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A COMPETITIVE MARKET STRUCTURE FOR REACTIVE POWER PROCUREMENT*

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Abstract– This paper first proposes a competitive market structure for reactive power procurement and then develops a methodology for incorporating voltage stability problems into the model. The owners of electric transactions should participate in this competitive framework and submit their own firmness bids in ($/MW) to the Independent System/Market Operator (ISO-IMO). ISO clears the market for reactive energy regarding the value of each transaction and utilization cost of reactive power on one hand, and the impact of transaction amount on the voltage security of the power system on the other hand. Here, the voltage stability margin is incorporated into the power flow equations so that the security of the power system is provided when a sudden change in load occurs. Applying the Karush- Kuhn-Tuker theorem to the proposed OPF-based reactive power market model gives the reactive power to be provided at each generation node and amount of each bilateral transaction allowed for physical operation. To illustrate an interesting feature of the proposed methodology, several case studies are carried out over the IEEE 14-bus test system using the well-known GAMS software (MINOS solver). The results show that the proposed structure can provide an incentive for both generators and consumers to support their own electricity contracts by supplying enough reactive power at each generator or load bus.

Keywords– Bilateral transactions, deregulation, market design, OPF, reactive power, voltage stability

1. INTRODUCTION

Nowadays, the topic of deregulation is the center of attention in many countries. Various types of market structures have, so far, been established around the world for trading electricity which can fall into three main categories. The first type is the decentralized market which is realized with bilateral or multilateral contracts; the second one is the centralized or power pool market, and finally the hybrid market, which is a composition of the two preceding models and belongs to the third category [1, 20]. In the deregulated power markets, reactive power management is under the responsibility of the Independent System Operator (ISO). ISO should dispatch reactive power in order to provide system security and to ensure that all voltage magnitudes are within their satisfactory limits. Voltage and reactive power support are linked to each other as far as the reactive power support has a profound impact on operation and voltage stability of the power system. In a free electricity market, reactive support is distinguished as an ancillary service, which can facilitate active power transportation [2-3]. Thus, an efficient provision of this kind of ancillary service becomes a major concern, especially when the power system is going to be operated close to its maximum power transfer capability. As a matter of fact, ISO needs to procure and dispatch reactive power optimally in order to make more transactions feasible over the power network. Although it may technically be possible for the ISO to confirm all electricity transactions when adequate reactive power resources are available and the power system network has no limitation for reactive power transmission,

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such conditions are rarely met in a real power system operation where some transactions might cause violations of security constrains, and they need to be revised. Therefore, lack of reactive power resources or voltage and transfer limits are the main reason why the bilateral contracts need to compete with each other for using the available capacities of transmission lines in the ancillary service markets.

In the literature [4-14], numerous articles on reactive power pricing and also creating a competitive framework for managing reactive power in the deregulated power systems have been published and various tools have been developed. These approaches can be divided into three distinct groups; in the first group, considering sufficient reactive power sources are available, ISO, on behalf of consumers, attempts to purchase reactive power at a minimum cost [4-5]. In this methodology, active power transactions are usually kept constant, and no modification on active power contracts is allowable. A method which incorporates voltage stability margin within this type of reactive market formulation is presented in [6].

In the second approach, both active and reactive power energies are dispatched simultaneously considering different objective functions such as minimizing the total generation cost of electricity [7], social welfare maximization in the active power markets [1, 8] and minimizing the total procurement costs of active and reactive power production [9, 11]. The costs associated with reactive power support are not integrated into the power pool electricity market formulation developed by these methods; however, the third approach proposes the reactive power market needs to be established as a complete part of the electricity market, which is dominated by bilateral contracts [11, 12]. In the proposed models, reactive power is dispatched based on the purposes of minimizing transmission losses, minimizing deviations from transaction requests made by market participants, minimizing costs of reactive power generation or even proper combinations of the mentioned objectives. Nevertheless, a good coordination between active and reactive power markets cannot be distinguished in the proposed procedures. Transactions are assumed to have the same priority, and consequently no clear competition is established among owners of transactions; furthermore, the voltage stability problem has not yet been considered.

In this paper, a competitive market-based mechanism for reactive power procurement is introduced. The proposed structure provides a good coordination with the electricity market, which is dominated by bilateral contracts for both technical and economic perspectives. Market equations are set up to include a voltage stability margin to prevent shipping reactive power over long transmission lines. This criterion causes reactive power to be locally dispatched as optimally as possible. The proposed methodology is implemented on the IEEE 14-bus test system, and different case studies are conducted to show the impact of available reactive power resources, as well as participant bids on the approval of the electricity contracts. Simulation results demonstrate that this structure can provide a vast incentive for generators and consumers to provide reactive power locally to maintain their contracts as much as possible.

2. MARGINAL PRICE OF REACTIVE POWER

There are different types of equipment having good potential to support the reactive power for voltage regulation in power networks. They usually have different characteristics in terms of VAR control mode and utilization cost as illustrated in [4]. In this paper, it is assumed that generators, synchronous condensers and static VAR compensators are the main reactive power suppliers. Utilization costs are composed of two parts: explicit and implicit costs. Explicit costs of facilities consist of capital and operating costs that are commonly compensated proportional to injected reactive power into the network; but the implicit part of production costs mainly refers to opportunity costs, which are usually evaluated for power generators.

a) Synchronous generators

Synchronous generators are the main source of active power generation; however, they are also able to provide reactive power for security purposes. The stable operating point of a generator is always restricted to its capability curve boundaries, which are defined according to armature and field winding heating limits. A typical diagram shown in Fig. 1 reveals that the maximum reactive power output of a generator is extremely linked to its operating point. For example, when generator's active power output is set to P_A , it can only provide the reactive power within the limits of [$Q_A^{\min}$, $Q_A^{\max}$].

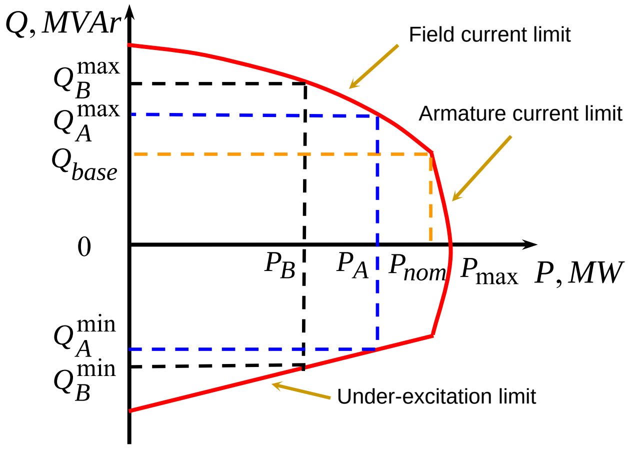

Fig.1. Typical capability curve of a generator [12]

In this condition, if more reactive power is required from the unit (for example Q_B), the active power generation should be shifted back from point P_A to P_B to relieve some portion of the generator's capacity. This action causes the generator to make less revenue in the energy market. Opportunity cost, as is currently being used in other economical systems, can be considered as a good option to compensate this loss value. The selection of a proper method for calculation of the opportunity cost of a generator is an important problem in deregulated power systems, and different approaches have been proposed for this critical issue [13-14]. In reference [12], a conceptual based reactive power bidding structure is proposed for being used in a competitive market; however, this structure is only applicable at a certain operating point. Nevertheless, it has not proposed an exact methodology to calculate the opportunity cost of a generator; hence, in this paper the reactive power production cost of a generator is approximated by [14]

$$C_{gqi}(Q_{gi}) = [C_{gpi}(\sqrt{P_{gi}^2 + Q_{gi}^2}) - C_{gpi}(P_{gi})].K_{gi} \tag{1}$$

where

$C_{gpi}(P_{gi}) = aP_{gi}^2 + bP_{gi} + c$: Cost function of the i th generator.

Q_{gi} : Reactive output power of the generator i.

$S_{gi} = \sqrt{P_{gi}^2 + Q_{gi}^2}$: Apparent output power of the generator i.

K_{gi} : Profit rate of the active power, which is usually chosen between 0.05~0.1.

