-plastic strain curve. The fitting was performed by minimizing the difference between the predicted and experimental stress-strain data over the considered strain range.

Subsequently, the weighting factor λ was determined by fitting the combined Swift-Voce model, using the previously obtained parameters, to the experimental true stress-plastic strain curve. The Swift-Voce hardening model exhibited excellent agreement with the experimental data, with a coefficient of determination (R^2) of 0.992, ensuring a reliable description of the material behavior for numerical simulations (see Fig. 1).

In sheet metal forming, material anisotropy critically influences part formability. Consequently, extensive research has investigated how anisotropy affects deformation behavior and forming limits [21]. The plastic anisotropy ratio (plastic strain ratio), also known as Lankford coefficients or r -values, measures a material's resistance to thinning or thickening during deformation. These coefficients are defined as the ratio of width strain to thickness strain during a uniaxial tensile test (Eq. (8)) and are crucial for predicting how a material will behave during forming processes. The r -values are determined through uniaxial tensile tests, measured at an engineering strain level of $\varepsilon_{eng} = 80\%$, and conducted along three directions: the rolling direction (RD, 0°), the transverse direction (TD, 90°), and the diagonal direction (DD, 45°). During testing, the major strain (length direction) ε_l and minor strain (width direction) ε_w are measured. These strains were determined using a caliper with an accuracy of 0.02 mm. The through-thickness strain is calculated ε_t using the

Table 1. The chemical composition of the AA1050 aluminum alloy sheet

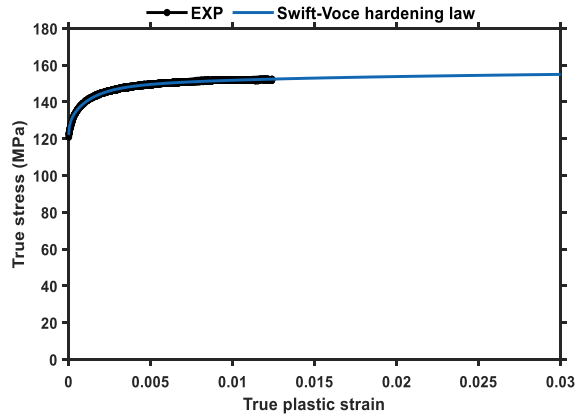
Element	Si	Fe	V	Mn	Cr	Zn	Ti	Others	Al
wt.%	0.03	0.05	0.03	0.03	<0.001	0.07	0.01	<0.005	Balance

Table 2. Mechanical properties of the AA1050 sheet obtained from a uniaxial tensile test conducted along the rolling direction

E (GPa)	YS (MPa)	UTS (MPa)	Poisson's ratio	Elongation (%)	
				Uniform	Fracture
70	120.9	152.3	0.33	1.45	2.3

Table 3. Calibrated parameters of the strain hardening law

K	n	λ	ε_0	A	B	c	R^2
181	0.037	0.46	8e-6	127.4	24	623.6	0.992

**Fig. 1.** The experimental true stress-true plastic strain curve with calibrated Swift-Voce hardening laws.

law of constant volume during deformation ($\varepsilon_l + \varepsilon_w + \varepsilon_t = 0$).

$$r = \frac{\varepsilon_w}{\varepsilon_t} \quad (8)$$

Where ε_w is the true strain perpendicular to the loading axis, and ε_t is the true strain through the thickness direction. Under the constant volume assumption, ε_t can be calculated by Eq. (9):

$$\varepsilon_t = -\varepsilon_l - \varepsilon_w \quad (9)$$

Due to its simplicity and high accuracy, the Hill's 1948 yield criterion (Eq. (10)) is widely used to predict the behavior of anisotropic sheets [22-24]. This quadratic yield function requires only a limited number of mechanical properties to determine the shape of the yield surface. Thus, the anisotropic behavior of the sheet metal was modeled using Hill's 1948 yield criterion [25, 26].

$$\sigma_h = \sqrt{\frac{F(\sigma_{22} - \sigma_{33})^2 + G(\sigma_{33} - \sigma_{11})^2 + H(\sigma_{11} - \sigma_{22})^2}{+ 2L\sigma_{23}^2 + 2M\sigma_{31}^2 + 2N\sigma_{12}^2}} \quad (10)$$

The constants F , G , H , L , M , and N are calculated based on the material's anisotropy parameters according to Eqs. (11-14):

$$F = \frac{1}{2} \left(\frac{1}{R_{22}^2} + \frac{1}{R_{23}^2} - \frac{1}{R_{11}^2} \right) \quad (11)$$

$$G = \frac{1}{2} \left(\frac{1}{R_{11}^2} + \frac{1}{R_{33}^2} - \frac{1}{R_{22}^2} \right) \quad (12)$$

$$H = \frac{1}{2} \left(\frac{1}{R_{11}^2} + \frac{1}{R_{22}^2} - \frac{1}{R_{33}^2} \right) \quad (13)$$

$$L = \frac{3}{2R_{23}^2}, M = \frac{3}{2R_{13}^2}, N = \frac{3}{2R_{12}^2} \quad (14)$$

Where R_{ij} are the anisotropy parameters, calculated based on the anisotropy coefficients obtained from the uniaxial tensile tests conducted in different directions, according to Eqs. (15-18):

$$R_{11} = R_{13} = R_{23} = 1 \quad (15)$$

$$R_{22} = \sqrt{\frac{r_{90}(r_0+1)}{r_0(r_{90}+1)}} \quad (16)$$

$$R_{33} = \sqrt{\frac{r_{90}(r_0+1)}{r_0+r_{90}}} \quad (17)$$

$$R_{12} = \sqrt{\frac{3r_{90}(r_0+1)}{(2r_{45}+1)(r_{90}+r_0)}} \quad (18)$$

After calculating the r -values using Eq. (8) and determining the anisotropy parameters from Eqs. (15-18), the constants of the Hill yield function were obtained from Eqs. (11-14) and are presented in Table 4, along with the Lankford coefficients.

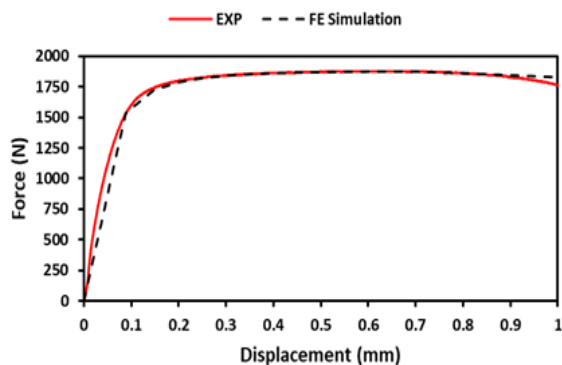
The quality of the material models was evaluated by comparing the numerical and experimental load-displacement curves of the uniaxial tensile test. As shown in Fig. 2, this comparison demonstrates good agreement between the two curves. Consequently, it can be concluded that the FE model of the uniaxial tensile test effectively predicts the material behavior.

3.2. Formability tests

Ductile fracture criteria rely on specific parameters that must be determined through experimental formability tests. For a multi-parameter criterion, it is essential to conduct experiments that cover a broad range of stress states [27]. To achieve this, uniaxial tension, in-plane shear, plane strain tension, and notched tensile tests along the rolling direction were conducted in this study

Table 4. Lankford coefficient and Hill 1948 parameters

Lankford coefficient, r			Hill 1948 parameters						
0°	45°	90°	F	G	H	N	L	M	
0.56	1.06	0.85	0.42	0.64	0.36	1.66	1.5	1.5	

**Fig. 2.** Comparison of the experimental and numerical force-displacement curves obtained from the uniaxial tensile test.

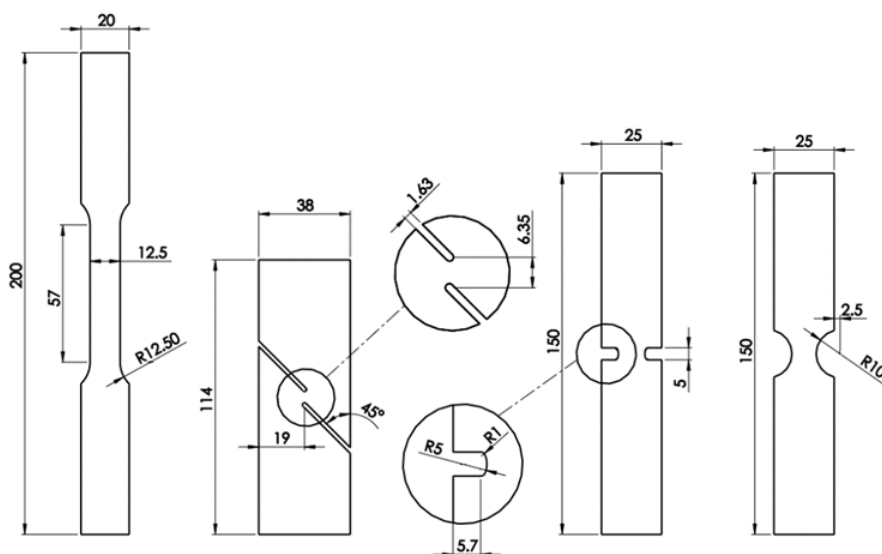
using specimens with various geometries (Fig. 3). The specimens were prepared using a laser cutting machine, and the tensile tests were performed on a Santam 40-ton universal testing machine. An extensometer with a gauge length of 50 mm was used to measure elongation in the longitudinal direction. The testing speed was set to 0.2 mm/min. At least three specimens were fabricated for each test to ensure the repeatability of the experiments.

