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Development of a Modified Relationship Between the Constant Friction Factor and the Coefficient of Friction in Extrusion Using Upper Bound Analysis

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ABSTRACT

This study evaluates friction in bulk metal forming processes, where frictional shear stress is commonly described by either Coulomb's friction law or the constant friction factor law. Despite their widespread use, establishing a precise correlation between the constant friction factor (m) and the coefficient of friction (μ) remains challenging. Building on previous work, a modified relationship between these parameters is proposed to improve consistency across practical friction ranges. The new equation is based on upper bound analysis, incorporating both the constant friction factor and Coulomb's hypothesis. Using regression on the extrusion process results, the equation was obtained and validated. The proposed equation satisfies the required boundary conditions and offers simplified implementation in forming simulations. Comparative analysis shows strong agreement, particularly under low-friction conditions typical of metal forming. The findings enhance the reliability of frictional behavior modeling in engineering applications.

1. Introduction

In metal forming processes, friction is a critical factor that significantly influences numerous parameters, including tool life, material formability, surface finish quality, internal microstructure, and ultimately the performance and durability of the final product. The interaction at the tool-workpiece interface generates frictional forces that can profoundly affect process efficiency and product integrity. Excessive friction not only increases heat generation but also accelerates tool wear, causes material pick-up on the tool surface, and may cause galling, all of which contribute to premature tool failure and higher operational costs. Moreover, high friction can induce non-uniform deformation in the workpiece, leading to defects such as surface cracking, waviness, and dimensional inaccuracies that compromise the quality [1–4].

To address these challenges, lubricants were

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extensively employed in metal forming operations to reduce the frictional resistance and thus mitigate its detrimental effects. However, achieving the lowest possible friction is not always the most desirable objective. In many metal forming processes, friction plays a beneficial role by influencing controlled material flow, thereby helping to achieve the desired product geometry more efficiently and with better mechanical properties. For instance, in rolling, conform extrusion, and extrusion forging, a certain level of friction is advantageous because it enhances grip between the tool and workpiece, facilitating effective material deformation and improving dimensional accuracy. [5-7].

Advancements in computational modeling, particularly the development of plastic finite element analysis (FEA) methods, have revolutionized the study of metal forming processes by enabling precise simulations of local stress, strain, and temperature distributions. The accuracy of these simulations depends heavily on reliable input data that characterize both the mechanical behavior of the work material and the frictional conditions at the tool-workpiece interface. Consequently, extensive research has been devoted to quantifying friction in metal forming, utilizing both real-world forming experiments and controlled laboratory simulations to gather reliable data.

Conventionally, frictional shear stress distributions in bulk metal forming are described using two primary models. The first is Coulomb's friction law, expressed as $\tau = \mu P$, where τ is the frictional shear stress, μ is the coefficient of friction, and P is the normal pressure. The second is the constant friction factor law, $\tau = mK$, where m is the friction factor and K represents the shear yield stress of the material. Each model presents distinct advantages in terms of applicability and simplicity, depending on the forming process or simulation environment [8-11].

Modern commercial finite element software packages such as Abaqus and Ansys are typically designed to incorporate friction either through the coefficient of friction or the constant friction factor approach. In practical metal forming operations, friction effects are often measured and reported based on the friction factor, m, due to its direct link with material properties. However, for analytical modeling and numerical simulation, friction is often expressed in terms of the coefficient of friction, μ. This discrepancy underscores the critical need for establishing a clear and reliable relationship between the friction factor and the coefficient of friction to ensure consistency and accuracy in both experimental and computational analyses. [12,13].

Given that some investigations focus on the constant friction factor approach while others rely on the coefficient of friction, and considering that commercial FEM software may operate with either parameter, the present research aims to bridge this gap by establishing a straightforward and practical correlation between the two. Developing such a relationship will enhance the applicability of friction data across different analytical platforms and improve the accuracy of metal forming simulations. Ultimately, this will facilitate more reliable process optimization and contribute to achieving higher product quality.