According to (1), each generator can declare its own marginal price for reactive power generation, which equals to at least:

$$W_{gqi}(Q_{gi}) = \frac{\partial C_{gqi}(Q_{gi})}{\partial Q_{gi}} \qquad (\$/MVArh) \tag{2}$$

b) Synchronous condensers

Although synchronous condensers have no opportunity cost, we have assumed that they will be paid according to Eq. (1) setting P_{gi} to zero.

C) Static VAR compensator

Static VAR compensators are generally used to regulate the voltage profile within the local areas.

Fixed capacitors and reactors have low installation and operation costs, as well as slow response to change their reactive outputs. Electronic based VAR compensators have a good response time to change their outputs compared with conventional ones; however, their installation and operation costs are moderately high. Regardless of the quality of reactive power resources, operational costs of static VAR compensators can be given proportional to their reactive outputs as follows [6].

$$C_{cj}(Q_{cj}) = r_{cj} Q_{cj} \tag{3}$$

Q_{cj}: injected reactive power at the bus j in *(MVAr-h)*.
r_{cj} is the price of reactive power per *MVAr-h*, depending on some factors such as capital cost , period of a lifetime and average utilization factor. For example, for a SVC with an investment cost of $22000/*MVAr*, lifetime of 30 years and average use of 2/3, r_{cj} can be calculated as follows:

$$r_{cj} = \frac{22000}{30 * 365 * 24 * \frac{2}{3}} = 0.1255(\$/MVAr\_h) \tag{4}$$

3. VOLTAGE STABILITY ANALYSIS

Voltage stability is defined as the ability of a power system to support voltages at desired levels when some perturbations in load or sudden equipment outage occur during operating condition. The advent of the restructured power system has put systems operators in a difficult situation in view of technical/economical management of the system and also its operation. Usually, the increase in the amount or number of bilateral transactions can lead the power system to get closer to voltage instability boundaries. Since it is an ISO responsibility to keep the system stable at different operating conditions, it is preferred that the ISO properly modify some transactions or dispatch adequate reactive power resources in advance to maintain the power system at a specific distance from its instability borders. Thus, providing the voltage stability margin, as a major concern of ISO, should be integrated into the reactive power market formulation.

Various static indices have been proposed to study the voltage stability problem in power systems. They can provide useful information for estimating proximity to voltage collapse. Some indices such as modal analysis, Fast Voltage Stability Index (FVSI) and minimum eigenvalue/singular value index can give some useful information for instability study around the current operating point; however, other techniques such as P-V and Q-V analysis can estimate voltage stability margin in a wide range of variation. Voltage stability margin is defined as a *MW* distance between the current operating point and maximum loading condition as illustrated graphically in Fig. 2. This margin is defined as $(P_{MAX} - P_o)$. P_{MAX} is the maximum permissible loadability and P_o is the current operating point of the power system.

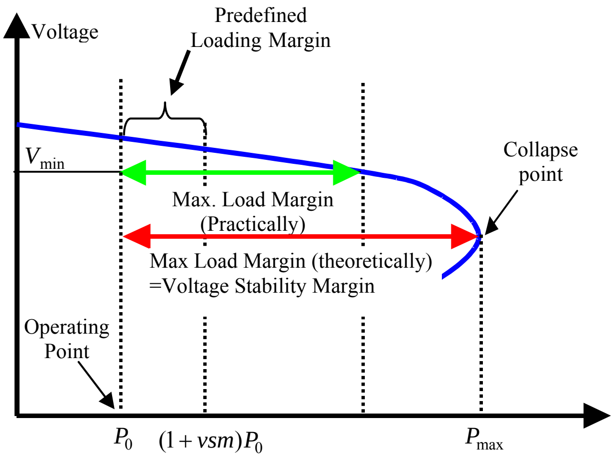

Fig. 2. Concept of voltage stability in a power system

In this paper, we have employed a loading margin index to ensure that the system security can be maintained around its operating point. There are also a lot of experiences with the incorporation of this index into an OPF formulation, which makes it a good candidate for steady state voltage stability studies [15].

4. REACTIVE POWER DEMAND FOR SUPPORTING TRANSACTIONS OF GENERATORS

Transmission lines are the unique paths connecting generation resources to loading points. Available Transfer Capability (ATC) of a transmission system depends on a number of factors such as system generation dispatch, system load level, load distribution the network, network topology and limits imposed on transmission lines due to thermal, voltage and stability considerations [16]. Maximum capacity of long transmission lines is usually restricted to voltage stability limits in as a consequence of inadequate reactive power compensation at sending or receiving points. Figure 3 shows this concept for a simple 2-bus test system. The generator should provide sufficient reactive power, Q_G, for maintaining the bus voltages at the desired values: V_1 and V_2. The P_G can securely be transferred to the load center when the following equations are satisfied

$$Q_G = \frac{|V_1| \cdot (|V_1| - |V_2| Cos\delta_{12})}{X} = \frac{|V_1|^2}{X} - \sqrt{\left(\frac{|V_1| \cdot |V_2|}{X}\right)^2 - P_G^2} \; ; \; \delta_{12} = \delta_1 - \delta_2 \; ; P_G = P_D = \frac{|V_1| \cdot |V_2|}{X} Sin\delta_{12} \; ; R = 0 \quad (5)$$

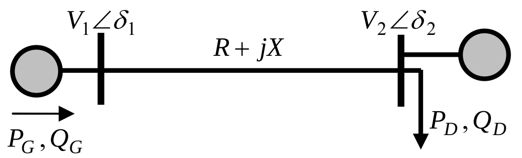

Fig. 3. A typical 2-bus test system

Equation (5) determines the exact value of reactive power (Q_G), which the generator should provide if it is willing to inject a specific value of active power (P_G) into the power network. This is an indispensable element of AC transmission grids. Optimal management of reactive power can increase overall power transfer capability of the system and enable the system operator to dispatch more power transactions over the existing infrastructure.

5. A COMPETITIVE MARKET STRUCTURE FOR REACTIVE POWER

Up to now, different methods have been proposed for reactive power management in which ISO, on behalf of the consumers, purchases reactive power and then allocates the incurred cost among customers accordingly. In this section, a competitive structure based on the concepts of market orientation is introduced in which the ISO is only responsible to secure the operation of the power systems, and has no interference with financial settlements. Different parts of the proposed market structure are introduced as follows:

a) Bilateral transaction matrix

It is assumed that electricity is totally traded via bilateral contracts. A bilateral transaction is defined between one generator and one electric consumer. It is assumed that the ISO only knows the quantities of bilateral contracts, which are represented by the Bilateral Transaction Matrix (BTM) as follows:

$$BTM = \begin{bmatrix} T_{11} & T_{12} & . & . & T_{1n} \\ T_{21} & T_{22} & . & . & T_{2n} \\ . & . & . & . & . \\ . & . & . & . & . \\ T_{m1} & T_{m2} & . & . & T_{mn} \end{bmatrix} \quad (6)$$

Where

m : number of generators.

n : number of consumers.

T_{mn} : bilateral contract level between the m th generator and n th consumer.

In this arrangement, total active power generation at the ith bus and total demand load at the jth bus can be calculated as:

$$P_{Gi} = \sum_{j \in \alpha_G} T_{ij} \text{ : Total active power generation at the i}^{th} \text{ bus} \tag{7}$$

$$P_{Dj} = \sum_{i \in \alpha_D} T_{ji} \text{ : Total active power demand at the j}^{th} \text{ bus.}$$

$\alpha_G=\{1,2\ldots m\}$ and $\alpha_D=\{1,2\ldots n\}$ are the sets corresponding to the generator and load buses ($m<n$).

b) The proposed bidding structure

Considering no constraints such as congestion over certain transmission corridors or lack of reactive power capacity, the ISO would easily accept all transactions for physical operation. However, this situation may rarely occur in practice, hence ISO needs to give the priority to each transaction in order to establish an explicit mechanism to confirm transactions. Different methods can be used to recognize the importance of the transactions. In one approach, priority of transactions is technically evaluated based on sensitivity factors, while in this approach it is assumed that all transactions have the same economic worth [1]. Transactions can also be prioritized based on sensitivity factors defined according to financial indices. This is the second approach, which considers all transactions have the same technical effects [11-12].