3.3. SPIF experiments

To evaluate the performance of the DF2014 criterion,

parts with different geometries were fabricated using an incremental forming process. The locations and depths of the cracks that occurred in these specimens were compared with those predicted by the DF2014 criterion in the FE simulations. The equipment used for forming these parts included a thick-walled cylinder serving as the base structure (Fig. 4(a)), a backing plate with a hole shaped like the cross-section of the specimen to prevent undesirable bending and improve geometric accuracy (Fig. 4(b)), a blank holder to prevent sheet movement by clamping the periphery of the sheet (Fig. 4(c)), and a 10-mm-diameter hemispherical tool mounted on a three axis CNC milling machine (Fig. 4(d)). For these tests, three specimens of each part were also prepared to verify their repeatability.

A grid of interlaced circles with a diameter of 2.5 mm was stamped onto the blank to measure major and minor in-plane strains. To create this pattern, the sheet's surface was first coated with black paint, and the grid was then produced using a laser. The parts formed using the SPIF method include a truncated cone and a pyramid with variable wall angles and a depth of 30 mm (Fig. 5). The sheets were cut to dimensions of 120 mm $\times$ 120 mm. Tool paths were generated using PowerMill software as 3D spiral paths with a vertical pitch or step down of 1 mm. The tool feed rate was set to 500 mm/min, with no rotation, and mineral oil was used to lubricate the contact

**Fig. 3.** Dimensions of the specimens used in the tensile tests: from left to right, uniaxial tension, shear tension, plane strain tension, and notched tension.

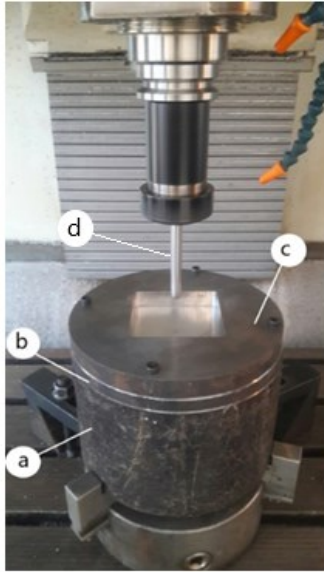


Fig. 4. Single-point incremental forming equipment: (a) base structure, (b) backing plate, (c) blank holder, and (d) hemispherical tool.

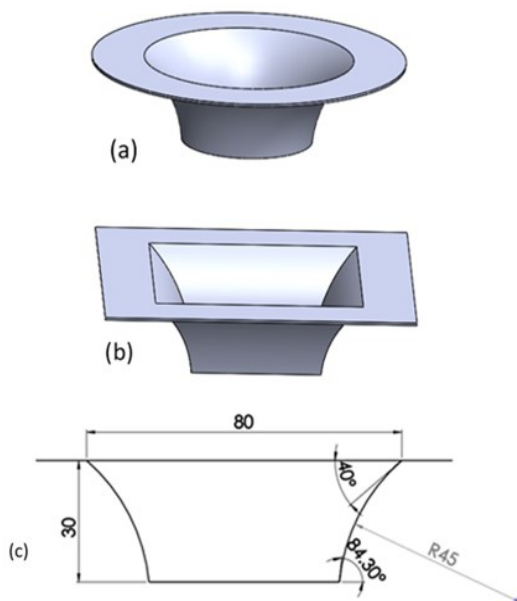


Fig. 5. Schematic presentation of parts formed by SPIF: (a) truncated cone, (b) truncated pyramid with variable wall angle, and (c) 2D sketch of both.

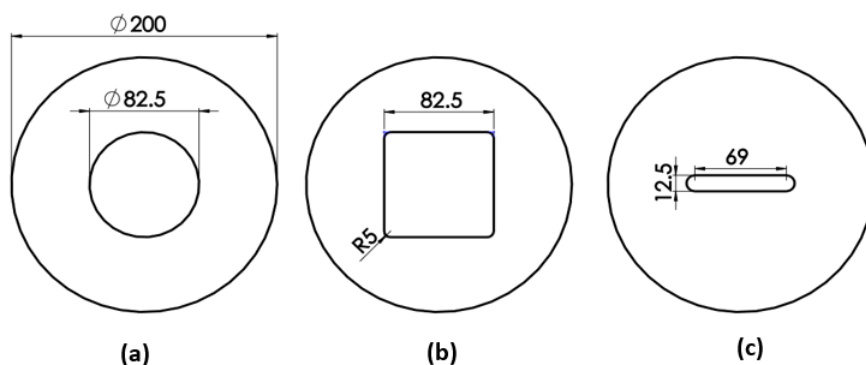


Fig. 6. The backing plates of (a) conic part, (b) pyramid part, and (c) straight groove part.

surface between the tool and the blank.

To investigate nonlinear damage accumulation caused by non-proportional loading in single-point incremental forming (SPIF), a straight-groove specimen was manufactured. A zigzag tool path with a length of 69 mm and a vertical pitch of 1 mm was employed for this specimen. Three backing plates were fabricated to generate three distinct part geometries (Fig. 6). In order to avoid tool-backing-plate interference during forming, the holes in the backing plates were designed with a 2.5 mm clearance relative to the final part dimensions.

Following the forming process, selected conical and pyramidal parts were sectioned, and their thickness distribution was measured using a micrometer with an accuracy of 0.01 mm. These experimental measurements were then compared with the thickness distribution predicted by the finite element (FE) simulation.

4. Finite Element Modeling

4.1. Formability tests

To obtain the material constants of the ductile fracture criterion considered in this study, FE modeling of tensile tests was performed using ABAQUS/Explicit 2022 software. All tests were fully modeled using 3D solid elements (C3D8R) with element sizes of 0.7 mm for the uniaxial test, 0.4 mm for the plane-strain and notched tensile tests, and 0.25 mm for the in-plane shear test, and six elements were used through the thickness for all tests in the critical area, based on the convergence of the fracture strain (Fig. 7). Elements outside the critical area were modeled with larger sizes to save analysis time. For all specimens, one side was fully constrained, while a slightly larger fracture displacement than that applied in

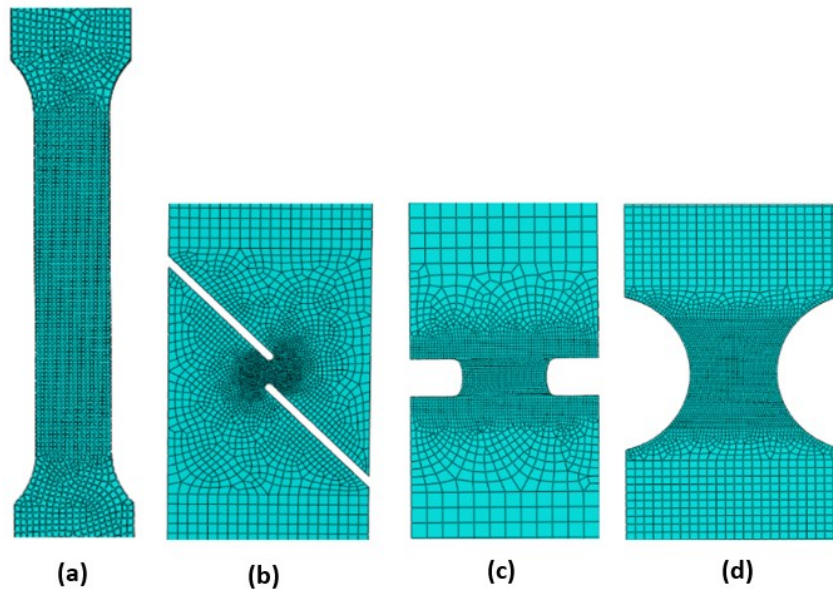


Fig. 7. The FE models of calibration tests: (a) uniaxial tension, (b) shear tension, (c) plane strain tension, and (d) notched tension.

the experimental tests was imposed on the other side. A dynamic explicit solver with a time scaling approach was employed to reduce simulation time, ensuring that the ratio of kinetic energy to internal energy was negligible. The material was modeled using an elastic-plastic model. The true stress-true plastic strain data, extrapolated to high strains using the Swift-Voce hardening law (Eq. (7)), were used to represent its plastic behavior. Additionally, the coefficients of Hill's 1948 yield criterion were employed to describe the anisotropy of the sheet during plastic deformation. Field variables such as damage indicator (D), stress triaxiality (η), and Lode parameter (L) were introduced into the model via the VUSDFLD subroutine.

