

A PULSE-TRAIN MIMO RADAR BASED ON THEORY OF INDEPENDENT COMPONENT ANALYSIS^{*}

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Abstract– The idea of co-located Multiple-Input Multiple-Output (MIMO) radars is based on transmission of orthogonal signals. In conventional co-located MIMO radars, usually a set of orthogonal code modulated pulses is transmitted. In this approach, finding orthogonal signals with a proper range side-lobe level is a problem. In this paper the approach of transmitting a set of proper pulse-trains is proposed. In the pulse-train signaling, pulse compression is achieved by the stepped frequency idea and so, the range side-lobes of the compressed codes are not a problem in the process of code selection anymore. To separate the received pulse trains, a new approach based on the Independent Component Analysis (ICA) is proposed. Compared to other presented approaches which use a set of filter banks, it is shown that the ICA-based approach is less sensitive to the Doppler effect and the orthogonality of signals. So, better beam-forming features and less error in Direction of Arrival (DOA) estimation is gained in this approach. According to this approach, an appropriate signal design method is presented, based on the separation performance of ICA algorithms. It is shown that independent random sequences are proper signals in this sense.

Keywords– MIMO radar, co-located antennas, ICA, pulse-train signaling

1. INTRODUCTION

The advent of phased array radars enables designers to improve estimation and detection performance of radar systems [1, 2]. The ability to exploit these performances is limited by the realizable number of receiver elements and the computational power available at the receiver. Recently, intensive research has focused on the development of a new radar structure that is known as Multiple-Input Multiple-Output (MIMO) radar [3–13]. In the MIMO radar concept, multiple antennas are employed for emitting several waveforms and multiple antennas are used to receive the echoes reflected by the targets.

Co-located MIMO radar structure is a common type of MIMO radar that is quite similar to the phased-array radars [8]. In this type, for a given number of receiver elements, signal diversity can virtually increase the effective number of array elements. In other words, the waveform diversity leads to increasing the virtual aperture of the receiving array [8]. In co-located MIMO radars, a point target model is usually assumed, and a far-field signal source and narrow-band transmitted signal are also considered. Co-located MIMO radars have many advantages compared to traditional approaches of phased array signaling. Improvement of angle estimation, better angular side-lobe level (SLL) and beam shape characteristics are some of these advantages [8]. These benefits can be obtained only with the assumption of ideal signal separation in the receiver. In other words, orthogonality should be considered for the transmitted signals [10]. But for MIMO radars with numerous transmitter elements, it is difficult to find fully orthogonal signals. Only frequency separated signals are offered as fully orthogonal signals for MIMO radar signaling [14–16].

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Another approach is to transmit m orthogonal base-band signals or codes on a single carrier frequency. In this signaling, a matched-filter or correlator is used to separate signals in each receiver [1, 17]. There are some problems in this approach that cannot be neglected. Range side-lobe level is the first problem that destroys the radar range resolution of strong and weak targets near each other. The most important problem is to find a set of orthogonal signals which fulfill all radar signaling restrictions. Even if it is possible to find orthogonal signals, the Doppler effect may destroy the orthogonality. Suppose that for the signal length of T , the transmitted signals are given by $s_k(t)$, $0 < t < T$ and the sampled signal is $s_k[n] = s_k(n\tau_p)$ where $k = 1, \dots, m$, n is an integer that indicates the sampling number, τ_p is the duration of a single chip and $c\tau_p/2$ is the radar resolution cell size. The ideal orthogonality condition of signals can be formulated as:

$$\langle s_k[n-p]^* s_\ell[n] \rangle = \delta[k-\ell]\delta[p] \quad k, \ell = 1, \dots, m \quad (1)$$

where δ is the Kroneckers delta, $\langle \cdot \rangle$ denotes the inner product and $(\cdot)^*$ denotes complex conjugate. Assume that $\tilde{s}_k(t) = s_k(t)e^{i\omega_k t}$, $k = 1, \dots, m$ are the Doppler shifted versions of these signals. Based on the Doppler frequency ω_k , if considerable phase shift exists in the signal duration ($\Delta\phi = \omega_k T$), the orthogonality is no longer valid, and we have:

$$\begin{aligned} \langle s_k[n]^* \tilde{s}_k[n] \rangle &\neq 1 & k = 1, \dots, m \\ \langle s_k[n]^* \tilde{s}_\ell[n] \rangle &\neq 0 & k, \ell = 1, \dots, m \end{aligned} \quad (2)$$

In some signal design methods for MIMO radars, the Doppler effect is not considered in the model, assuming that its value is not high enough to change the phase of signal samples. In this case, an optimized waveform design based on the range side lobe level is presented in [7]. In this method a set of single pulse signals is considered and the Doppler effect is ignored in the signal model. Some signaling methods that are used in co-located MIMO radars are set to approximate a given transmit beam-pattern, and also to minimize the cross-correlation of the signals reflected by different targets [18]. The problem of range side lobe level and Doppler effect is not considered in these signalings.

In this paper we present a novel signaling technique for co-located MIMO radars. In the former techniques, an inter-pulse code modulation is employed in the dimension of range-delay (Fig. 1a). But, in this technique an intra-pulse code modulation is employed and a pulse-train signaling is proposed (Fig. 1b). In this approach, each radar transmitter sends a unique batch of pulses that are coded in phase and/or amplitude. In ordinary techniques, by applying the intra-pulse code modulation, the range resolution would be limited by the pulse duration ($c\tau_p/2$). To resolve this restriction, in our presented signaling, different frequency variations are applied to the signals of the transmitters. Applying the frequency difference of $\Delta f = 1/\tau_p$ to all m transmitters, the range resolution can be improved to $c\tau_p/2m$ according to the theory of ‘‘Stepped Frequency Radars’’ [19, 20]. In the inter-pulse modulation signaling, the codes should be designed according to the criteria of good separability and proper pulse compression property. So, range side-lobe of the compressed code is a problem in the inter-pulse modulation. But in the proposed signaling the codes should be designed only based on the separability criterion and the pulse compression is achieved by the stepped frequency idea. So the range side-lobe is not a problem in the process of code selection anymore. Therefore, the complexity of code design is decreased in the proposed signaling scheme.