In this paper, it is assumed that owners of bilateral contracts would offer the value they are willing to pay for utilization from reactive power resources. These values would prevent their contracts from being curtailed. In other words, these values inherently declare the priority of the transactions in the reactive power allocation procedure. ISO does not share this information with other participants to prevent market power occurrence. Transactions Firmness Bids, *TFB*, should be presented to ISO in (\$/MW) corresponding to each element of *BTM* matrix. For example $W_{ij} = 1$ (\$/MW) means that the owner of the transaction likes to pay 1\$ for approving each MW of its transaction.

$$TFB = \begin{bmatrix} W_{11} & W_{12} & . & . & W_{1n} \\ W_{21} & W_{22} & . & . & W_{2n} \\ . & . & . & . & . \\ . & . & . & . & . \\ W_{m1} & W_{m2} & . & . & W_{mn} \end{bmatrix} \tag{8}$$

Where

W_{mn} : Willingness to pay for transaction held between the generator *m* and the load *n*.

c) The proposed reactive power market structure

In this section, we define a two-part auction market mechanism for reactive power procurement. Using this model, the ISO is able to maximize the total surplus for both consumers and producers at the same time. This is the core operation of fully competitive and transparent market mechanism, which is usually used for hybrid electricity market structures. The main purpose of this section is to define a competitive market structure for reactive power procurement incorporating voltage stability criteria. The model can be formulated as the following optimization problem:

1. Objective function: The objective of reactive power market operation is the social welfare maximization that is given as:

$$Max\left[\left(\sum_{i\in\alpha_G}\sum_{j\in\alpha_D}W_{ij}T_{ij}\right)-\left(\sum_{i\in\alpha_G}\int_0^{Q^\circ_{Gi}}W_{gqi}(q)dq+\sum_{j\in\alpha_C}r_{cj}Q^\circ_{cj}\right)\right] \tag{9}$$

2. Equality constraints at normal condition: Nodal load flow equations can be written as follows:

-Active power equations:
For i=1

$$\left[\sum_{j\in\alpha_D}T_{1j}+P^{\circ Slack}_{reg}-\sum_{j\in\alpha_G}T'_{1j}\right]=f_1^\circ\left(\underline{V}^\circ,\underline{\theta}^\circ\right) \tag{10.1}$$

For $i\neq 1$

$$\left[\sum_{j\in\alpha_D}T_{ij}-\sum_{j\in\alpha_G}T'_{ij}\right]=f_i^\circ\left(\underline{V}^\circ,\underline{\theta}^\circ\right)\quad 2\le i\le n \tag{10.2}$$

-Reactive power equations

$$\left[Q^\circ_{Gi}+Q^\circ_{ci}-\tan\left(\varphi^\circ_i\right)\sum_{j\in\alpha_D}T'_{ij}\right]=g_i^\circ\left(\underline{V}^\circ,\underline{\theta}^\circ\right)\quad 1\le i\le n \tag{10.3}$$

3. Equality constraints at increased generation/load conditions:

-Active power equations:
For i=1

$$\left[(1+vsm)\left(\sum_{j\in\alpha_D}T_{1j}-\sum_{j\in\alpha_G}T'_{1j}\right)+P^{vsmSlack}_{reg}\right]=f_1^{vsm}\left(\underline{V}^{vsm},\underline{\theta}^{vsm}\right) \tag{10.4}$$

For $i\neq 1$

$$\left[(1+vsm)\left(\sum_{j\in\alpha_D}T_{ij}-\sum_{j\in\alpha_G}T'_{ij}\right)\right]=f_i^{vsm}\left(\underline{V}^{vsm},\underline{\theta}^{vsm}\right)\quad 2\le i\le n \tag{10.5}$$

-Reactive power equations: $1\le i\le n$

$$\left[Q^{vsm}_{Gi}+Q^\circ_{ci}-(1+vsm)\tan\left(\varphi^\circ_i\right)\sum_{j\in\alpha_D}T'_{ij}\right]=g_i^{vsm}\left(\underline{V}^{vsm},\underline{\theta}^{vsm}\right) \tag{10.6}$$

4. Resource limitations and operational constraints:

- For normal operation

$$\begin{aligned}&0\le P^{\circ Slack}_{reg}\le P^{\max Slack}_{reg}\\&0\le T_{ij}\le T^{\max}_{ij}\\&Q^{\min}_{Gi}\le Q^\circ_{Gi}\le Q^{\max}_{Gi}\qquad i\in\alpha_G\\&Q^{\min}_{ci}\le Q^\circ_{ci}\le Q^{\max}_{ci}\qquad i\in\alpha_C\\&V^{\min}_j\le V^\circ_j\le V^{\max}_j\qquad j\in\alpha_D;\qquad V^\circ_i=V^{spec.}_i\qquad i\in\alpha_G\end{aligned} \tag{11.1}$$

- For increased load condition

$$0 \le P_{reg}^{vsmSlack} \le P_{reg}^{\max Slack} \tag{11.2}$$

$$Q_{Gi}^{\min} \le Q_{Gi}^{vsm} \le Q_{Gi}^{\max}$$

$$V_j^{\min} \le V_j^{vsm} \le V_j^{\max} \qquad j \in \alpha_D \, ; \; V_i^{vsm} = V_i^{set} \qquad i \in \alpha_G$$

Assumptions:

- The electricity transactions are provided only through the bilateral contracts.
- The slack generator is the only supplier of transmission losses.
- Thermal limits of transmission lines are greater than their voltage stability limits. Therefore, thermal limits of transmission lines have not been included in the optimization model.
- In this framework, it is assumed that maximum reactive power capacities have been determined previously by considering credible (N-1) contingency analysis by the ISO, and hence, no equipment outage is considered into the model, which is presented for procuring reactive energy. This is a rational assumption for optimal dispatch of reactive power at a specified operating point. To establish a market for reactive power capacity, readers are referred to [17].

Also, in the above equations, superscript "$\circ$" denotes variables used for normal conditions, while superscript "vsm" indicates the variables used in increased loading conditions. The sign "$'$" is the vector/matrix transpose operator, P_{reg}^{slack} is the active power regulation supplied by the reference generator. Q_G and Q_c are reactive power generations of spinning and static VAR compensators. The constraint of the voltage stability margin is implemented by introducing the factor $(1+vsm)$ into the Eqs. (10.4), (10.5) and (10.6). These equations guarantee that there is a feasible solution for the power flow equations when amounts of all transactions increase proportional to the factor $(1+vsm)$. All demands have distinct power factors, which are assumed to be constant during the normal and stressed conditions ($\tan(\varphi_i^\circ) = const.$). The functions $f_i(\circ)$ and $g_i(\circ)$ represent nodal active and reactive power flow equations that can be stated as:

$$f_i\left(\underline{V}\ ,\underline{\theta}\ \right) = +|V_i| \sum_{j\in\alpha_D} |V_j||Y_{ij}|\cos(\theta_j - \theta_i + \angle Y_{ij}) \tag{12}$$

$$g_i\left(\underline{V}\ ,\underline{\theta}\ \right) = -|V_i| \sum_{j\in\alpha_D} |V_j||Y_{ij}|\sin(\theta_j - \theta_i + \angle Y_{ij})$$

Where

$\underline{V}$ is the bus voltage magnitude vector , $\underline{\theta}$ is the corresponding angle vector and Y_{ij} is the admittance of a line connected between node i and node j.