4.2. Single point incremental forming

The incremental forming process was simulated using ABAQUS/Explicit 2022 software, and the results were analyzed to evaluate the effectiveness of the DF2014 criterion in predicting fractures in the formed parts. The S4R shell element was employed to model the sheet because of its capability to capture large deformations, accurately predicting thickness, and capture the stress state, while significantly reducing the computational cost compared with solid elements [8, 9]. In the truncated cone specimen, the edges of the sheet were fully

clamped, with no displacement or rotation allowed, and the backing plate was modeled as an analytical rigid surface. In contrast, the pyramidal and straight groove specimens were simulated without a backing plate, with the sheet clamped by applying appropriate boundary constraints that prevented any displacement or rotation (Fig. 8). The forming tool was modeled as an analytical rigid surface, and the interaction between the forming tool and the sheet was defined using the penalty contact formulation. Due to the use of mineral oil as a lubricant between the tool and the sheet, a low friction coefficient of 0.1 was assumed according to the Coulomb friction law [25]. The tool was moved without rotation along a path defined by coordinate points generated using PowerMill software in the X, Y, and Z directions. To reduce the computational time, the time scaling technique was employed so that the ratio of kinetic energy to internal energy remained very small. The element size of 0.5 mm was used for the blank, based on a convergence analysis of the mesh. The ductile fracture criterion was introduced into the model via the VUSDFLD subroutine, in the same manner as the field variables, and crack initiation was modeled using the element deletion technique when the damage function reached unity. Due to the low tool feed rate and the use of lubricant during the process, thermal effects were

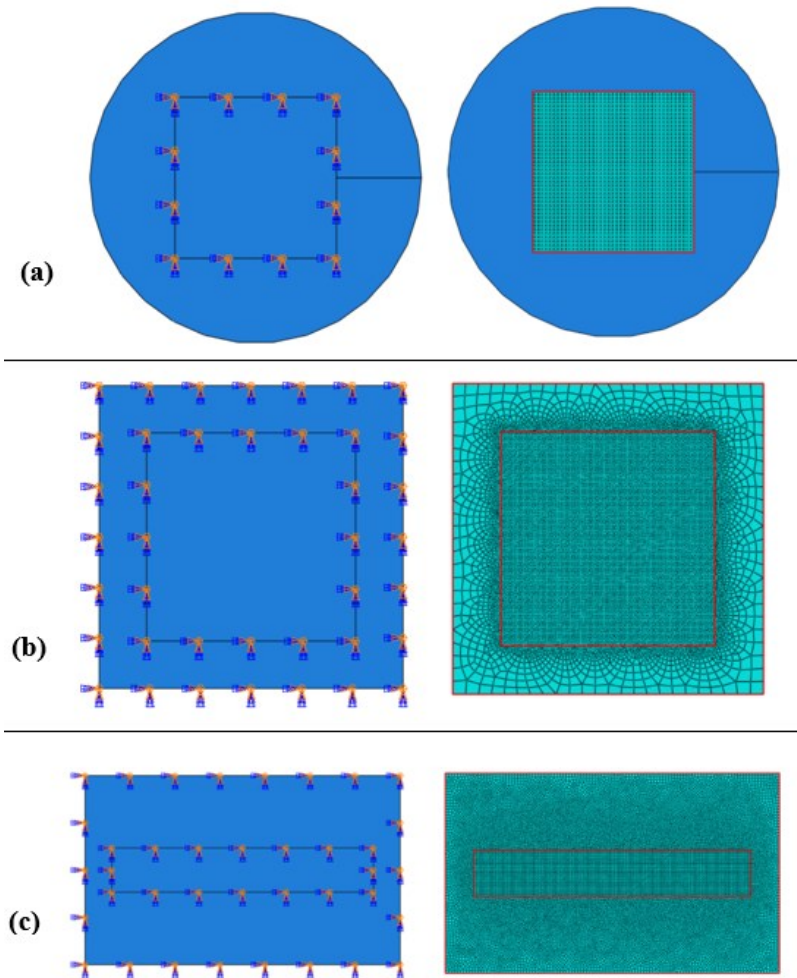


Fig. 8. Boundary constraints and finite models of the blanks for (a) conical, (b) pyramidal, and (c) straight groove specimens.

considered negligible and were therefore not included in the numerical model. It is worth noting that the material was modeled as described in section 3.1.

5. Calibration Method

In this study, a hybrid method combining experimental and numerical approaches was employed to calibrate the DF2014 criterion. After conducting the formability tests, the tests were simulated without considering the fracture criterion, up to the experimental fracture displacement determined by the extensometer during the experiments. Then, the force-displacement curve of each test was extracted and compared with the corresponding experimental force-displacement curve to evaluate the accuracy of the specimen modeling. As shown in Fig. 9, good agreement is observed between the experimental and FE-simulated tensile test results, demonstrating the

accuracy of the finite element simulations.

In the next step, the highest equivalent plastic strain was identified as the fracture strain. The element with the highest equivalent strain is critical for assessing material failure, as it represents the region with the most severe deformation and extreme conditions. Evaluating the Lode parameter and stress triaxiality in this element enables the prediction of failure initiation and progression, ensuring that the material models and fracture criterion reflect actual behavior under critical conditions. The evolution of stress triaxiality η and the Lode parameter L as functions of the equivalent plastic strain in the element with the highest equivalent strain is presented in Fig. 10.

As can be seen, both the Lode parameter and stress triaxiality are not constant but evolve with increasing equivalent plastic strain. Therefore, to properly calibrate

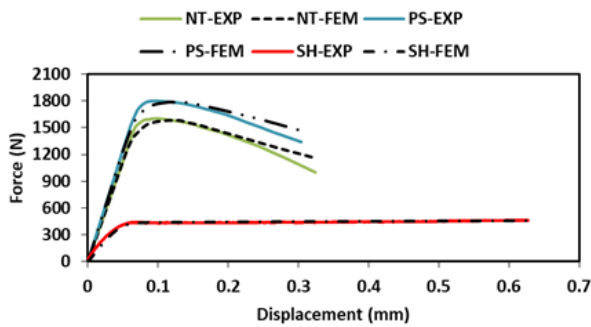


Fig. 9. Comparison between experimental and numerical force-displacement curves in the tensile tests.

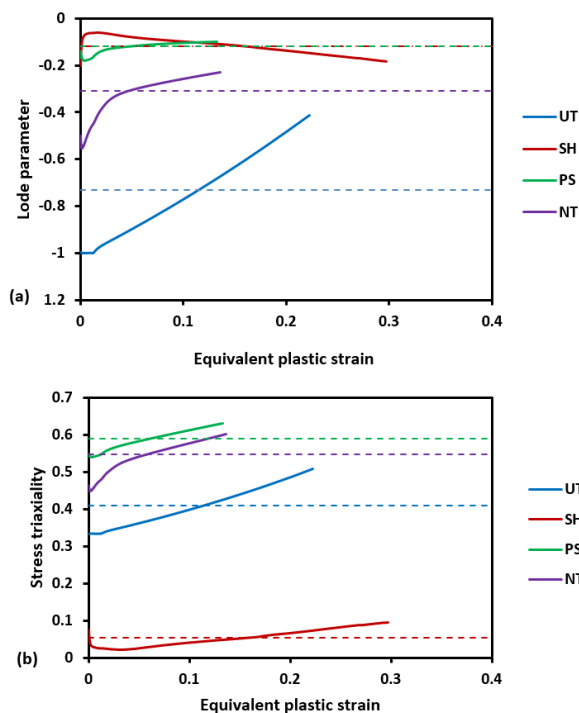


Fig. 10. Evolution of the Lode parameter (a) and stress triaxiality, (b) with their average values obtained from the tensile tests.

the ductile fracture criterion, it is necessary to use the average values of these parameters over the deformation history (Eqs. (19) and (20)).

$$\bar{\eta} = \frac{1}{\bar{\epsilon}_f} \int_0^{\bar{\epsilon}_f} \eta(\bar{\epsilon}^p) d\bar{\epsilon}^p \quad (19)$$

$$\bar{L} = \frac{1}{\bar{\epsilon}_f} \int_0^{\bar{\epsilon}_f} L(\bar{\epsilon}^p) d\bar{\epsilon}^p \quad (20)$$

Where $\bar{\epsilon}^p$ and $\bar{\epsilon}_f$ represent the equivalent plastic strain and fracture strain, respectively. The obtained average values for the two parameters, along with the

fracture strain used for calibration points, are presented in Table 5.