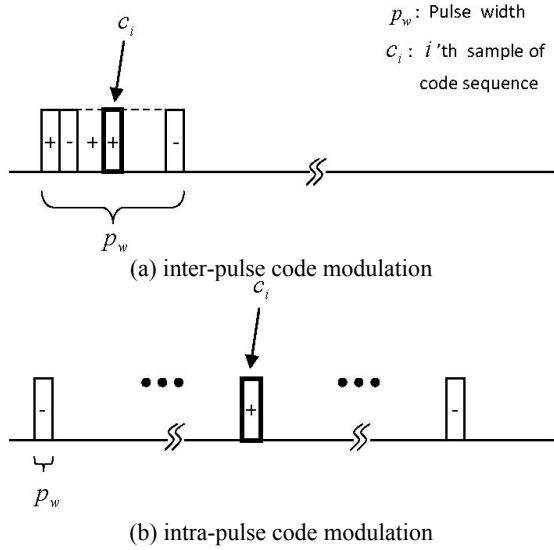


Fig. 1. Comparison of the inter-pulse and the intra-pulse code modulations

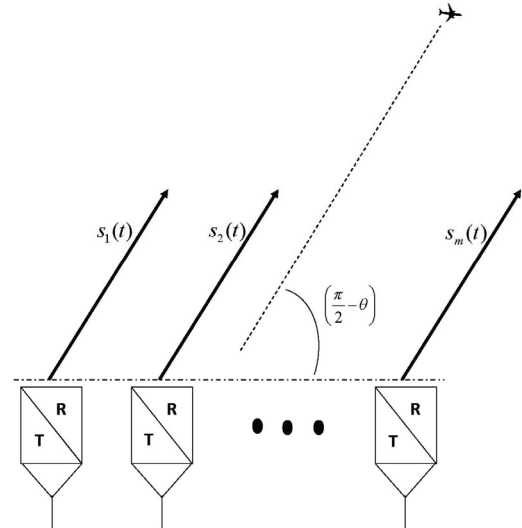


Fig. 2. A uniform linear array by a set of m jointly transmitter/receiver antenna elements

The exact model of the received signal is presented in Section 2. It is shown that estimating the array matrix in this model is the key to beam-forming and extracting target parameters, such as Direction of Arrival (DOA) and Doppler value. In Section 3 the Maximum Likelihood estimator of array mixing matrix for both known and unknown Doppler conditions is proposed. In Section 4, using the advantages of Independent Component Analysis (ICA), a new approach for estimation of the receiver array matrix is presented. The ICA model is introduced in Section 4 and a brief review on ICA techniques is also offered. The performance of ICA techniques for separation of different signals is our constraint to design a set of codes for the proposed pulse-train signaling. This signal design technique is presented in Section 5. Finally, in Section 6 simulation results for the performance evaluation of the proposed estimators is presented.

2. SIGNAL MODEL

Without loss of generality, in this paper we consider a uniform linear array (ULA) that is formed by a set of m jointly transmitter/receiver antenna elements which are separated by 0.5λ (Fig. 2), where $\lambda = c/f_c$ and f_c is the center frequency of the transmitted signals. According to Fig. 1b, we consider a fixed pulse width of $P_w = \tau_p$, where $c\tau_p/2$ is the radar resolution cell size. For a pulse repetition interval (PRI) of T_p , the narrow-band transmitted signals are given by:

$$\begin{cases} s_k(t) \neq 0 & q \times T_p \leq t \leq q \times T_p + \tau_p \\ s_k(t) = 0 & \text{else} \end{cases} \quad (3)$$

where $k = 1, \dots, m$, $q = 0, 1, 2, \dots, L$ and L denotes the number of transmitted pulses. As described in the previous section, to eliminate the problem of range side-lobes, in this signaling, a single chip pulse with suitable duration (τ_p), according to the desired range resolution is transmitted during each T_p . In each transmitted pulse train, the amplitude and phase of the signal can be changed in a suitable manner to make all $s_k(t)$'s separable. We assume that the transmitted signals have different carrier frequencies, but the difference is small compared to the signal band-width. So the reflected signal $\mathbf{x}_k(t)$'s cannot be separated in the frequency band-width. We also assume that the RCS is constant for all carrier frequencies, because of the point target assumption in this model. If we consider f_c as the center carrier frequency, each

transmitted signal has a small frequency shift of Δf_k related to f_c . So, $\mathbf{x}_k(t)$, which is the base-band reflected signal from a far-field point target for the k th transmitted signal is given by:

$$\mathbf{x}_k(t) = \alpha \mathbf{e}_k(\theta) b_k(\theta) s_k(t - \tau) \cdot \exp(2\pi i(f_{d_k} + \Delta f_k)t + i\phi_k) \quad (4)$$

where θ is the direction of target, τ is the received signal delay and $\phi_k = \phi_0 - 2\pi\Delta f_k \times \tau$ is the phase related to this delay that depends on the frequency of the transmitter, and $\phi_0 = -2\pi f_c \tau$ is equal for all transmitted frequencies. Completely similar to the theory of ‘‘Stepped Frequency Radars’’ [19, 20], this range dependent phase variation can be used to improve the range resolution of this system. It is assumed that the delay of the target whose DOA is to be estimated is known. So, in this paper the values of ϕ_k ’s are assumed to be known. According to (4), α is a complex constant, containing the effect of power, RCS and propagation loss. For a single point target in this model, α is equal for all the transmitted signals due to the co-located antenna assumption. $\mathbf{e}(\theta)$ is the $m \times 1$ array factor of receiving antennas for the direction of target, $b_k(\theta)$ is the k th transmitter factor and depends on the direction of the target, and f_{d_k} is the target Doppler frequency. With radial velocity V , f_{d_k} is given by:

$$f_{d_k} = \frac{2V}{\lambda_k} = \frac{2V}{\lambda} \times \frac{\lambda}{\lambda_k} = f_d \times \frac{\lambda}{\lambda_k} \quad (5)$$

where λ_k is the wavelength of the k th transmitted signal and f_d is the Doppler value related to λ , which is the center wavelength. We define $f_k = (f_{d_k} + \Delta f_k)$ for each transmitted signal and so we can rewrite (4) as:

$$\mathbf{x}_k(t) = \alpha \mathbf{e}_k(\theta) b_k(\theta) s_k(t - \tau) \exp(2\pi i f_k t + i\phi_k) \quad (6)$$

The j th element of $\mathbf{e}_k(\theta)$ is given by:

$$\{\mathbf{e}_k(\theta)\}_j = \exp\left(2\pi i \frac{d_j}{\lambda_k} \sin(\theta)\right) \quad j, k = 1, \dots, m \quad (7)$$

Here d_j stands for the relative position of the j th receiver element from the array center. Also, the k th transmitter factor $b_k(\theta)$ can be formulated as:

$$b_k(\theta) = \exp\left(2\pi i \frac{d_k}{\lambda_k} \sin(\theta)\right) \quad k = 1, \dots, m \quad (8)$$

Considering $\mathbf{n}(t)$ as the vector of additive Gaussian noise of receiver elements, the vector of received signal($\mathbf{x}(t)$) would be:

$$\begin{aligned} \mathbf{x}(t) &= \sum_{k=1}^m \mathbf{x}_k(t) + \mathbf{n}(t) = \sum_{k=1}^m \alpha \mathbf{e}_k(\theta) b_k(\theta) \exp(i\phi_k) s_k(t - \tau) \exp(2\pi i f_k t) + \mathbf{n}(t) \\ &= \sum_{k=1}^m \mathbf{a}_k(\theta) \tilde{s}_k(t - \tau) + \mathbf{n}(t) = \mathbf{A}(\theta) \tilde{\mathbf{s}}(t - \tau) + \mathbf{n}(t) \end{aligned} \quad (9)$$

where the $m \times 1$ vector $\mathbf{a}_k(\theta)$, which is the k th column of the $m \times m$ mixing matrix $\mathbf{A}(\theta)$, is given by:

$$\{\mathbf{a}_k(\theta)\}_j = \alpha \{\mathbf{e}_k(\theta)\}_j b_k(\theta) \exp(i\phi_k) \quad j = 1, \dots, m \quad (10)$$

$\tilde{\mathbf{s}}(t - \tau)$ is the $m \times 1$ vector of delayed and frequency shifted source signals. In this model, the direction of target (θ) and the Doppler value of signal (f_d) are both assumed to be unknown. Considering the pulse repetition interval of T_p , after range sampling of the received array signal at times $t = \ell \times T_p + \tau$ for $\ell = 1, \dots, L$, we can form an $m \times L$ matrix of the received data in which L denotes the number of

transmitted/received pulses. In other words, L is the length of the transmitted code sequences in the pulse-train signaling.

$$\mathbf{X}_{m \times L} = \mathbf{A}(\theta) \tilde{\mathbf{S}}_{m \times L} + \mathbf{N}_{m \times L} = \sum_{k=1}^m \mathbf{a}_k(\theta) \tilde{\mathbf{s}}_k^T + \mathbf{N}_{m \times L} \quad (11)$$

where $\mathbf{X}$ is the $m \times L$ matrix of the received data and $\mathbf{N}$ contains the independent samples of receiver Gaussian noise. In this notation, $(.)^T$ denotes transpose and the $L \times 1$ vector $\tilde{\mathbf{s}}_k$, which is the k th column of $\tilde{\mathbf{S}}_{m \times L}$, contains the samples of $\tilde{\mathbf{s}}_k(t - \tau)$ at the desired delay for L consecutive pulses. Each element of $\tilde{\mathbf{s}}_k$ can be written as:

$$\{\tilde{\mathbf{s}}_k\}_\ell = \{\mathbf{s}_k\}_\ell \times \exp(2\pi j f_k (\ell \times T_p + n \tau_p)) \quad (12)$$

where $\{\mathbf{s}_k\}_\ell = s_k(\ell \times T_p + \delta_\tau)$. Now, in order to estimate the direction of the target, beam-forming and extracting the target information, the mixing matrix $\mathbf{A}(\theta)$ or equivalently $\mathbf{a}_k(\theta)$ for $k = 1, \dots, m$ should be estimated. In the next sections of the paper, different estimators for this problem are proposed.