In this model, the control variables are composed of the levels of the bilateral transactions, reactive power output of generators, synchronous condensers and electronic based static VAR compensators. Voltage magnitudes of generators are assumed to be fixed during simulations. Lagrange equations associated with the reactive power market modeling should be solved to obtain the optimal equilibrium points, i.e. reactive power generation and levels of the transactions. The Lagrange function of the proposed optimization model can be written as follows:

$$Max\ \ L = \left[\left(\sum_{i\in\alpha_G}\ \sum_{j\in\alpha_D} W_{ij}T_{ij}\right) - \left(\sum_{i\in\alpha_G} \int_0^{Q_{Gi}^\circ} W_{gqi}(q)dq + \sum_{j\in\alpha_C} r_{cj}Q_{cj}^\circ\right)\right] \tag{13}$$

$$-\lambda_1^{\circ P}\left[\sum_{j\in\alpha_D} T_{1j} + P_{reg}^{\circ Slack} - \sum_{j\in\alpha_G} T'_{1j} - f_1^\circ\left(\underline{V}^\circ,\underline{\theta}^\circ\right)\right] - \sum_{i=2}^{n} \lambda_i^{\circ P}\left[\sum_{j\in\alpha_D} T_{ij} - \sum_{j\in\alpha_G} T'_{ij} - f_i^\circ\left(\underline{V}^\circ,\underline{\theta}^\circ\right)\right]$$

$$-\sum_{i=1}^{n} \lambda_i^{\circ Q}\left[Q_{Gi}^\circ + Q_{ci}^\circ - \tan\left(\varphi_i^\circ\right) \sum_{j\in\alpha_D} T'_{ij} - g_i^\circ\left(\underline{V}^\circ,\underline{\theta}^\circ\right)\right]$$

$$
\begin{aligned}
&-\lambda_1^{vsmP}\left[(1+vsm)\left(\sum_{j\in\alpha_D}T_{1j}-\sum_{j\in\alpha_G}T'_{1j}\right)+P_{reg}^{vsmSlack}-f_1^{\circ}\left(\underline{V}^{\circ},\underline{\theta}^{\circ}\right)\right]\\
&-\sum_{i=2}^{n}\lambda_i^{vsmP}\left[(1+vsm).\left(\sum_{j\in\alpha_D}T_{ij}-\sum_{j\in\alpha_G}T'_{ij}\right)-f_i^{vsm}\left(\underline{V}^{vsm},\underline{\theta}^{vsm}\right)\right]\\
&-\sum_{i=1}^{n}\lambda_i^{vsmQ}\left[Q_{Gi}^{vsm}+Q_{ci}^{\circ}-(1+vsm).\tan\left(\varphi_i^{\circ}\right)\sum_{j\in\alpha_D}T'_{ij}-g_i^{vsm}\left(\underline{V}^{vsm},\underline{\theta}^{vsm}\right)\right]\\
&-\mu_{P_{reg}^{\circ Slack}}^{\max}\left[P_{reg}^{\max}-P_{reg}^{\circ Slack}-s_{P_{reg}^{\circ Slack}}^{\max^2}\right]-\sum_{i=1}^{m}\sum_{j=1}^{n}\mu_{T_{ij}^{\circ}}^{\max}\left[T_{ij}^{\max}-T_{ij}-s_{T_{ij}^{\circ}}^{\max^2}\right]\\
&-\sum_{i=1}^{m}\mu_{Q_{Gi}^{\circ}}^{\max}\left[Q_{Gi}^{\max}-Q_{Gi}^{\circ}-s_{Q_{Gi}^{\circ}}^{\max^2}\right]-\sum_{i=1}^{m}\mu_{Q_{Gi}^{\circ}}^{\min}\left[Q_{Gi}^{\circ}-Q_{Gi}^{\min}-s_{Q_{Gi}^{\circ}}^{\min^2}\right]\\
&-\sum_{i=1}^{n_c}\mu_{Q_{ci}^{\circ}}^{\max}\left[Q_{ci}^{\max}-Q_{ci}^{\circ}-s_{Q_{ci}^{\circ}}^{\max^2}\right]-\sum_{i=1}^{n_c}\mu_{Q_{ci}^{\circ}}^{\min}\left[Q_{ci}^{\circ}-Q_{ci}^{\min}-s_{Q_{ci}^{\circ}}^{\min^2}\right]\\
&-\sum_{i=1,i\notin\alpha_G}^{n}\mu_{V_{ci}^{\circ}}^{\max}\left[V_i^{\max}-V_i^{\circ}-s_{V_i^{\circ}}^{\max^2}\right]-\sum_{i=1,i\notin\alpha_G}^{n}\mu_{V_i^{\circ}}^{\min}\left[V_i^{\circ}-V_i^{\min}-s_{V_i^{\circ}}^{\min^2}\right]\\
&-\mu_{P_{reg}^{vsmSlack}}^{\max}\left[P_{reg}^{\max}-P_{reg}^{vsmSlack}-s_{P_{reg}^{vsmSlack}}^{\max^2}\right]\\
&-\sum_{i=1}^{m}\mu_{Q_{Gi}^{vsm}}^{\max}\left[Q_{Gi}^{\max}-Q_{Gi}^{vsm}-s_{Q_{Gi}^{vsm}}^{\max^2}\right]-\sum_{i=1}^{m}\mu_{Q_{Gi}^{vsm}}^{\min}\left[Q_{Gi}^{vsm}-Q_{Gi}^{\min}-s_{Q_{Gi}^{vsm}}^{\min^2}\right]\\
&-\sum_{i=1}^{n_c}\mu_{Q_{ci}^{vsm}}^{\max}\left[Q_{ci}^{\max}-Q_{ci}^{vsm}-s_{Q_{ci}^{vsm}}^{\max^2}\right]-\sum_{i=1}^{n_c}\mu_{Q_{ci}^{vsm}}^{\min}\left[Q_{ci}^{vsm}-Q_{ci}^{\min}-s_{Q_{ci}^{vsm}}^{\min^2}\right]\\
&-\sum_{i=1,i\notin\alpha_G}^{n}\mu_{V_{ci}^{vsm}}^{\max}\left[V_i^{\max}-V_i^{vsm}-s_{V_i^{vsm}}^{\max^2}\right]-\sum_{i=1,i\notin\alpha_G}^{n}\mu_{V_i^{vsm}}^{\min}\left[V_i^{vsm}-V_i^{\min}-s_{V_i^{vsm}}^{\min^2}\right]
\end{aligned}
$$

Where the Lagrange multipliers can be classified as:

$$
\begin{aligned}
&\lambda^{\circ}=\left[\lambda_i^{\circ P},\lambda_i^{\circ Q}\right] && \mu^{\circ}=\left[\mu_{P_{reg}^{\circ Slack}}^{\max},\mu_{T_{ij}^{\circ}}^{\max},\mu_{Q_{Gi}^{\circ}}^{\max},\mu_{Q_{ci}^{\circ}}^{\max},\mu_{V_i^{\circ}}^{\max},\mu_{Q_{Gi}^{\circ}}^{\min},\mu_{Q_{ci}^{\circ}}^{\min},\mu_{V_i^{\circ}}^{\min}\right]\\
&\lambda^{vsm}=\left[\lambda_i^{vsmP},\lambda_i^{vsmQ}\right] && \mu^{vsm}=\left[\mu_{P_{reg}^{vsmSlack}}^{\max},\mu_{T_{ij}^{vsm}}^{\max},\mu_{Q_{Gi}^{vsm}}^{\max},\mu_{Q_{ci}^{vsm}}^{\max},\mu_{V_i^{vsm}}^{\max},\mu_{Q_{Gi}^{vsm}}^{\min},\mu_{Q_{ci}^{vsm}}^{\min},\mu_{V_i^{vsm}}^{\min}\right]
\end{aligned}
$$

and the slack variables are as follows:

$$
s^{\circ}=\left[s_{P_{reg}^{\circ Slack}}^{\max},s_{T_{ij}^{\circ}}^{\max},s_{Q_{Gi}^{\circ}}^{\max},s_{Q_{ci}^{\circ}}^{\max},s_{V_i^{\circ}}^{\max},s_{Q_{Gi}^{\circ}}^{\min},s_{Q_{ci}^{\circ}}^{\min},s_{V_i^{\circ}}^{\min}\right],\qquad s^{vsm}=\left[s_{P_{reg}^{vsmSlack}}^{\max},s_{T_{ij}^{vsm}}^{\max},s_{Q_{Gi}^{vsm}}^{\max},s_{Q_{ci}^{vsm}}^{\max},s_{V_i^{vsm}}^{\max},s_{Q_{Gi}^{vsm}}^{\min},s_{Q_{ci}^{vsm}}^{\min},s_{V_i^{vsm}}^{\min}\right]
$$

Necessary condition of Lagrange theorem implies that:

$$
\frac{\partial L}{\partial T_{ij}}=0=W_{ij}-\lambda_i^{\circ P}+\lambda_j^{\circ P}+\left(\tan(\varphi_j^{\circ})\right)\lambda_j^{\circ Q}-\mu_{T_{ij}^{\circ}}^{\max}-(1+vsm)\left(\lambda_i^{vsmP}-\lambda_j^{vsmP}-\left(\tan(\varphi_j^{\circ})\right)\lambda_j^{vsmQ}\right) \quad (14)
$$