On the other hand, the average stress of the tensile tests in the stress triaxiality and Lode parameter space are shown in Fig. 11. These results demonstrate that the formability tests align well with the plane stress condition and confirm their distinct stress states, ranging from plane strain to shear. This validates the performed tensile tests and confirms the adequacy of the stress states for model calibration.

The material constants of the C_1, C_2, C_3 , and C of DF2014 criterion were determined by fitting the fracture envelopes to the calibration points using the curve fitting toolbox in MATLAB software. These constants are presented in Table 6.

The fracture locus of the calibrated DF2014 criterion with the calibration points is plotted in Fig. 12.

The fracture loci indicate that, as stress triaxiality increases, the equivalent plastic strain at fracture decreases due to enhanced void growth. Conversely, a reduction in stress triaxiality leads to an increase in the equivalent plastic strain at fracture. Under negative stress triaxiality, where void growth is suppressed, the equivalent plastic strain at fracture is higher than under

Table 5. Calibration points obtained from the formability tests

Formability test	$\bar{\epsilon}_f$	$\bar{\eta}$	$\bar{L}$
UT	0.22	0.41	-0.73
ST	0.3	0.054	-0.12
PS	0.13	0.59	-0.12
NT	0.136	0.55	-0.31

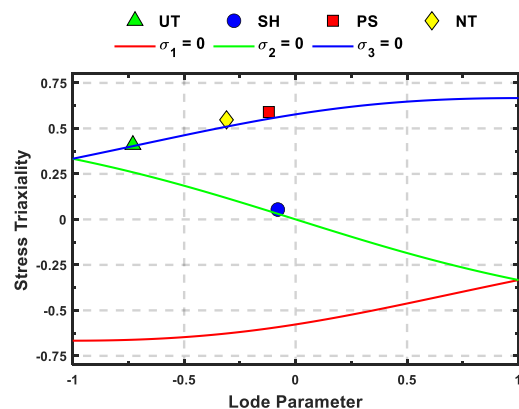


Fig. 11. Stress states of the tensile tests in the stress triaxiality and Lode parameter space, along with curves representing the plane stress conditions.

Table 6. Material constants for the DF2014 criterion

C_1	C_2	C_3	C
4.7	1.9	0.325	1/3

positive stress triaxiality. This effect is more pronounced under compressive stress states with positive Lode parameters than under tensile stress states with negative Lode parameters.

Fig. 13 presents the two-dimensional fracture loci under plane stress conditions together with the calibration points. The DF2014 criterion shows good agreement with the calibration data, indicating that the identified parameters adequately represent the fracture locus over the investigated calibration stress states. The criterion can predict failure within the stress triaxiality range of $-2/3 \leq \eta \leq 2/3$, covering a wide range of stress states from compression to tension.

The calibrated model, together with the identified material parameters, was used to simulate the tensile tests in order to assess its capability to predict fracture initiation. Fig. 14 presents the fractured specimens predicted by the finite element simulations.

The good agreement between the predicted and experimental fracture displacements and crack shapes, as shown in Table 7, indicates successful calibration of the DF2014 fracture criterion. Therefore, this provides the necessary confidence to use this model for predicting

Table 7. Prediction error of the DF2014 criterion in fracture displacement of calibration specimens compared to the experimental results

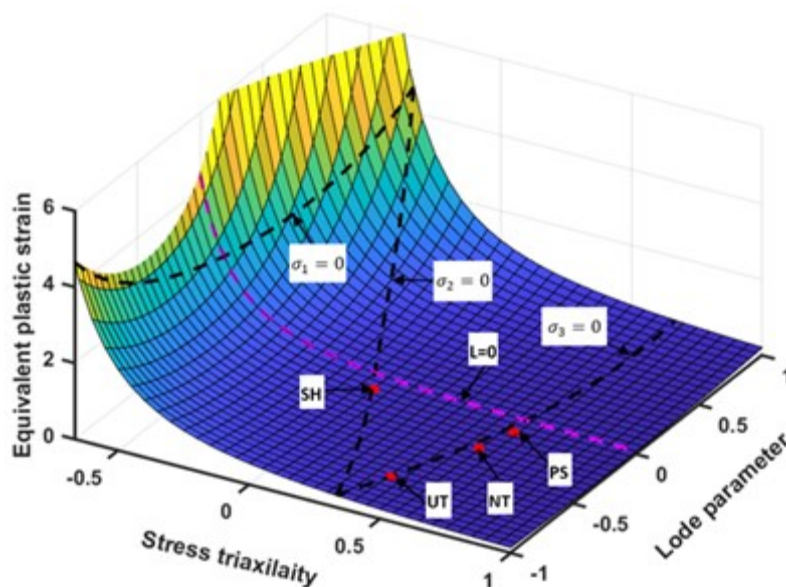
Test	UT	SH	PS	NT
Exp. (mm)	1.58	0.65	0.32	0.33
FE (mm)	1.66	0.62	0.315	0.32
Error %	5.1	4.9	1.5	3.1

fracture in specimens formed through the incremental forming process.

6. Results and Discussion

6.1. Validation of FE model of SPIF

To evaluate the calibrated model and its ability to predict fracture in incremental forming, FE models of parts formed experimentally using the incremental forming process were simulated in ABAQUS/Explicit 2022. The predicted fractures were then compared with those observed experimentally on the formed parts. In the experiments, the AA1050 alloy was formed using the SPIF method into a truncated cone and a pyramid with variable wall angles to a depth of 30 mm to observe cracks formation. To measure the thickness distribution and in-plane strains, specimens were formed to a depth of 25 mm, then cut at the center along the rolling direction, as shown in Fig. 15. The in-plane principal strains and thickness distributions of the formed specimens, along with their corresponding predictions

**Fig. 12.** Fracture locus of the AA1050 alloy sheet for the DF2014 criterion.

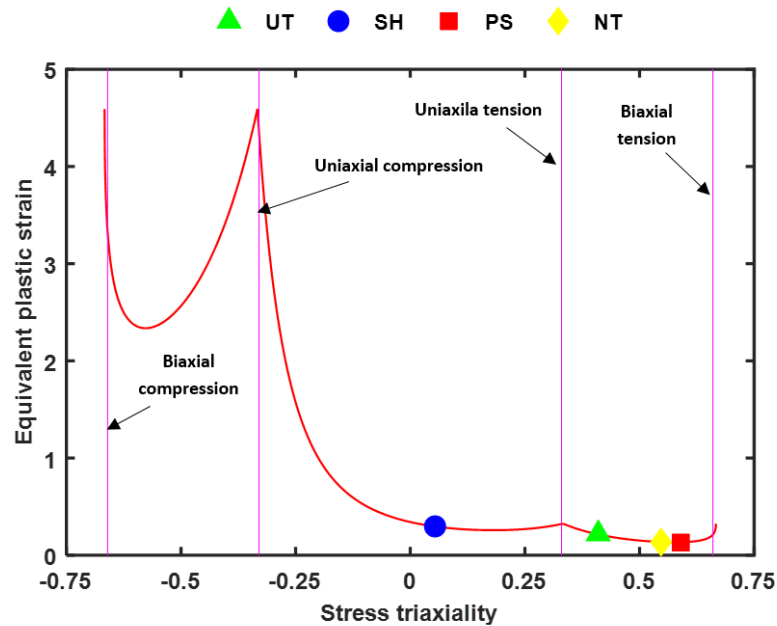


Fig. 13. The fracture loci under plane stress constructed using the DF2014 ductile fracture criterion.

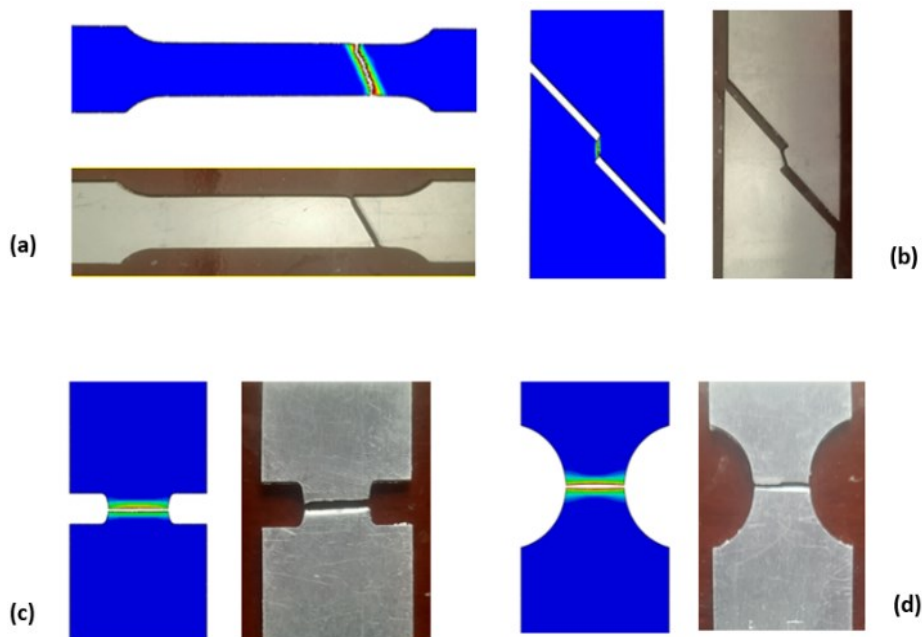


Fig. 14. Fracture in the FE models of (a) uniaxial, (b) in-plane shear, (c) plane strain, and (d) notched tensile tests.

from the FE models, are presented in Figs. 16 and 17. The experimental major and minor in-plane strains were determined by measuring the major and minor diameters of ellipses formed from the deformation of the circular grid after the sheet's plastic deformation. Since the minor strain is negligible or zero, the stress state in the walls of both shapes corresponds to a plane strain condition.