3. MAXIMUM LIKELIHOOD ESTIMATOR

Considering the problem formulation of (11), the ideal Maximum Likelihood (ML) estimator for θ and f_d is derived in [21] and is:

$$\hat{f}_d, \hat{\theta} = \underset{\theta, f_d}{\text{Arg Max}} \frac{\left| \text{tr}(\mathbf{A}(\theta) \tilde{\mathbf{S}}(f_d) \mathbf{X}^H) \right|^2}{\left\| \mathbf{A}(\theta) \tilde{\mathbf{S}}(f_d) \right\|^2} \quad (13)$$

where $\|\cdot\|$ denotes the Frobenius norm and $\text{tr}(\cdot)$ denotes the trace of matrix. It is also shown in [21] that the computational load of this estimator is very high and so, the practical application of this detector is not realizable. So, we propose a two-step ML estimator in this paper, instead of the ideal estimator of (13). In the two-step ML estimator, first the array matrix of $\mathbf{A}(\theta)$ is estimated. Then, this estimated array matrix is used to determine the value of target direction. It is shown in Appendix I that the ML estimator of $\mathbf{A}(\theta)$ is:

$$\hat{\mathbf{A}}(\theta) = \mathbf{X} \tilde{\mathbf{S}}^H(f_{d_0}) (\tilde{\mathbf{S}}(f_{d_0}) \tilde{\mathbf{S}}^H(f_{d_0}))^{-1} \quad (14)$$

where $f_{d_0} = \underset{f_d}{\text{Arg Max}} \left\| \mathbf{X} \tilde{\mathbf{S}}^H(f_{d_0}) (\tilde{\mathbf{S}}(f_{d_0}) \tilde{\mathbf{S}}^H(f_{d_0}))^{-1} \tilde{\mathbf{S}}(f_{d_0}) \right\|$

Another estimator that can be used to estimate the array matrix is based on the conventional technique of matched-filtering. If f_{d_k} (and hence f_k) or equivalently the signal matrix $\tilde{\mathbf{S}}$ is known, the matched-filter estimator is given by [1]:

$$\hat{\mathbf{A}}(\theta) = \mathbf{X} \tilde{\mathbf{S}}^H \quad (15)$$

But in practice, the Doppler values and so the signal matrix $\tilde{\mathbf{S}}$ is unknown. In this case a bank of matched-filters can be used for different Doppler values. So, this ad-hoc estimator is given by:

$$\hat{\mathbf{A}}(\theta) = \mathbf{X} \tilde{\mathbf{S}}^H(f_{d_0}) \quad (16)$$

where $f_{d_0} = \underset{f_d}{\text{Arg Max}} \left\| \mathbf{X} \tilde{\mathbf{S}}^H(f_{d_0}) \right\|$

If n_f is the number of desired Doppler values in the interval of $[0, PRF)$, this structure consists of n_f filters in each receiver. The frequency space of these matched filters is very important in this method. On the other hand, even if we design a suitable orthogonal signal set i.e $\mathbf{S} \mathbf{S}^H = \mathbf{I}$, due to Doppler effect and the difference of λ_k 's, the orthogonality condition ($\tilde{\mathbf{S}} \tilde{\mathbf{S}}^H = \mathbf{I}$) no longer holds. So the matched filter bank of

(16) is not the ML estimator, even for orthogonal signals. In the rest of this paper we call the ad-hoc estimator of (16) the “MF estimator” and the two-step ML estimator of (14) the “ML estimator”.

4. ICA BASED ESTIMATOR

As mentioned before, in the pulse-train technique, for each transmitter unit one can change the amplitude and/or phase of the signal in a certain manner. In other words, pulse to pulse coding technique can be employed for the transmitted signals and the applied code to each unit is different. Random or pseudo-random pulse to pulse coding is a suitable simple example. The randomness and independency of the transmitted signals from pulse to pulse lead us to consider ICA algorithms as a suitable solution in the estimation of array mixing matrix. Before describing the complete solution and deriving the ICA estimator, a brief review of ICA is presented in the next subsection.

a) ICA model

The theory of Independent Component Analysis is described in [22–25]. In the standard ICA model, we suppose a Linear Structure of the form:

$$\mathbf{X} = \mathbf{A}\mathbf{S} + \mathbf{N} \quad (17)$$

where $\mathbf{S}$ is the $m \times L$ dimensional matrix of source signals. The row of $\mathbf{S}$, named $\mathbf{s}_k^T$, $k = 1, \dots, m$ are all independent and each $\mathbf{s}_k^T$ is a vector of L i.i.d samples. $\mathbf{A}$ is the unknown $m \times m$ mixture matrix and $\mathbf{N}$ is the additive Gaussian noise. Also, $\mathbf{X}$ is the available data matrix in which any row ($\mathbf{x}_k^T$) is a linear mixture of independent sources ($\mathbf{s}_k^T$)’s. Different ICA methods are all designed to estimate $\mathbf{A}$ or equivalently $\mathbf{W} = \mathbf{A}^{-1}$ and the source signals, $\mathbf{S}$. Since $\mathbf{A}$ and $\mathbf{S}$ are both unknown, without the assumption of independent source signals, the problem cannot be solved in general. Different ICA methods use different available statistical properties of signals to solve this estimation problem. ICA finds the desired sources by maximizing the statistical independence of the estimated components. There are many ways to define independency and each way may result in a different form of the ICA algorithm. The two major definitions for independence in ICA are based on “Minimization of Mutual Information” and “Maximization of non-Gaussianity”.

The family of ICA algorithms that are based on maximization of non-Gaussianity originate from the idea of central limit theorem. Higher order statistics are proper tools to measure the non-Gaussianity of signals. JADE, introduced by Cardoso [26], is the most commonly used method in this class. Because of the proper performance for separation of complex signals, in this paper we focus on using the complex form of the JADE algorithm [27]. ICA methods have two common restrictions. The first is that they cannot determine the power and phase of source signals, and the second is that the separation is not possible for signals with Gaussian distribution. Due to ambiguity in signal power, it would be better to assume that the variances of source signals are equal to one. Also, in most ICA methods, without loss of generality, it is assumed that both the data and source signals have zero mean. Because of the second restriction of ICA methods, no mixture of Gaussian signals can be estimated by ICA algorithms. This restriction is caused by the structure of ICA algorithms that are usually based on the measurement of non-Gaussianity of mixed signals. In order to simplify the ICA problem for the practical iterative algorithms, typical ICA methods use centering, whitening, and dimension reduction as preprocessing steps [22].