This is an important equation, which gives some useful and clear information about the cost of transactions executed on the power system network. The concept of a supply-demand market is distinguished clearly in (14) where W_{ij} is placed in front of the operation cost of transaction T_{ij}. The transaction T_{ij} will be totally accepted if $\mu_{T_{ij}^{\circ}}^{\max}$ has a positive value. In other words, the following equations are to be fulfilled:

$$
\begin{aligned}
&\mu_{T_{ij}^{\circ}}^{\max}=W_{ij}-\lambda_i^{\circ P}+\lambda_j^{\circ P}+\left(\tan(\varphi_j^{\circ})\right)\lambda_j^{\circ Q}-(1+vsm)\left(\lambda_i^{vsmP}-\lambda_j^{vsmP}-\left(\tan(\varphi_j^{\circ})\right)\lambda_j^{vsmQ}\right)\geq 0 \quad \Rightarrow\\
&\hspace{30em}(15)\\
&W_{ij}\geq\lambda_i^{\circ P}-\lambda_j^{\circ P}-\left(\tan(\varphi_j^{\circ})\right)\lambda_j^{\circ Q}+(1+vsm)\left(\lambda_i^{vsmP}-\lambda_j^{vsmP}-\left(\tan(\varphi_j^{\circ})\right)\lambda_j^{vsmQ}\right)
\end{aligned}
$$

The proposed objective for reactive power procurement represented in Eqs. (9) and (13) is composed of two distinct terms. The right hand term, $\sum_{i\in\alpha_G}\sum_{j\in\alpha_D} W_{ij}T_{ij}$, indicates the total outlays that owners of transactions are willing to pay to confirm their own contracts, while the remaining terms are associated with reactive power procurement costs. The difference between these two components is defined as a proper index representing the social benefits of reactive power market participants. In this manner, reactive power is procured in accordance with the prices proposed by market participants and reactive power allocation cost, such that the social welfare function gets its maximum value. The salient feature of the proposed structure is its potential for transaction modification not only based on a rival's economic tendency, but also based on their technical effects on the power system. In brief, reactive power, as well as transactions, is dispatched through a direct competition mechanism where the technical concerns are included as soft constraints. Thus, an optimal solution is obtained so that both economical and technical constraints are met.

6. SIMULATION AND RESULTS

The IEEE 14-bus test system is used to demonstrate the proposed methodology. A one-line diagram of the system is shown in Fig.4. The detailed data of the transmission lines as well as the system components are given in [18]. The slack bus is Bus 1, which supplies both the loads it has a contract with and the power losses of the integrated network. In some cases, the reactive power capacity limits of the generators are intentionally changed to show how lack of sufficient reactive power capacity can decrease approved amounts of transactions.

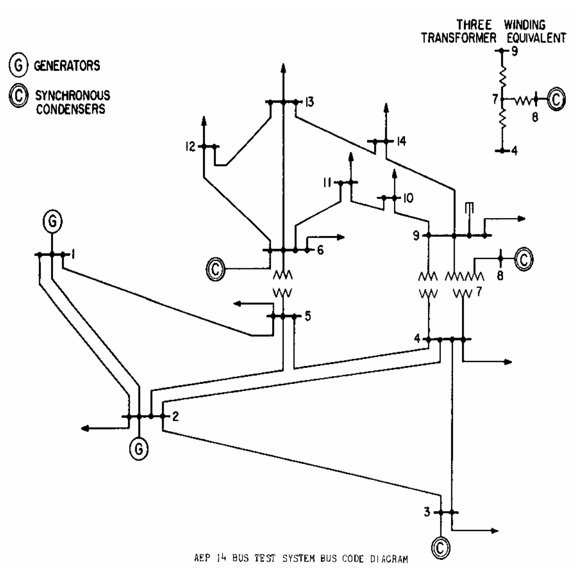

Fig. 4. IEEE 14 bus test system

Case1. In this case, we have assumed that the consumers located at buses 2, 3, 12 and 14 make a contract with G2 and the rest with G1. Synchronous condensers are placed at buses 3, 6 and 8. Here, we assume that the firmness bid for each transaction is \$2.4/MW. This means that all transactions have the same priority in using reactive energy. The market is simulated for different predefined loading margins and corresponding results are reported in Table 1. As Table 1 indicates, The ISO can only approve 0.077p.u from total amounts of transactions (0.09p.u) held between generator 1 and load 10 to keep the $vsm = 0.09$p.u. If the ISO plans more value for vsm, this transaction is totally rejected. This is also true for transactions $T_{1\text{-}9}$ and $T_{2\text{-}3}$ for $vsm = 0.14$(p.u.). As shown in Fig. 5, generator 3 reaches its maximum reactive power capacity and this is a good reason for these curtailments. In this framework, transactions are modified according to their offers and available reactive power resources in such a way that the social benefit function takes the maximum value. Variation of the value of the social welfare and also the buses voltage magnitudes are shown in Figs. 6 and 7, respectively.

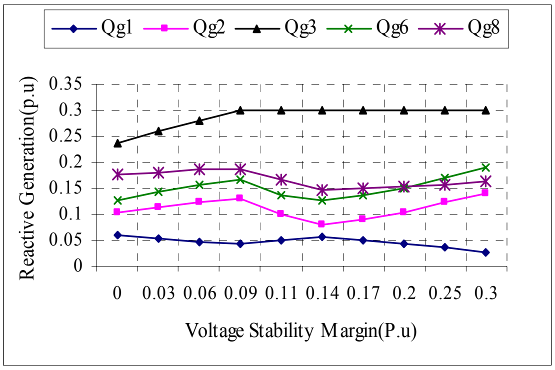

Fig. 5. Reactive power outputs of the generators (Case 1)

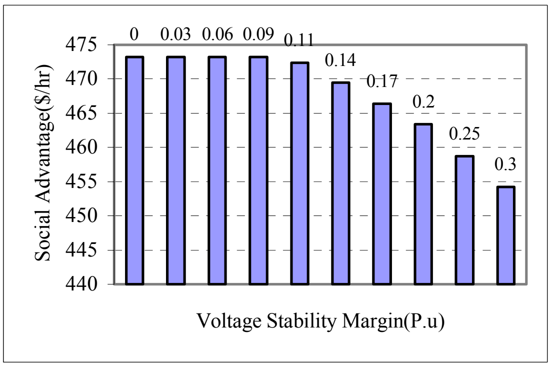

Fig. 6. Variarion of social benefits (Case 1)

Table 1. Results of transactions dispatch incorporating voltage stability margin (case 1)

	vsm =0.0	*vsm* =0.03	*vsm* =0.06	*vsm* =0.09	*vsm* =0.11	*vsm* =0.14	*vsm* =0.17	*vsm* =0.20	*vsm* =0.25	*vsm* =0.30
$T_{1\text{-}4}$	0.478	0.478	0.478	0.478	0.478	0.478	0.478	0.478	0.478	0.478
$T_{1\text{-}5}$	0.076	0.076	0.076	0.076	0.076	0.076	0.076	0.076	0.076	0.076
$T_{1\text{-}6}$	0.112	0.112	0.112	0.112	0.112	0.112	0.112	0.112	0.112	0.112
$T_{1\text{-}9}$	0.295	0.295	0.295	0.295	***0.292***	***0.224***	***0.221***	***0.219***	***0.215***	***0.21***
$T_{1\text{-}10}$	0.09	0.09	0.09	***0.077***	***0***	***0***	***0***	***0***	***0***	***0***
$T_{1\text{-}11}$	0.035	0.035	0.035	0.035	0.035	0.035	0.035	0.035	0.035	0.035
$T_{1\text{-}12}$	0.027	0.027	0.027	0.027	0.027	0.027	0.027	0.027	0.027	0.027
$T_{1\text{-}13}$	0.135	0.135	0.135	0.135	0.135	0.135	0.135	0.135	0.135	0.135
	vsm =0.0	*vsm* =0.03	*vsm* =0.06	*vsm* =0.09	*vsm* =0.11	*vsm* =0.14	*vsm* =0.17	*vsm* =0.20	*vsm* =0.25	*vsm* =0.30
$T_{2\text{-}2}$	0.217	0.217	0.217	0.217	0.217	0.217	0.217	0.217	0.217	0.217
$T_{2\text{-}3}$	0.942	0.942	0.942	0.942	0.942	***0.931***	***0.903***	***0.877***	***0.836***	***0.799***
$T_{2\text{-}12}$	0.034	0.034	0.034	0.034	0.034	0.034	0.034	0.034	0.034	0.034
$T_{2\text{-}14}$	0.149	0.149	0.149	0.149	0.149	0.149	0.149	0.149	0.149	0.149