With increasing part depth, the in-plane strain increases while the part thickness decreases uniformly, such that near the end of the depth, the highest strain and the lowest thickness are observed. In the bottom section of the part, which remains undeformed, no strain is generated, and the thickness remains unchanged. In conclusion, the comparison between simulation and experimental results shows good agreement, confirming

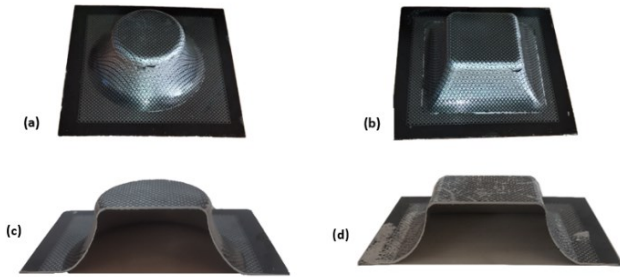


Fig. 15. (a) The truncated conical, (b) pyramidal parts with variable wall angles, formed by SPIF, along with the cut section views of the, (c) conical, and (d) pyramidal parts.

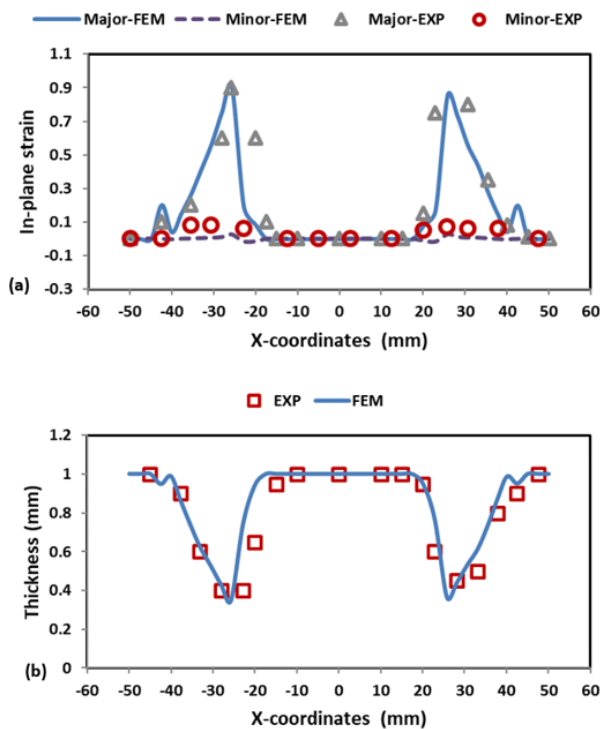


Fig. 16. (a) In-plane principal strains and (b) thickness distribution along the rolling direction for the truncated cone with a variable wall angle.

that the FE simulation was conducted effectively. The progressive increase in in-plane strain and reduction in thickness with increasing forming depth can be attributed to the localized plastic deformation induced by the continuous movement of the forming tool. As the tool advances along the forming path, the material in the vicinity of the tool undergoes repeated plastic deformation, leading to the accumulation of plastic strain. Consequently, the deformation becomes increasingly localized near the forming zone, while the undeformed bottom region remains almost unaffected. This accumulation of plastic strain and the associated

thickness reduction progressively increase the susceptibility of the material to damage initiation and fracture.

6.2. Damage accumulation rule

In an uncoupled damage model, fracture prediction can be represented in three ways: (1) damage threshold (DT), which corresponds to damage accumulation; (2) fracture strain (FS), which depends on stress triaxiality and the Lode parameter; and (3) the fracture forming limit diagram (FFLD), which relates to principal strains [28]. Damage in sheet metal forming processes is often evaluated using models that account for the material's response under different loading conditions. For proportional loading where the stress state remains constant throughout the deformation, the damage is calculated using the following integral-based equation (Eq. (21)):

$$D = \int_0^{\bar{\varepsilon}^p} \frac{d\bar{\varepsilon}^p}{\bar{\varepsilon}_f^p(\eta, L)} \quad (21)$$

Where D is the accumulated damage, $\bar{\varepsilon}^p$ is the equivalent plastic strain, $\bar{\varepsilon}_f^p(\eta, L)$ is the critical failure strain, which depends on stress triaxiality η and the Lode parameter L . This approach assumes that the stress state, defined by the parameters η and L , remains constant as deformation progresses. Under these conditions, the material's damage evolves steadily, and the critical value of D (typically 1) signals the onset of fracture.

The evaluation of the linear damage accumulation rule indicates that applying this rule to predict the onset of fracture in incremental forming processes leads to sheet failure at an early stage, before any significant forming has occurred. Fig. 18 shows the fracture prediction using the DF2014 criterion with the linear damage accumulation rule in straight groove, truncated cone and pyramid specimens at a tool depth of 4 mm. As can be seen, fracture in these specimens occurs extensively and before reaching the depth observed in the experimental specimens. Therefore, the use of a nonlinear damage rule in forming processes with non-proportional loading is inevitable.

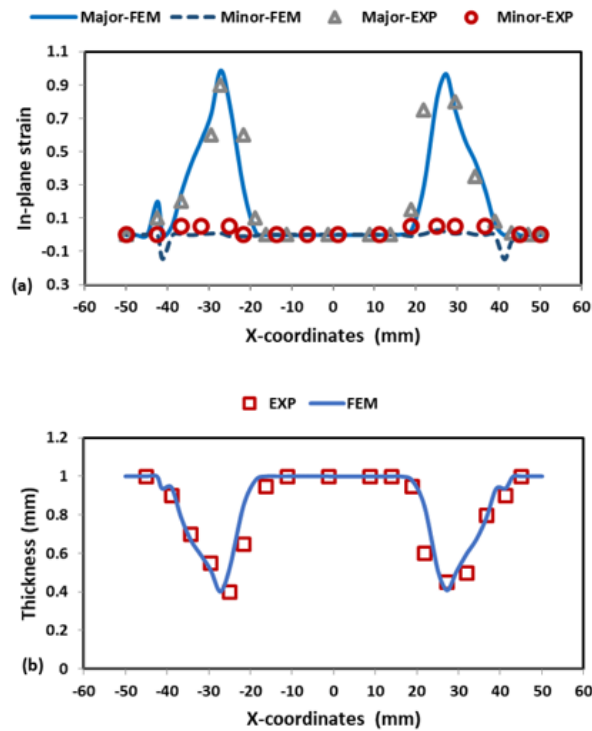


Fig. 17. (a) In-plane principal strains and (b) thickness distribution along the rolling direction for the truncated pyramid with a variable wall angle.

The early fracture predicted by the linear damage accumulation rule can be attributed to its inability to account for the changing deformation history during SPIF. In the linear formulation, damage increases at a nearly constant rate with increasing plastic strain, regardless of the changes in the stress state experienced by the material. However, in SPIF, a material point is subjected to continuously changing loading conditions as the forming tool moves along its path. Consequently, the stress triaxiality and Lode parameter evolve during deformation, resulting in changes in the critical fracture strain and, therefore, in the rate of damage accumulation. The nonlinear damage accumulation rule provides a more appropriate representation of this behavior by allowing the damage evolution to depend on the accumulated plastic strain and the deformation history. This explains the substantially larger fracture depth predicted by the nonlinear rule compared with the linear formulation.