One important property of ICA methods is that, most of the criteria that are used as the base of these techniques get no effect from the additive Gaussian noise. For example, the higher order cumulants are not affected by the additive Gaussian signals. Therefore, theoretically JADE and other algorithms that are based on higher order cumulants are immune to Gaussian noise for estimating the mixing matrix. But in practice, because of the limited number of samples, the approximated value of cumulants that are affected

by Gaussian noise are used, leading to increase of error in estimating the mixing matrix. So to decrease the noise effect, a noise reduction filter is usually applied before ICA estimation [22]. A wiener filter is the ideal structure for noise cancelation in this case (Appendix II).

ICA estimated signals usually have three ambiguities.

- 1) The actual number of source signals.
- 2) The proper scaling or phase of the source signals.
- 3) A uniquely correct ordering of the source signals.

Comparing Eqs. (17) and (11), it can be seen that the MIMO radar problem can be considered as an ICA problem. In this problem, the first ambiguity does not exist, because the number of sources is equal to m and is known. But the other two ambiguities cannot be ignored and should be solved in the MIMO radar problem. So, as it will be described in the following section, a proper solution is proposed for removing these ambiguities.

b) Derivation of ICA estimator

As described in the previous section, we confront an ICA problem of the form demonstrated in Eq. (11). We can easily transform this time domain equation to an equivalent frequency domain one by a matrix multiplication as:

$$\mathbf{X}^f = \mathbf{X} \cdot \mathbf{F} = \tilde{\mathbf{A}} \tilde{\mathbf{S}} \cdot \mathbf{F} + \mathbf{N} \cdot \mathbf{F} = \tilde{\mathbf{A}} \tilde{\mathbf{S}}^f + \mathbf{N}^f \quad (18)$$

where $\mathbf{F}$ is the $L \times L$ matrix of discrete Fourier transform, $\mathbf{X}^f$ is the $m \times L$ matrix of the received signal in the frequency domain and $\tilde{\mathbf{S}}^f$ contains the Fourier transform samples of $\tilde{\mathbf{S}}$. In the time domain, the frequency shift of the signal may change the estimated values of the covariance matrix, higher order statistics and other statistical terms that are required in ICA estimation. Considering the Doppler effect in the frequency domain, it can be seen that the effect may cause only a circular shift in the frequency samples of the signal. So the estimation of data statistics is more robust against the Doppler variation in the frequency domain. Therefore, the frequency domain modeling of the signal as (18) leads to better results in ICA estimation. Solving this ICA problem, we will find a linear estimator named $\hat{\mathbf{W}}$ that ideally should be equal to $\mathbf{A}^{-1}$. Then, for a zero mean signal we have:

$$\hat{\mathbf{Y}}^f = \hat{\mathbf{W}} \mathbf{X}^f = \hat{\mathbf{W}} \mathbf{A} \tilde{\mathbf{S}}^f + \hat{\mathbf{W}} \mathbf{N}^f \quad (19)$$

To estimate the source signals, the above estimated matrix can also be applied to the time domain signal as:

$$\hat{\mathbf{Y}} = \hat{\mathbf{W}} \mathbf{X} = \hat{\mathbf{W}} \mathbf{A} \mathbf{S} + \hat{\mathbf{W}} \mathbf{N} \quad (20)$$

Even if we consider an ideal solution and a noise-free signal model, there is an inevitable ambiguity in the phase and ordering of signals that can be formulated as:

$$\begin{aligned} \hat{\mathbf{y}}_k &= \exp(i\varphi_k) \times \tilde{\mathbf{s}}_j \\ \hat{\xi}_k &= \exp(-i\varphi_k) \times \mathbf{a}_j \end{aligned} \quad (21)$$

where the $L \times 1$ vector $\hat{\mathbf{y}}_k$ is the k th estimated signal (k th row of $\hat{\mathbf{Y}}$) which does not normally correspond to the k th signal $\hat{\mathbf{s}}_k$. Also, $\hat{\xi}_k$ is the k th column of $\hat{\mathbf{W}}^{-1}$ and normally is not equal to the k th column of $\mathbf{A}$. The reason for the incorrect ordering is that the inverse of the estimated mixing matrix $\hat{\mathbf{W}}^{-1}$ can be equal to $\mathbf{A} \mathbf{P}^{-1}$ instead of $\mathbf{A}$, where $\mathbf{P}$ is a permutation matrix. The phase ambiguity $\exp(j\varphi_k)$ can be neglected for order correction. But in order to achieve the beam-forming features of co-located MIMO radars, this phase should be compensated.

1. Arrangement and phase correction: We rewrite (21) as:

$$\hat{\mathbf{y}}_k = \mathbf{h}_j(f_d) \rho_k \quad (22)$$

where $\rho_k = \exp(i\varphi_k)$, $\mathbf{h}_j(f_d)$ is a $L \times 1$ vector that $[h_j(f_d)]_\ell = [\tilde{\mathbf{s}}_j]_\ell = [\mathbf{s}_j]_\ell \times \exp\{2\pi i(f_d \cdot \frac{\lambda}{\lambda_j} + \Delta f_j)t_\ell\}$. Then the Least-Squares (LS) estimator for the unknown parameters minimizes J , as shown below:

$$J(\rho_k, j, f_d) = (\hat{\mathbf{y}}_k - \mathbf{h}_j(f_d) \rho_k)^H (\hat{\mathbf{y}}_k - \mathbf{h}_j(f_d) \rho_k) \quad (23)$$

To solve this minimization problem, first we suppose that f_d and j or equivalently $\mathbf{h}_j(f_d)$ is known, then we have:

$$\hat{\rho}_k = (\mathbf{h}_j^H(f_d) \mathbf{h}_j(f_d))^{-1} \mathbf{h}_j^H(f_d) \hat{\mathbf{y}}_k \quad (24)$$