2. Relationship Between the Constant Friction Factor and the Coefficient of Friction

Bowden and Tabor [14] derived a theoretical relationship between the friction factor (m) and the coefficient of friction (μ):

$$\mu = \frac{m}{\sqrt{27(1-m^2)}} \tag{1}$$

By definition, $0 \leq m \leq 1$; $m = 0$ corresponds toa frictionless condition, and $m = 1$ represents the opposite extreme of sticking friction.

Similarly, based on ring test experiments, another approximate empirical relationship between these two parameters was suggested [15]:

$$\mu = \frac{m}{2\sqrt{3}} \tag{2}$$

A more precise analysis was conducted by Molaei et al. [16] using finite element analysis (FEA) of the double cup extrusion (DCE) process. The DCE process combines forward and backward extrusion, in which the upper punch moves downward while the container and lower punch remain fixed. Consequently, the frictional conditions at the top and bottom regions differ, producing unequal heights of the upper and lower extruded cups. The ratio of these two heights (H1/H2) is therefore governed by the prevailing frictional state [16, 17]. Molaei et al. examined the influence of different values of m and μ on the H1/H2 ratio and demonstrated that increasing frictional severity leads to a higher H1/H2 ratio. To further clarify this relationship, values of μ and m corresponding to identical cup-height ratios were plotted [16]. Following the form introduced in Eq. (1), a comparable expression was proposed to fit the resulting data [16], with constants determined through regression analysis:

$$\mu = \frac{m^{0.9}}{2.72(1-m)^{011}} \tag{3}$$

This proposed equation demonstrates excellent agreement with the FEA results [16].

3. Results and Discussion

The results of the three equations presented in the previous section are plotted in Fig. 1. The theoretical Eq. (1) and the semi-empirical Eq. (3) both indicate that $\mu = 0$ at $m = 0$ and (μ = unknown) μ approaches infinity as $m = 1$, whereas the empirical Eq. (2) has the advantage of simplicity within the practical range of friction in metal forming processes.

Furthermore, based on robust upper bound theory, a relationship can be established between the friction factor (m) and the coefficient of friction (μ). Upper bound analysis of the extrusion process, assuming a constant friction factor and power minimization, provides the following relationship [18]:

$$m = \frac{2}{3\,ln\left(\frac{R_o}{R_f}\right)}\alpha_{opt}^2 \tag{4}$$

Where R_o and R_f are the initial and final radii of the workpiece, respectively, and α_{opt} represents the optimum die angle.

Similarly, upper bound analysis of the extrusion process, when using Coulomb's hypothesis and the principle of power minimization, yields a more complex relationship between the coefficient of friction (μ), with R_o, R_f, and α_{opt} as follows [18]:

$$\mu = 2\frac{sin\,\alpha_{opt}\left(\sqrt{1-\frac{11}{12}sin^2\alpha_{opt}}-(cos\,\alpha_{opt})f(\alpha_{opt})\right)ln\left(\frac{R_o}{R_f}\right)+\frac{1}{\sqrt{3}}\left(1-\frac{\alpha_{opt}}{tan\,\alpha_{opt}}\right)}{\left(1+ln\left(\frac{R_o}{R_f}\right)\right)ln\left(\frac{R_o}{R_f}\right)} \tag{5}$$

Where:

$$f(\alpha_{opt}) = \frac{1}{sin^2\alpha_{opt}}\left(1-cos\,\alpha_{opt}\sqrt{1-\frac{11}{12}sin^2\alpha_{opt}} + \frac{1}{\sqrt{11\times 12}}ln\left(\frac{1+\sqrt{\frac{11}{12}}}{\sqrt{\frac{11}{12}}cos\,\alpha_{opt}+\sqrt{1-\frac{11}{12}sin^2\alpha_{opt}}}\right)\right) \tag{6}$$

Since the optimal die angle is the same in both Eqs. (4) and (5), eliminating the angle and plotting the two equations over the same range allows the establishment of a numerical relationship between the coefficient of friction (μ) and the constant friction factor (m) for various die angles ranging from 0° to 90°. The resulting data are also plotted in Fig. 1. The upper bound analysis results can be observed to be in good agreement with previously proposed relationships, particularly in the sliding friction region, that is, for small values of m and μ.