The non-proportional loading involves a varying stress state during the deformation process, making damage prediction more complex. The evolution of η

and L parameters impacts the material's damage and failure behavior, requiring advanced integration techniques and more detailed stress-strain history tracking. In the context of incremental forming processes, predicting fracture accurately is crucial due to the complex loading conditions experienced by the material. Due to the non-proportional loading inherent in the SPIF process, a nonlinear damage accumulation rule (Eq. (22)) was employed to predict failure. Where m is the damage exponent, which controls how damage accumulates with plastic strain.

$$D = \int_0^{\bar{\epsilon}^p} m \left(\frac{\bar{\epsilon}^p}{\bar{\epsilon}_f^p(\eta, L)} \right)^{m-1} \frac{d\bar{\epsilon}^p}{\bar{\epsilon}_f^p(\eta, L)} \quad (22)$$

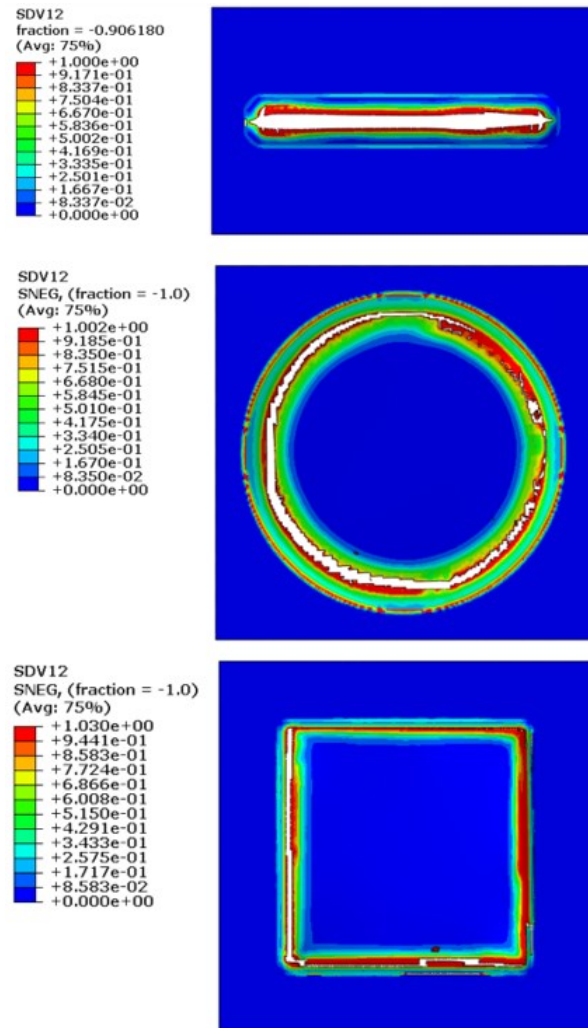


Fig. 18. Fracture prediction using the DF2014 criterion and the linear damage accumulation rule with the damage indicator for specimens: (a) straight groove, (b) truncated cone, and (c) truncated pyramid.

The parameter m was determined through iterative simulations of the incremental forming process of the straight groove shape because of its simplicity and reduced numerical analysis time, with the aim of matching the predicted fracture depth with the experimentally obtained fracture depth as closely as possible. This forming was carried out to a depth of 8 mm, with experimental fracture depths of 7 and 8 mm observed at the two corners of the groove. Subsequently, the straight-groove specimen was simulated iteratively using different values of m until the predicted crack depth matched the experimentally measured crack depth as closely as possible (Fig. 19). The corresponding value of m was identified as $m = 0.102$.

In Fig. 20, the damage evolution as a function of plastic strain for damage function of the DF2014 criterion is presented for the critical element. The curves for the AA1050 alloy indicate that damage accumulation

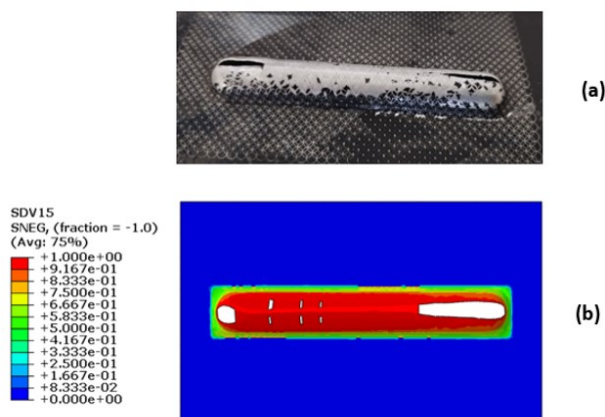


Fig. 19. Fracture location in the straight groove shape: (a) experimental and (b) DF2014.

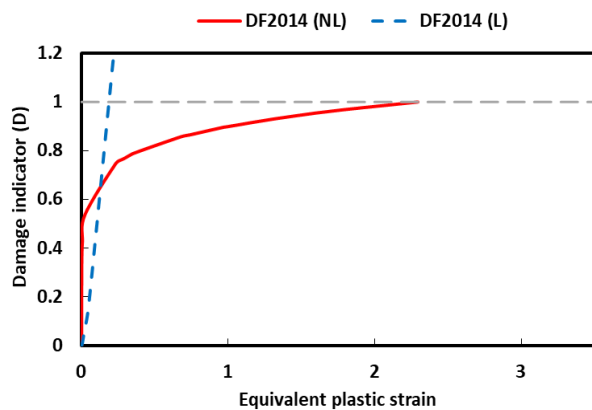


Fig. 20. Damage evolution with increasing $\bar{\epsilon}^p$ for DF2014 ($m = 0.102$) with the linear and nonlinear damage accumulation rules.

following the linear rule (Eq. (21)) results in an almost constant gradient, yields a more conservative prediction, and leads to a significantly smaller fracture depth. In contrast, the nonlinear rule (Eq. (22)) exhibits a variable gradient and predicts significantly higher plastic strain compared to the linear damage. The variable damage gradient obtained from the nonlinear rule reflects the changing loading history of the material during incremental forming. At different stages of tool movement, the critical element experiences different combinations of stress triaxiality, Lode parameter, and plastic strain. Therefore, the rate at which plastic deformation contributes to damage is not constant throughout the forming process. In contrast, the approximately constant gradient of the linear rule does not adequately represent these changes and consequently results in a more conservative estimation of the fracture condition. The nonlinear formulation therefore provides a more realistic description of damage evolution under the non-proportional loading conditions characteristic of SPIF. Thus, the nonlinear damage accumulation rule is necessary to capture the progressive damage evolution and obtain a realistic prediction of fracture initiation and depth in SPIF.

6.3. Strain paths in AA1050 SPIF

Another set of diagrams derived from the simulation results includes the FFLD diagrams, along with the strain paths for the first deleted element in both the tensile tests and the incremental forming processes. The FFLD diagrams are derived by transforming the 2D fracture loci (Fig. 13) into major and minor in-plane strain space using Eq. (23) [8], where the equivalent plastic strain is calculated based on the DF2014 criterion through a code written in MATLAB software.

$$\epsilon_1 = \frac{\bar{\epsilon}^p}{\sqrt{\frac{(1+r_0)r_{90}(1+r_{90})}{1+r_0+r_{90}} + \frac{1+r_0}{r_{90}}\alpha^2 + 2\alpha}} \quad (23)$$

In this equation r_0 and r_{90} are r-values corresponding to the rolling direction and the direction perpendicular to the rolling direction, respectively. $\bar{\epsilon}^p$ is the plastic strain, which is obtained from each DF2014

criterion (Eq. (4)). After calculating ε_1 , the ε_2 is obtained using the equation of $\varepsilon_2 = \alpha \varepsilon_1$, where α denotes the ratio of the minor to major in-plane strains. For tensile tests, these strain paths are linear, whereas in the incremental forming process, the non-proportional nature of the process results in completely nonlinear strain paths.

Fig. 21 presents the FFLD diagram for the DF2014 criterion along with the strain path of the first deleted element from the tensile test simulations. This diagram is based on the criterion calibrated using tensile tests under proportional loading. As shown, due to the proportional loading in the tensile tests, the resulting strain path diagrams are linear and exhibit good agreement with the theoretical path of these tests. The diagram indicates that the DF2014 model tends to underestimate fracture for the in-plane shear test, while fracture is overestimated for the other states resulting from the simulations. Nevertheless, the minor discrepancies between the simulation results of the tensile tests and the fracture loci obtained from the FFLD diagrams suggest that this criterion is reasonably accurate in predicting in-plane strains at the moment of fracture under proportional loading conditions.

The strain path during incremental forming is

analyzed below. Fig. 22 shows the evolution of the strain path for the critical elements up to the moment of fracture, as predicted by the DF2014 criterion. These paths are entirely nonlinear and extend significantly beyond the FFLD diagram obtained from tensile tests under proportional loading conditions. This behavior can be attributed to the localized forming and the non-proportional loading characteristics of the incremental forming process. The nonlinear strain paths are important for fracture prediction because the material does not experience a single proportional stress state throughout the forming process. As the tool moves, the local stress state continuously changes, resulting in variations in both stress triaxiality and the Lode parameter. Consequently, the fracture condition of a material point is governed not only by its final strain state but also by the sequence of loading conditions experienced before fracture initiation. This explains why the strain paths obtained from SPIF extend beyond the fracture forming limit diagram calibrated from proportional tensile tests.