Replacing (24) in (23) we have:

$$\begin{aligned} J(j, f_d) &= \left(\hat{\mathbf{y}}_k - \mathbf{h}_j(f_d) (\mathbf{h}_j^H(f_d) \mathbf{h}_j(f_d))^{-1} \mathbf{h}_j^H(f_d) \hat{\mathbf{y}}_k \right)^H \times \left(\hat{\mathbf{y}}_k - \mathbf{h}_j(f_d) (\mathbf{h}_j^H(f_d) \mathbf{h}_j(f_d))^{-1} \mathbf{h}_j^H(f_d) \hat{\mathbf{y}}_k \right) \\ &= \hat{\mathbf{y}}_k^H \left(\mathbf{I} - \mathbf{h}_j(f_d) (\mathbf{h}_j^H(f_d) \mathbf{h}_j(f_d))^{-1} \mathbf{h}_j^H(f_d) \right) \hat{\mathbf{y}}_k \end{aligned} \quad (25)$$

since $(\mathbf{I} - \mathbf{h}(\mathbf{h}^H \mathbf{h})^{-1} \mathbf{h}^H)$ is an idempotent matrix [28], minimization of $J(j, f_d)$ is equivalent to maximizing $J'(j, f_d)$ as:

$$J'(j, f_d) = \hat{\mathbf{y}}_k^H \mathbf{h}_j(f_d) (\mathbf{h}_j^H(f_d) \mathbf{h}_j(f_d))^{-1} \mathbf{h}_j^H(f_d) \hat{\mathbf{y}}_k \quad (26)$$

then the estimator is given by:

$$(\hat{j}, \hat{f}_d^k) = \underset{j, f_d}{\text{Arg Max}} (J'(j, f_d)) = \underset{j, f_d}{\text{Arg Max}} \left(\frac{\left| \sum_{\ell=1}^n \hat{\mathbf{y}}_k[\ell] \mathbf{s}_j^*[\ell] \exp\{-2\pi i(f_d \cdot \frac{\lambda}{\lambda_j} + \Delta f_j)t_\ell\} \right|^2}{\sum_{\ell=1}^n \left| \mathbf{s}_j[\ell] \exp\{-2\pi i(f_d \cdot \frac{\lambda}{\lambda_j} + \Delta f_j)t_\ell\} \right|^2} \right) \quad (27)$$

where $t_\ell = \ell \times T_p$. Considering equal power for the transmitted signals, this estimator can be simplified as a windowed periodogram estimator as:

$$(\hat{j}, \hat{f}_d^k) = \underset{j, f_d}{\text{Arg Max}} (J'(j, f_d)) = \underset{j, f_d}{\text{Arg Max}} \left(\frac{1}{N} \left| \sum_{\ell=1}^n \hat{\mathbf{y}}_k[\ell] \mathbf{s}_j^*[\ell] \exp\{-2\pi i(f_d \cdot \frac{\lambda}{\lambda_j} + \Delta f_j)t_\ell\} \right|^2 \right) \quad (28)$$

Applying this estimator, the arrangement is complete. According to (27, 28), the arrangement of signals can be performed with a weighted FFT in practice. In this method the number of frequency test points (n_f) in maximizing $J'(j, f_d)$, specifies the computational load of this algorithm. After signal arrangement, the estimated vector $\hat{\mathbf{h}} = \mathbf{h}_j(\hat{f}_d^k)$ should be applied to (24). So the phase estimator is given by:

$$\hat{\rho}_k = (\hat{\mathbf{h}}^H \hat{\mathbf{h}})^{-1} \hat{\mathbf{h}}^H \hat{\mathbf{y}}_k \quad (29)$$

where $[\hat{\mathbf{h}}]_\ell = [\mathbf{s}_j]_\ell \times \exp\{2\pi i(\hat{f}_d^k \cdot \frac{\lambda}{\lambda_j} + \Delta f_j)t_\ell\}$ and $\hat{\mathbf{h}}$ is a $L \times 1$ vector. Applying arrangement and phase correction, the estimated column of array matrix $\hat{\mathbf{A}}$ is obtained as:

$$\hat{\mathbf{a}}_j = \hat{\rho}_k \hat{\xi}_k \quad (30)$$

Now a beam-former and DOA estimator can be applied to $\hat{\mathbf{A}}$ to extract the spatial information of the target. For a general review, the total processing steps of the proposed algorithm are presented in Fig. 3.

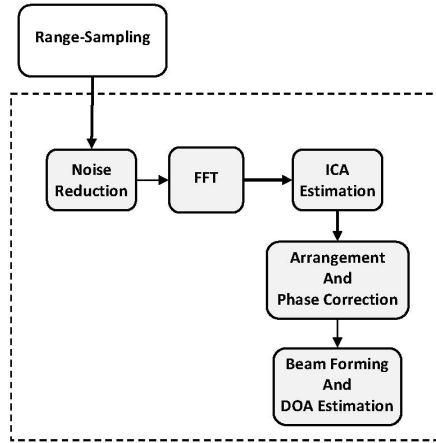


Fig. 3. Block diagram of the presented ICA-based estimator

2. Beam forming and DOA Estimation: After arrangement and phase correction, the estimation of the mixing matrix is available. If the target is in direction θ_t , the estimated mixing matrix is $\hat{\mathbf{A}}(\theta_t)$. From (9) we can see that the k th column of this matrix is the estimated array response of the k th transmitted signal which is:

$$\hat{\mathbf{a}}_k(\theta_t) = (\alpha b_k(\theta_t) \mathbf{e}_k(\theta_t) \exp(i\phi_k)) \quad (31)$$