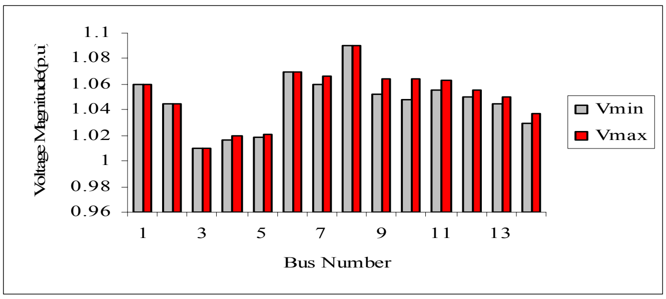

Fig. 7. Voltages variation respect to increasing vsm (Case 1)

Case 2. In this case, we have assumed that load 3 sets a contract with generator 1 instead of generator 2. The maximum capacity of generator 1 and 3 are set to 50 and 30MVAr, respectively. The same as the previous case, all contracts have the same offers, but are equal to 2.8$/MW. Simulation results are reported in Table 2 and Figs. 8, 9 and 10. $T_{1\text{-}10}$ is restricted more than before when $vsm = 0.09$p.u. For

$vsm = 0.11$p.u and more, $T_{1\text{-}3}$ should also be curtailed due to the limited capacity of injected reactive power at buses 1 and 3. This is shown in Fig. 8. A comparison between the approved quantities of $T_{2\text{-}3}$ and $T_{1\text{-}3}$ in case 1 and case 2 shows that it would be better for load 3 to purchase its own active power from generator 2 instead of generator 1, since in this situation, $T_{2\text{-}3}$ is totally approved for the vsm values less than 0.14p.u. This also shows that the main reason of transaction curtailment in case 1 is related to the shortage of reactive power at bus 3. In case 2, however, the shortage of reactive power at bus 1 can cause some transactions held with G1 to be rejected. In other words, if generator 1 had more reactive power capacity, more transactions could be approved.

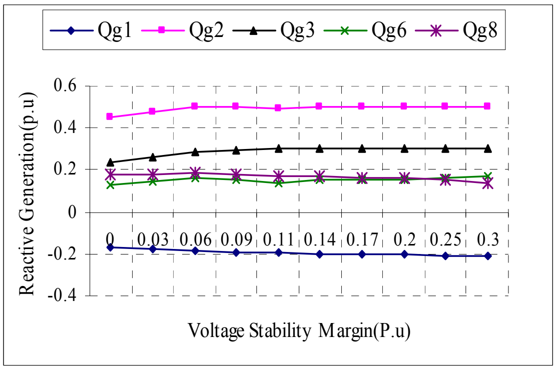

Fig. 8. Reactive power outputs of the generators (Case 2)

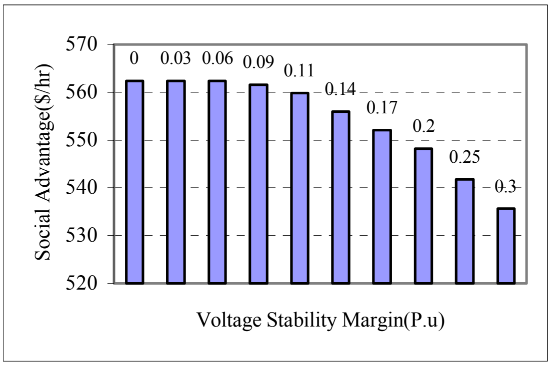

Fig. 9.Variarion of social benefits (Case 2)

Table 2. Results of transactions dispatch incorporating voltage stability margin (Case 2)

	vsm =0.0	vsm =0.03	vsm =0.06	vsm =0.09	vsm =0.11	vsm =0.14	vsm =0.17	vsm =0.20	vsm =0.25	vsm =0.30
$T_{1\text{-}3}$	0.942	0.942	0.942	0.942	***0.94***	***0.911***	***0.891***	***0.871***	***0.841***	***0.813***
$T_{1\text{-}4}$	0.478	0.478	0.478	0.478	0.478	0.478	0.478	0.478	0.478	0.478
$T_{1\text{-}5}$	0.076	0.076	0.076	0.076	0.076	0.076	0.076	0.076	0.076	0.076
$T_{1\text{-}6}$	0.112	0.112	0.112	0.112	0.112	0.112	0.112	0.112	0.112	0.112
$T_{1\text{-}9}$	0.295	0.295	0.295	0.295	0.295	***0.29***	***0.262***	***0.234***	***0.191***	***0.151***
$T_{1\text{-}10}$	0.09	0.09	0.09	***0.05***	***0***	***0***	***0***	***0***	***0***	***0***
$T_{1\text{-}11}$	0.035	0.035	0.035	0.035	0.035	0.035	0.035	0.035	0.035	0.035
$T_{1\text{-}12}$	0.027	0.027	0.027	0.027	0.027	0.027	0.027	0.027	0.027	0.027
$T_{1\text{-}13}$	0.135	0.135	0.135	0.135	0.135	0.135	0.135	0.135	0.135	0.135
Sum (p.u)	***2.19***	***2.19***	***2.19***	***2.15***	***2.098***	***2.064***	***2.016***	***1.968***	***1.895***	***1.827***
	vsm =0.0	vsm =0.03	vsm =0.06	vsm =0.09	vsm =0.11	vsm =0.14	vsm =0.17	vsm =0.20	vsm =0.25	vsm =0.30
$T_{2\text{-}2}$	0.217	0.217	0.217	0.217	0.217	0.217	0.217	0.217	0.217	0.217
$T_{2\text{-}12}$	0.034	0.034	0.034	0.034	0.034	0.034	0.034	0.034	0.034	0.034
$T_{2\text{-}14}$	0.149	0.149	0.149	0.149	0.149	0.149	0.149	0.149	0.149	0.149
Sum (p.u)	***0.4***	***0.4***	***0.4***	***0.4***	***0.4***	***0.4***	***0.4***	***0.4***	***0.4***	***0.4***

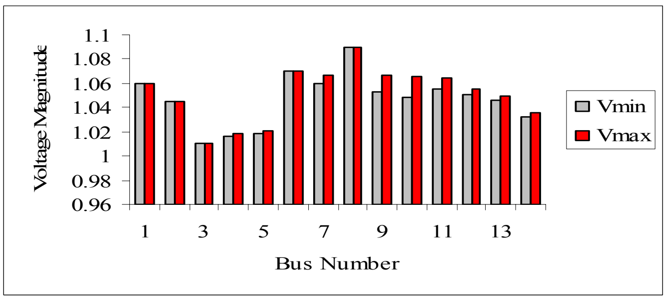

Fig. 10. Voltages variation with respect to increasing vsm (Case 2)

Case 3. There is no significant difference between this case and the previous one. We only increase the maximum available reactive power of generator 1 from 50MVAr to 60MVAr. Simulation results are tabulated in Table 3, and reactive power dispatch of generators is shown in Fig. 11. Investigations show that in this case, more transactions are approved in contrast to the previous case. For example, for $vsm = 0.3$p.u the total amount of approved transactions of generator 1 is 1.932p.u, while this value is 1.827p.u for the same situation in case 2. This indicates that in this structure, generators attempt to provide reactive power to get more benefit from approving their active power contracts. This may cause less reactive market power in the proposed structure than those models which are currently being used for reactive power procurement where generators are treated as a reactive power seller and only electrical energy consumers are charged for the consumption of reactive energy. This strategy also provides an incentive for owners of transactions to supply reactive power locally in order to maintain all amounts of their required power. Social benefit does not have a high variation because reactive power cost also increases with increasing of approved amounts of transactions. This is shown in Fig. 12 while voltage variation is given in Fig. 13.