The FFLD diagrams of the DF2014 model, together with the strain distribution of all blank elements and the damage contour maps at the final frame prior to fracture

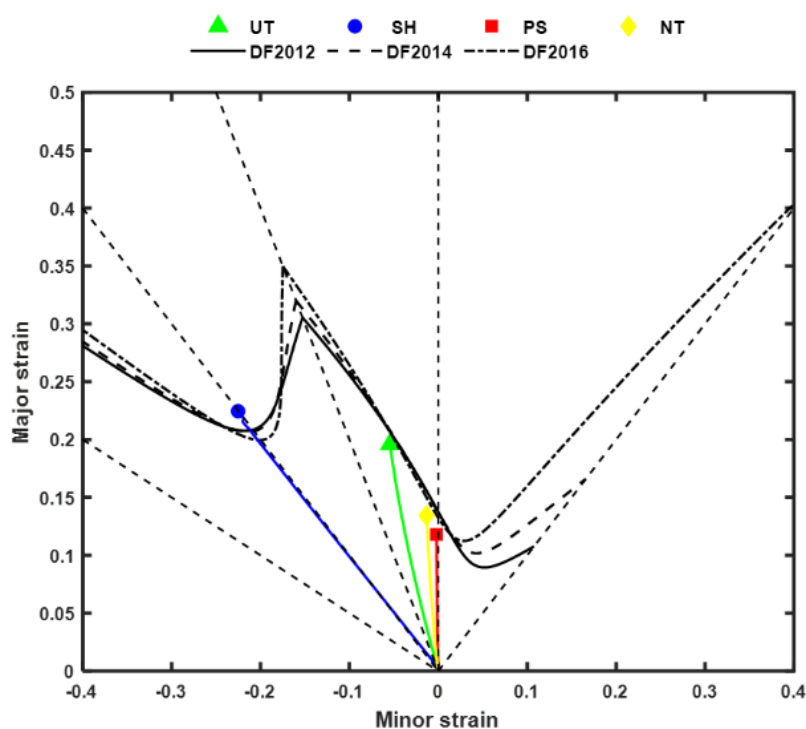


Fig. 21. FFLD diagrams of the DF2014 criterion alongside the strain paths of the first deleted element from the tensile tests.

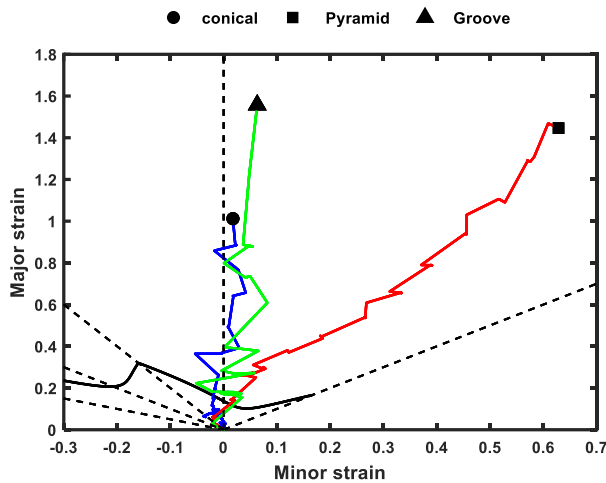


Fig. 22. Variation of the strain paths for the first deleted element in the FE model of the formed parts.

initiation, are shown in Figs. 23 (a-c). The elements with critical damage are highlighted in black. For the conical part, the strain states of the elements are closer to plane strain, and fractures occur in the wall under this condition. In the pyramidal part, the strain states are distributed between plane strain and biaxial tension. The critical elements leading to fracture exhibit a biaxial tensile stress state, which occurs in the corner region. In this region, the tool changes direction and maximum thinning is observed. In the straight groove part, the elements experience strain states ranging from plane strain to biaxial tension, with critical elements approaching the plane strain state. The different strain states observed in the formed geometries are associated with different local stress states and, consequently, different damage evolution mechanisms. Changes in the Lode parameter modify the relative contribution of tensile and shear related deformation mechanisms to fracture. In addition, stress triaxiality and the Lode parameter evolve during the forming process due to its non-proportional loading nature, affecting void growth and ductile damage accumulation. Therefore, the combination of stress triaxiality and the Lode parameter, together with the accumulated plastic strain and deformation history, plays an important role in determining the initiation time and location of fracture in the formed parts.

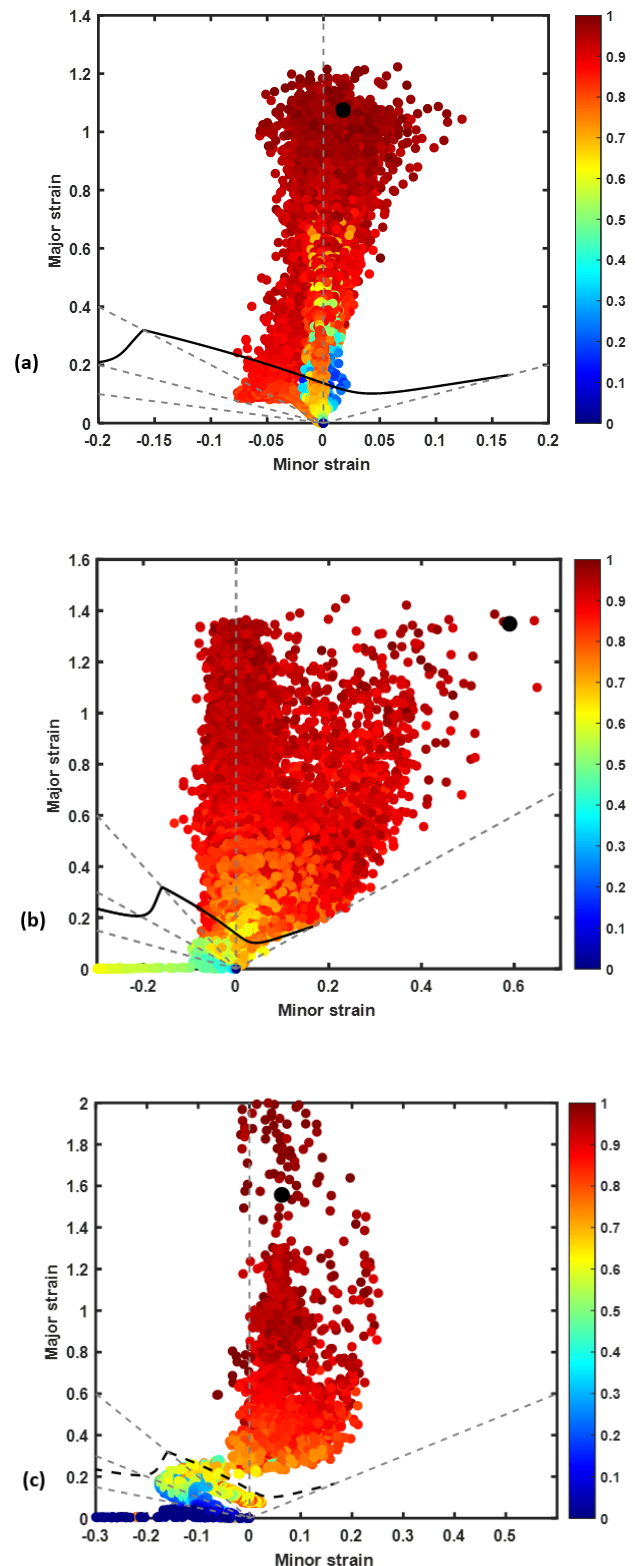


Fig. 23. In-plane strain distribution for: (a) cone, (b) pyramid, and (c) straight groove; the location of the critical element for different the DF2014 criterion is marked by a solid black color.

6.4. The cut-off value for the stress triaxiality of the DF2014 criterion

Fig. 24 presents the cut-off values for the stress triaxiality of the DF2014 criterion, together with the plane stress conditions. As observed, the DF2014 criterion predicts significantly lower cut-off values compared with the $\sigma_1 = 0$ condition, depending on the Lode parameter. These reduced cut-off values, relative to the $\sigma_1 = 0$ condition, indicate that the DF2014 criterion is able to account for compressive in addition to tensile stress states arising during the forming process. Varying the value of C changes the cut-off value for the stress triaxiality with respect to the σ_1 line. In particular, when $C = 0$, the cut-off value for the stress triaxiality becomes identical to the σ_1 line. Given the wide range of stress triaxiality represented by the DF2014 criterion, which can be adjusted through different values of the parameter C for both compressive and tensile stress states, this model can be applied to predict ductile fracture in a variety of metal forming processes, including single point incremental forming.

6.5. Fracture prediction in SPIF

The depth of the formed parts for the truncated cone and pyramid is 30 mm, and the wall angle at this depth is 84.30° (Fig. 5). Cracks were observed in the specimens at tool depths of 23-24 mm for both the truncated cone

and truncated pyramid. For the straight groove specimen, cracks formed at the corners of the groove, at a depth of approximately 7 and 8 mm, respectively. By obtaining the damage exponent m for the nonlinear damage accumulation rule, the simulation of truncated conical and pyramidal parts was performed. The predicted crack depths and positions were then compared with experimental results for the truncated pyramidal and conical parts, as shown in Figs. 25 and 26, respectively.