In co-located MIMO radar, target localization is achieved by computing the beam pattern for all possible θ as:

$$|g(\theta)| = \frac{|\mathbf{d}^H(\theta) \hat{\mathbf{d}}(\theta_t)|}{\|\mathbf{d}(\theta)\|} \quad (32)$$

where $\mathbf{d}(\theta)$ is the MIMO ideal array response to direction θ and $\hat{\mathbf{d}}(\theta_t)$ is the estimated array response of the target at θ_t . For an ordinary MIMO design with no shift in the carrier frequency of the m transmitters, we have:

$$\mathbf{d}(\theta) = [b_1(\theta) \cdots b_m(\theta)]^T \otimes \mathbf{e}(\theta) \quad (33)$$

where $\otimes$ denotes the Kronecker matrix product. But in our presented signaling we have:

$$\mathbf{d}(\theta) = \text{Vec}(\mathbf{A}(\theta)) \quad \text{and} \quad \hat{\mathbf{d}}(\theta_t) = \text{Vec}(\hat{\mathbf{A}}(\theta_t)) \quad (34)$$

where $\text{Vec}(\cdot)$ denotes the vectorization of a desired matrix. Considering a unit norm for each column of $\mathbf{A}(\theta)$, we have $\|\mathbf{d}(\theta)\| = \sqrt{m}$, and then (32) can be written as:

$$|g(\theta)| = \frac{1}{\sqrt{m}} \left| \sum_{k=1}^m \mathbf{a}_k^H(\theta) \hat{\mathbf{a}}_k(\theta_t) \right| \quad (35)$$

Now the DOA estimator is derived by maximizing the conventional beam pattern over θ as:

$$\hat{\theta}_t = \text{Arg Max}_{\theta} \left| \sum_{k=1}^m \mathbf{a}_k^H(\theta) \hat{\mathbf{a}}_k(\theta_t) \right| \quad (36)$$

5. SIGNAL DESIGN

Independent component analysis is a powerful tool to solve our estimation problem. In this estimation problem, we use some information about mixed signals that enables us to solve the problem. Independency is the most important granted property, but other parameters about signals such as

probability density function (PDF) should be considered too. Now the question is, which signals are better separated in this model? In other words, the performance of estimation with ICA should be evaluated by a quantitative parameter such as variance of error or in the more general case, Cramer Rao Bound (CRB). This question is the basis for designing the transmitted pulse trains. In the first step, we neglect the additive noise in the model to derive a measure of performance for our estimation problem. In a noise-free model the de-mixing matrix $\hat{\mathbf{W}}$ is estimated and the estimated signal is given by:

$$\hat{\mathbf{S}} = \hat{\mathbf{W}}\mathbf{X} = \hat{\mathbf{W}}\mathbf{A}\mathbf{S} = \mathbf{G}\mathbf{S} \quad (37)$$

The matrix $\mathbf{G}$, is called the gain matrix and ideally should be equal to the identity matrix [29, 30]. But in practice, its non-diagonal elements have nonzero values. The (k, j) th element of $\mathbf{G}$ represents the residual components of the k th source signal in the j th estimated signal. A quantity of performance of ICA is “Signal to Interference Ratio”(SIR) that is defined as:

$$SIR_j = \frac{E[\mathbf{G}_{jj}^2]}{E[\sum_{k=1, k \neq j}^m \mathbf{G}_{jk}^2]} \quad (38)$$

It is asserted in [29, 30] that the gain matrix is independent of mixing matrix ($\mathbf{A}$). Considering unity variance for diagonal elements, Eq. (38) can be written as:

$$SIR_j = \frac{1}{\sum_{k=1, k \neq j}^m Var(\mathbf{G}_{jk}^2)} \quad (39)$$

Also, in [29] a CRB is derived for this variance that is:

$$CRB(\mathbf{G}_{jk}) = \frac{1}{L} \frac{\kappa_k}{\kappa_j \kappa_k - 1} \quad (40)$$

where κ_j is defined as:

$$\kappa_j = E[\psi^2(\mathbf{s}_j)] \quad (41)$$

In this notation $\psi(s_j)$ is the score function of s_j whose PDF is $f(s_j)$ and is defined as $\psi(s_j) = -f'(s_j)/f(s_j)$. Now we have a measure of separability that is not necessarily realizable. This measure can be derived for different classes of radar signals.

a) Bounded magnitude signals.

Most radar signals such as “Binary Phase Shift Keying” (BPSK) and Linear FM, are bounded in magnitude. It is shown in [29], [30] that the CRB is zero for these signals and also for the random signals that have finite support such as uniformly distributed signals.

b) Random signals

Random signals are widely used as the transmitted radar signals because of the LPI advantages and good ambiguity functions. A wide variety of distributions can be considered to form a random radar signal. However, it is not possible to cover every possible density function. In this paper we consider a category of distributions that contain almost all of the different probability densities. Generalized Gaussian Distributions or “exponential power family”, as completely introduced in [31], have a variable parameter α , whose variation changes the PDF shape and characteristics. For $\alpha = 2$ it is the Normal distribution. $\alpha > 2$ makes distributions that are flatter than the Normal PDF. For example, $\alpha \rightarrow \infty$ results in the uniform PDF. Also, $\alpha < 2$ results in supergaussian or long tail distributions such as laplacian PDF. The significant

property of this class is the existence of the CRB for $\alpha > 0.5$ [31]. For example, for two assumptive systems with $L=200$ and $L=1000$, the CRB of $\text{var}[\mathbf{G}_{jk}]$ in equation (39) has been shown in Fig. 4 for this family of distributions. In this figure, we can see that for both small α (supergaussian PDF's) and very large values of α , better separation performance in the sense of CRB can be expected. For $\alpha \rightarrow \infty$ (Uniform distributions), the $\text{CRB} \rightarrow 0$ as asserted in [29], [30]. Although the separation property of supergaussian PDF's is good, bounded magnitude signals such as BPSK and uniformly distributed signals have better performance according to the CRB. So we consider them for radar signal design in the next subsection.