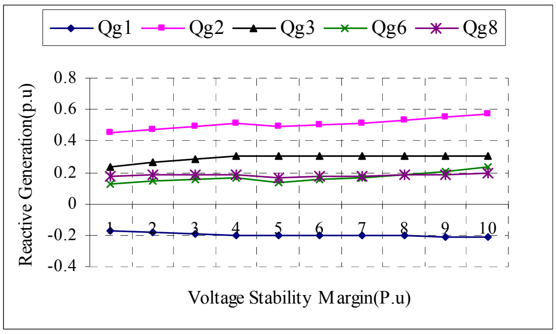

Fig. 11. Reactive power outputs of the generators (Case 3)

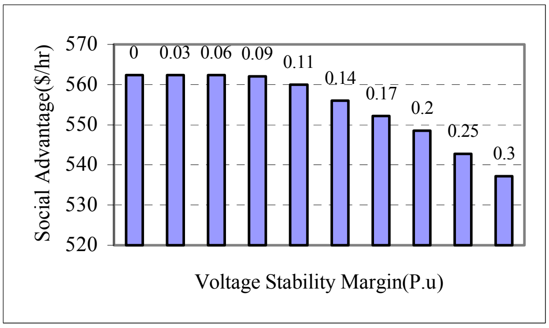

Fig. 12. Variarion of social benefits (Case 3)

Table 3. Results of transactions dispatch incorporating voltage stability margin (Case 3)

	vsm =0.0	*vsm* =0.03	*vsm* =0.06	*vsm* =0.09	*vsm* =0.11	*vsm* =0.14	*vsm* =0.17	*vsm* =0.20	*vsm* =0.25	*vsm* =0.30
$T_{1\text{-}3}$	0.942	0.942	0.942	0.942	***0.94***	***0.91***	***0.882***	***0.855***	***0.813***	***0.774***
$T_{1\text{-}4}$	0.478	0.478	0.478	0.478	0.478	0.478	0.478	0.478	0.478	0.478
$T_{1\text{-}5}$	0.076	0.076	0.076	0.076	0.076	0.076	0.076	0.076	0.076	0.076
$T_{1\text{-}6}$	0.112	0.112	0.112	0.112	0.112	0.112	0.112	0.112	0.112	0.112
$T_{1\text{-}9}$	0.295	0.295	0.295	0.295	0.295	0.295	0.295	0.295	0.295	0.295
$T_{1\text{-}10}$	0.09	0.09	0.09	***0.071***	***0***	***0***	***0***	***0***	***0***	***0***
$T_{1\text{-}11}$	0.035	0.035	0.035	0.035	0.035	0.035	0.035	0.035	0.035	0.035
$T_{1\text{-}12}$	0.027	0.027	0.027	0.027	0.027	0.027	0.027	0.027	0.027	0.027
$T_{1\text{-}13}$	0.135	0.135	0.135	0.135	0.135	0.135	0.135	0.135	0.135	0.135
Sum (p.u)	***2.19***	***2.19***	***2.19***	***2.171***	***2.098***	***2.068***	***2.04***	***2.013***	***1.971***	***1.932***
	vsm =0.0	*vsm* =0.03	*vsm* =0.06	*vsm* =0.09	*vsm* =0.11	*vsm* =0.14	*vsm* =0.17	*vsm* =0.20	*vsm* =0.25	*vsm* =0.30
$T_{2\text{-}2}$	0.217	0.217	0.217	0.217	0.217	0.217	0.217	0.217	0.217	0.217
$T_{2\text{-}12}$	0.034	0.034	0.034	0.034	0.034	0.034	0.034	0.034	0.034	0.034
$T_{2\text{-}14}$	0.149	0.149	0.149	0.149	0.149	0.149	0.149	0.149	0.149	0.149
Sum (p.u)	***0.4***	***0.4***	***0.4***	***0.4***	***0.4***	***0.4***	***0.4***	***0.4***	***0.4***	***0.4***

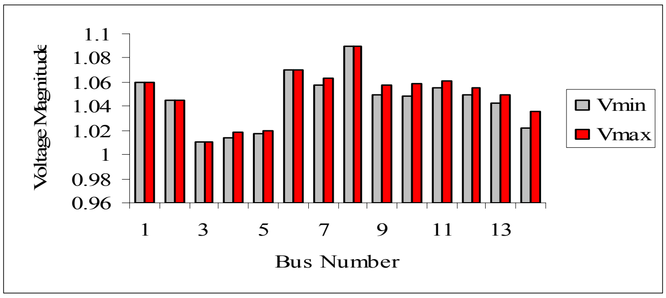

Fig.13. Voltages variation with respect to increasing vsm (Case 3)

Case 4. The impacts of static VAR compensators on the performance of the market are investigated in this case. Thus, we assume that there exist 3 SVCs installed at buses 4, 7and 10 with maximum values of 300MVAr. Their marginal costs are assumed to be 0.13$/MVAr, 0.1$/MVAr and 0.07$/MVAr, respectively. Other parameters are similar to case 2. The simulation results for this case can be found in Table 4 and Figs. 14, 15 and 16. As shown, for $Q_{c4} = 0.854$p.u, $Q_{c7} = 0.179$ and $Q_{c10} = 0.0$ all transactions will be approved for different values of vsm. This means that the voltage stability margin is increasingly enhanced using the static VAR compensators in the network. The value of reactive power provided by Q_{c4} is much more than the rest of the compensators, although, its price is moderately higher than the others. This indicates two important points: 1-Bus 4 and its vicinity are the best locations for new reactive power capacities installation. 2- The competitive market does not aim at minimizing the reactive power to be purchased, but it attempts to maximize benefits of market participants.

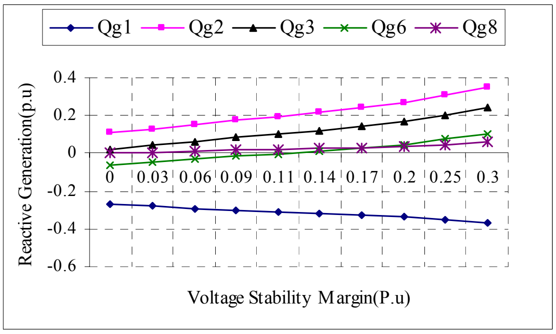

Fig. 14. Reactive power outputs of the generators (Case 4)

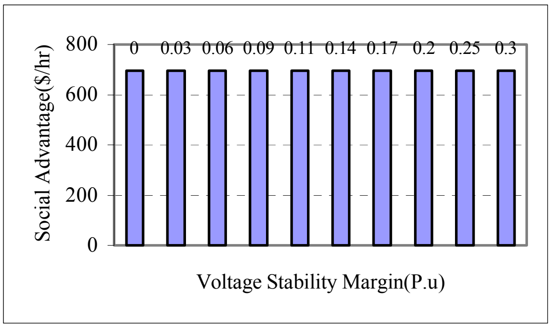

Fig.15.Variarion of social benefits (Case 4)

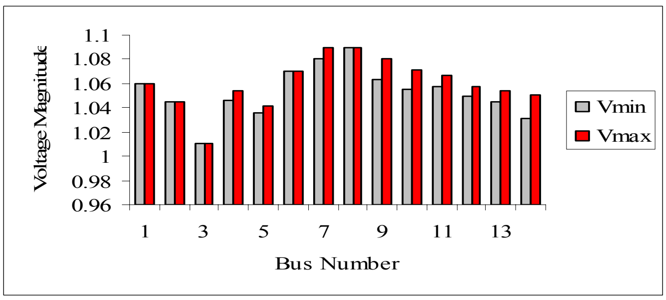

Fig. 16. Voltages variation with respect to increasing vsm (Case 4)

Case 5. The main purpose of this last case is to investigate the impact transactions offer in their approving process. Therefore, we assume that all situations are the same as case 2, but $T_{1\text{-}9}$ and $T_{1\text{-}10}$ have different offers as 3.8 and 5.6 \$/MVAr, respectively. The simulation results are reported in Table 5 and Figs 17and 18. As Table 5 shows, T_{1-10} will be approved in all cases; however, T_{1-9} will be curtailed for $vsm = 0.09$ and more. The restriction on reactive power support provided by generators 2 and 3 is the main cause of this phenomenon. From this simulation, one can conclude that the proposed model not only acts based on receiving offers from competitors, but also considers the effect of each transaction on the social benefit index which is the most important feature of this structure.