Overall, the DF2014 model provides good predictions. For the truncated pyramid with a variable wall angle, the experimentally observed fractures were located on one side of an edge and at two opposite corners of the pyramid. In the simulations, however, crack initiation was predicted at all four corners as well as on both sides of the edges. The prediction of multiple potential fracture locations in the numerical model, compared with the more localized fracture sites observed experimentally, can be attributed to the similar local stress and strain states predicted at these regions. Small differences in the actual material condition, local thickness, surface condition, or experimental boundary conditions may cause one of these potential sites to become critical in the experiment. Nevertheless, the numerical model successfully identified the critical regions and provided a close prediction of the fracture

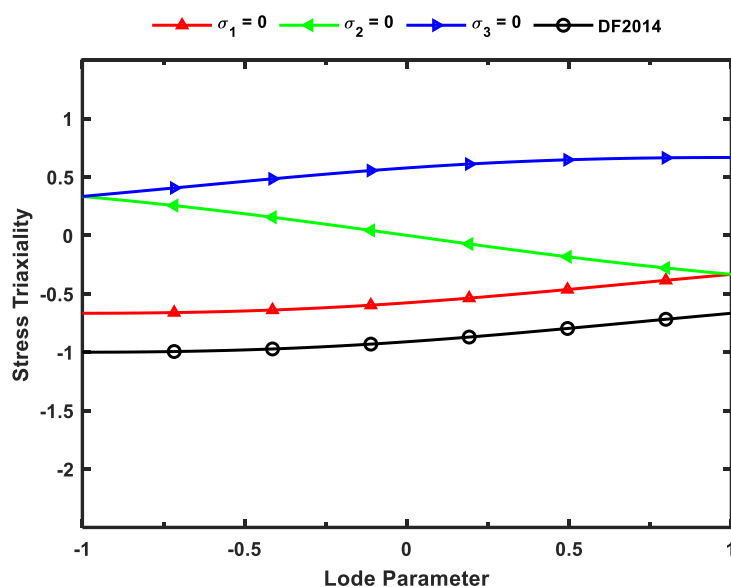


Fig. 24. The cut-off values for the stress triaxiality of the DF2014 criterion along with the plane stress conditions.

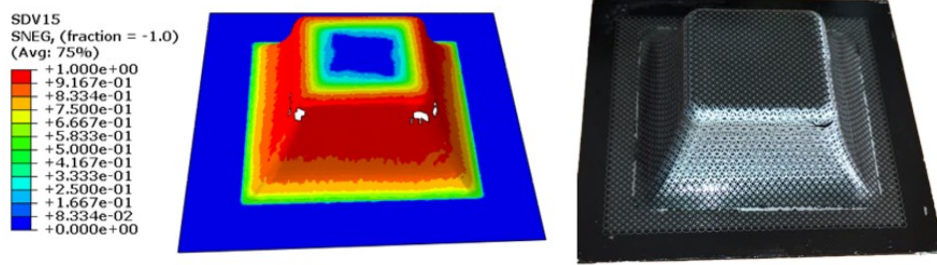


Fig. 25. Experimental and numerical predictions of fracture in the truncated pyramid with a variable wall angle using the damage indicator based on the DF2014 criterion.

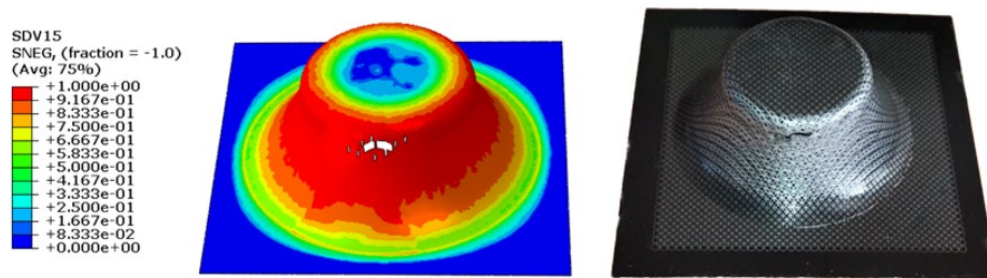


Fig. 26. Experimental and numerical predictions of fracture in the truncated cone with a variable wall angle using the damage indicator based on the DF2014 criterion.

depth. Likewise, for the truncated cone with a variable wall angle, the DF2014 model predicted both the crack depth and its location, further demonstrating its strong predictive capability. The crack depth for the conical and pyramidal parts is presented in Table 8.

As mentioned, fracture in the truncated conical specimen occurs along the wall under plane strain stress state, while in the truncated pyramidal specimen it occurs at the corners or edges under biaxial stress state. The difference in fracture location between the truncated cone and pyramid can be explained by their distinct local stress states. In the truncated pyramid, the tool changes direction at the corners, producing a localized biaxial tensile state and severe thinning. The resulting increase in stress triaxiality promotes void growth and accelerates

ductile damage accumulation, making the corners critical locations for fracture initiation.

Under biaxial tension, stress triaxiality, and consequently void growth, is higher than in plane strain. Therefore, it is expected that fracture occurs at a lower depth in the pyramidal part. Nevertheless, the greater role of shear coalescence due to the maximum shear stress in truncated cone forming leads to lower strain in the conical part, and consequently less forming depth at the moment of fracture initiation for this component. In the incremental forming of aluminum sheets for truncated cone and pyramidal parts, where plane strain and biaxial tensile states dominate, the DF2014 criterion can predict the fracture strain in this process.

Overall, the fracture behavior observed in SPIF is governed by the combined effects of plastic strain accumulation, stress triaxiality, the Lode parameter, and the local deformation history. The continuous movement of the tool produces a non-proportional loading path, causing the local stress state to evolve throughout the forming process. This evolution directly affects the critical fracture strain and the rate of damage accumulation. The results demonstrate that considering

Table 8. FE predicted and experimental fractured depth measured in SPIF tests

SPIF parts	Truncated cone		Truncated pyramid	
	DF2014	24.5	DF2014	24.9
Exp. (mm)		23.1		23.8
Error %	DF2014	6.1	DF2014	4.6

only the final strain or stress state is insufficient for accurately predicting fractures in SPIF. The nonlinear damage accumulation rule, together with the stress-state dependence of the DF2014 criterion, provides a more appropriate description of the progressive damage evolution and explains the good agreement between the numerical predictions and experimental fracture depths.

7. Conclusion

This study presented an experimental and numerical investigation of ductile fracture prediction during the single point incremental forming (SPIF) of AA1050 aluminum alloy sheets using the DF2014 ductile fracture criterion. The criterion was implemented in ABAQUS/Explicit through the VUSDFLD user subroutine and coupled with a nonlinear damage accumulation rule. The main conclusions are summarized as follows:

- The SPIF process involves localized plastic deformation and continuously evolving stress states, resulting in non-proportional loading paths. Under these conditions, the nonlinear damage accumulation rule provided a more appropriate representation of damage evolution than the linear rule and prevented the premature fracture prediction observed with the linear formulation.
- The numerical model reproduced the experimentally measured in-plane principal strain distributions and thickness variations with good agreement. The DF2014 criterion combined with the nonlinear damage accumulation rule also predicted the fracture depths of the truncated cone and truncated pyramid with errors of 6.1% and 4.6%, respectively.
- The nonlinear damage accumulation rule was found to be necessary for obtaining realistic fracture predictions in SPIF. In contrast, the linear damage accumulation rule predicted fracture at an early forming stage, demonstrating that the loading history and continuously changing stress state should be considered in damage accumulation under the non-proportional loading conditions of SPIF.

- The fracture predictions obtained using DF2014 are associated with its consideration of the combined effects of stress triaxiality and the Lode parameter on the critical fracture strain. The formulation of the DF2014 criterion is motivated by micromechanical ductile fracture mechanisms and accounts for the combined effects of stress triaxiality and the Lode parameter on the critical fracture strain.
- Although the DF2014 criterion incorporates a Lode-parameter-dependent cut-off value for stress triaxiality, the stress states experienced during the SPIF of the investigated geometries were predominantly within the tensile region. Therefore, the cut-off stress triaxiality had a limited influence on the fracture predictions obtained in this study.
- The results demonstrate that combining the DF2014 criterion with a nonlinear damage accumulation rule provides a suitable approach for predicting fracture initiation in the investigated SPIF geometries. The applicability of the approach to other materials, geometries, and forming conditions requires further investigation.

Authors' contributions

S. A. Teymouri Sendesi: Material preparation, Data collection & analysis, Simulations, Experiments, Writing - original draft

M. M. Kasaei: Conceptualization, Methodology, Supervision, Data analysis, Writing - review & editing

M. Majidi: Supervision, Writing - review & editing

S. J. Hashemi: Investigation, Data analysis, Writing - review & editing

Conflict of interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Data availability

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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