In light of the previous discussion and based on the upper bound results, a new modified equation relating the friction factor (m) and the coefficient of friction (μ)is proposed using regression analysis of the resulting data:

$$\mu = \frac{m}{2\sqrt{3(1-m^2)}} \tag{7}$$

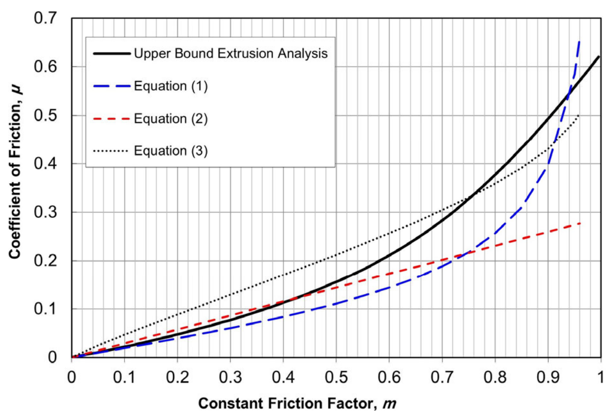

Fig. 1. Comparison of the theoretical Eq. (1), empirical Eq. (2), and semi-empirical Eq. (3) with numerical results of upper bound analysis of open die extrusion.

Despite its simplicity, Eq. (6) offers two significant advantages. First, it rigorously satisfies the essential boundary conditions: when the friction factor $m = 0$, the coefficient of friction μ correctly equals zero, reflecting a frictionless scenario, and when $m = 1$, μ approaches an undefined or limiting value consistent with physical expectations. This ensures that the equation remains valid and meaningful across the entire range of friction factor values. Second, the numerical results derived from Eq. (6) closely match those obtained from the more complex upper bound analysis, as demonstrated in Fig. 2. This close agreement validates the accuracy of the simplified equation, confirming it as a reliable approximation. Moreover, the equation is semi-empirical in nature, meaning it is grounded in theoretical principles while also being calibrated using regression on empirical or numerical data. This combination allows it to satisfy theoretical boundary conditions and maintain practical relevance, making it highly useful for engineering applications where a balance between accuracy and simplicity is essential.

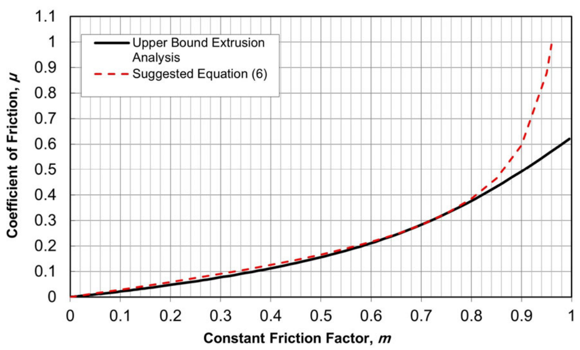

Fig. 2. Comparison of the new Eq. (6) with numerical results of upper bound analysis of open die extrusion.

4. Conclusions

This study proposed and developed a new modified relationship between the constant friction factor (m) and the coefficient of friction (μ), offering improved accuracy and practicality for bulk metal forming processes. Unlike traditional models, the proposed equation is valid across a wide range of friction conditions and maintains mathematical consistency by satisfying known boundary conditions. It bridges the gap between theoretical and empirical formulations, combining simplicity with physical relevance. The relationship was validated using results from a well-established upper bound analysis of an extrusion process, which confirmed its reliability. A comparative analysis showed that the new equation aligns closely with existing theoretical and semi-empirical models, especially under low-friction conditions typical of sliding-dominant forming operations. The numerical results derived from upper bound theory further reinforce the robustness of the proposed correlation. This enhanced understanding of the relationship between the constant friction factor and the coefficient of friction contributes to a more accurate modeling of frictional effects in metal forming simulations. Consequently, the proposed relationship can serve as a useful tool for engineers and researchers in designing and optimizing forming processes.

Conflict of interest

The author declares that there are no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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