Table 4.Results of transactions dispatch incorporating voltage stability margin (Case 4)

	vsm =0.0	vsm =0.03	vsm =0.06	vsm =0.09	vsm =0.11	vsm =0.14	vsm =0.17	vsm =0.20	vsm =0.25	vsm =0.30
T_{1-3}	0.942	0.942	0.942	0.942	0.942	0.942	0.942	0.942	0.942	0.942
T_{1-4}	0.478	0.478	0.478	0.478	0.478	0.478	0.478	0.478	0.478	0.478
T_{1-5}	0.076	0.076	0.076	0.076	0.076	0.076	0.076	0.076	0.076	0.076
T_{1-6}	0.112	0.112	0.112	0.112	0.112	0.112	0.112	0.112	0.112	0.112
T_{1-9}	0.295	0.295	0.295	0.295	0.295	0.295	0.295	0.295	0.295	0.295
T_{1-10}	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09
T_{1-11}	0.035	0.035	0.035	0.035	0.035	0.035	0.035	0.035	0.035	0.035
T_{1-12}	0.027	0.027	0.027	0.027	0.027	0.027	0.027	0.027	0.027	0.027
T_{1-13}	0.135	0.135	0.135	0.135	0.135	0.135	0.135	0.135	0.135	0.135
Sum (p.u)	***2.19***	***2.19***	***2.19***	***2.19***	***2.19***	***2.19***	***2.19***	***2.19***	***2.19***	***2.19***
	vsm =0.0	vsm =0.03	vsm =0.06	vsm =0.09	vsm =0.11	vsm =0.14	vsm =0.17	vsm =0.20	vsm =0.25	vsm =0.30
T_{2-2}	0.217	0.217	0.217	0.217	0.217	0.217	0.217	0.217	0.217	0.217
T_{2-12}	0.034	0.034	0.034	0.034	0.034	0.034	0.034	0.034	0.034	0.034
T_{2-14}	0.149	0.149	0.149	0.149	0.149	0.149	0.149	0.149	0.149	0.149
Sum (p.u)	***0.4***	***0.4***	***0.4***	***0.4***	***0.4***	***0.4***	***0.4***	***0.4***	***0.4***	***0.4***

Table 5. Results of transactions dispatch incorporating voltage stability margin (Case 5)

	vsm =0.0	vsm =0.03	vsm =0.06	vsm =0.09	vsm =0.11	vsm =0.14	vsm =0.17	vsm =0.20	vsm =0.25	vsm =0.30
T_{1-3}	0.942	0.942	0.942	0.942	***0.932***	***0.91***	***0.89***	***0.87***	***0.84***	***0.812***
T_{1-4}	0.478	0.478	0.478	0.478	0.478	0.478	0.478	0.478	0.478	0.478
T_{1-5}	0.076	0.076	0.076	0.076	0.076	0.076	0.076	0.076	0.076	0.076
T_{1-6}	0.112	0.112	0.112	0.112	0.112	0.112	0.112	0.112	0.112	0.112
T_{1-9}	0.295	0.295	0.295	***0.284***	***0.261***	***0.231***	***0.203***	***0.176***	***0.133***	***0.092***
T_{1-10}	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09
T_{1-11}	0.035	0.035	0.035	***0***	***0***	***0***	***0***	***0***	***0***	***0***
T_{1-12}	0.027	0.027	0.027	0.027	0.027	0.027	0.027	0.027	0.027	0.027
T_{1-13}	0.135	0.135	0.135	0.135	0.135	0.135	0.135	0.135	0.135	0.135
Sum (p.u)	***2.19***	***2.19***	***2.19***	***2.144***	***2.111***	***2.059***	***2.011***	***1.964***	***1.891***	***1.822***
	vsm =0.0	vsm =0.03	vsm =0.06	vsm =0.09	vsm =0.11	vsm =0.14	vsm =0.17	vsm =0.20	vsm =0.25	vsm =0.30
T_{2-2}	0.217	0.217	0.217	0.217	0.217	0.217	0.217	0.217	0.217	0.217
T_{2-12}	0.034	0.034	0.034	0.034	0.034	0.034	0.034	0.034	0.034	0.034
T_{2-14}	0.149	0.149	0.149	0.149	0.149	0.149	0.149	0.149	0.149	0.149
Sum (p.u)	***0.4***	***0.4***	***0.4***	***0.4***	***0.4***	***0.4***	***0.4***	***0.4***	***0.4***	***0.4***

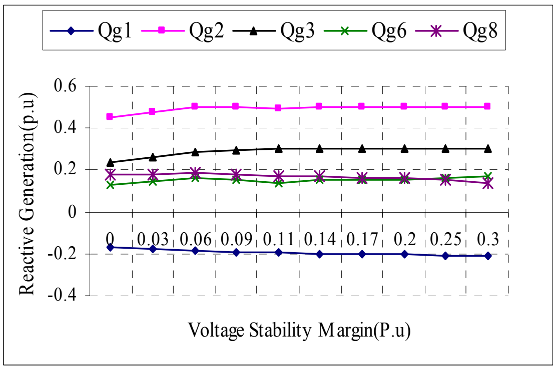

Fig. 17. Reactive power outputs of the generators (Case5)

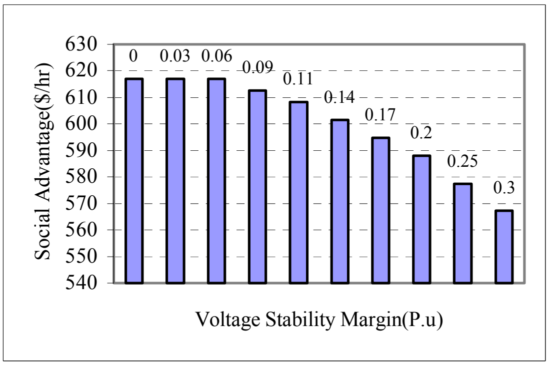

Fig. 18. Variarion of social benefits (Case5)

7. CONCLUSION

In this paper, a methodology for reactive power procurement in a competitive market structure is proposed. In this market, a proper social welfare cost is defined as the main objective function of the market operation. The market equations are arranged to include a voltage stability problem in the reactive power dispatch process. Maximizing the social benefits of market players reveals many aspects which rarely appear in current market mechanisms for reactive power procurement in the deregulated power systems. In our proposed structure, which has a good coherency with supply-demand concepts, reactive power is procured based on economical features. Market equations are written in the form of an OPF-based formulation to incorporate the voltage stability margin and are solved by using the high level-programming platform GAMS choosing MINOS solver [19]. The proposed methodology has been tested on IEEE 14 bus test system, and simulation results for several case studies are presented to show the different aspects of market performances. Simulation results show the efficiency of the proposed structure for reactive power market design and simulation.

NOMECLATURE

$C_{gpi}(\circ)$	operation cost of the ith generator for active power generation
K_{gi}	profit rate of active power
r_{cj}	bid price of the jth SVC for providing reactive energy in ($/MVAr-h)
T'_{ij}	ij th element of *BTM* transposed
W_{ij}	ijth element of the matrix *TFB*
α_G	set of generator buses
α_C	set of buses indicating the location of SVCs
α_D	set of load buses
$P_{reg}^{\circ Slack}$	regulation active power output of the slack generator at normal condition
$P_{reg}^{vsmSlack}$	regulation active power output of the slack generator at increased load condition
λ°	Lagrange multiplier vector associated with active and reactive power flow equations at normal condition
μ°	Lagrange multiplier vector associated with the inequality constraints at normal condition
s°	slack variable vector associated with the bounded variables at normal condition
$C_{gqi}(\circ)$	operation cost of the ith generator for reactive power generation
$W_{gqi}(\circ)$	bid price of the ith generator for providing reactive energy in ($/MVAr-h)

BTM	bilateral transaction matrix, where the ij element of this matrix, T_{ij}, denotes the bilateral transaction level held between the i[th] generator and j[th] consumer
TFB	transaction firmness bid matrix
Q_{Gi}°	reactive power output of the i[th] generator at normal condition
Q_{Cj}°	reactive power output of the j[th] SVC at normal condition
Q_{Gi}^{vsm}	reactive power output of the i[th] generator at increased load condition
V_i°	i[th] bus voltage magnitude at normal condition
V_i^{vsm}	i[th] bus voltage magnitude at increased load condition
λ^{vsm}	Lagrange multiplier vector associated with active and reactive power flow equations at stressed condition
μ^{vsm}	Lagrange multiplier vector associated with the inequality constraints at normal condition
s^{vsm}	slack variable vector associated with the bounded variables at normal condition

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