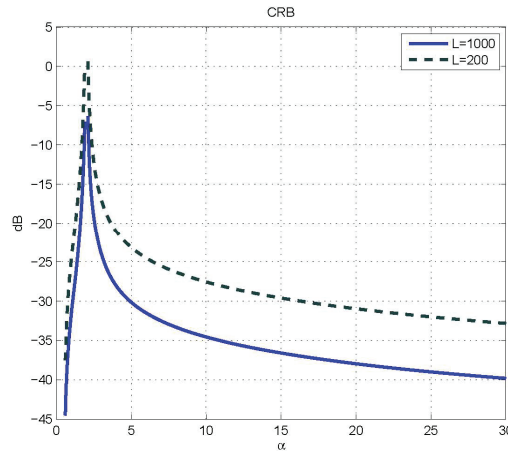


Fig. 4. The CRB of $\text{var}[\mathbf{G}_{jk}]$ for the generalized Gaussian PDF family, assuming $L=1000$ and $L=200$

c) Code sequence design

As mentioned in Section 4.b, even for transmitted signals that have proper separation performance via ICA algorithms, their doppler-shifted version of them may have different characteristics in general. But in frequency domain, the Doppler effect causes only a circular shift in the frequency samples, which does not change the statistical characteristics of the data. This forces us to apply our ICA estimator in the frequency domain. Therefore, the applied amplitude and phase code to the transmitted signal should be designed in a special manner to form a BPSK or uniformly distributed signal in the frequency domain. In this article a computer search was used to select proper signals for the ICA technique. According to Fig. 5, different random BPSK or uniformly distributed signals are mixed by an array-matrix of some arbitrary directions and then an ICA estimator is applied to this mixture. The signal which has the highest SIR is selected as the frequency samples of the transmitted signal. In the next step, applying an inverse FFT to the samples, the $m \times L$ code of the transmitted pulse train is obtained.

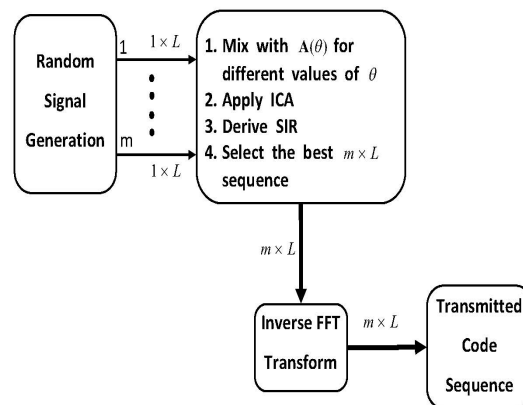


Fig. 5. The structure used to select m batches of transmitted codes for the ICA-based estimator

6. SIMULATION RESULTS

In this section we consider a ULA array of omni-directional transmit/receive antennas separated by 0.5λ . As described in Section 2, in the presented system a small shift is applied to the frequency of different narrowband transmitted signals. Considering a center frequency of f_c , in these simulations, m transmitted frequencies are uniformly spaced in the interval of $f_c \pm 5\%$. Here, a $m=6$ MIMO system with $L=200$ is considered and the obtained sequences, according to Fig. 5, are used for simulation. In this section, a far-field model for target is considered and the fluctuation model as described in [32] is not taken into account. For a hypothetical point target in the direction of $\theta=20^\circ$, the beam patterns of the ideal MIMO radar (single frequency) and the ideal phased-array radar is compared with the resulting beam pattern of the proposed signaling. The comparison for $m=6$ is presented in Fig. 6. It is shown that the beam pattern of the signaling has the maximum Peak to Side-Lobe Level (PSLL) ratio and its 3dB beamwidth is proposed equal to that of the ideal MIMO. The superiorities of co-located MIMO technique compared to the phased array signaling that are “the more PSLL ratio” and “the less 3dB beamwidth” of the beam pattern, are asserted in this figure.

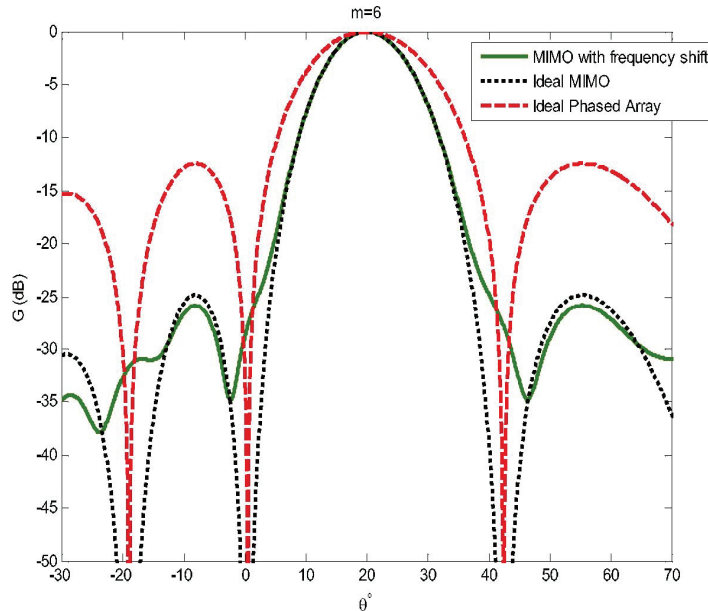


Fig. 6. Beam-patterns of frequency-shift signaling, ideal MIMO and ideal phased-array radars, considering $m=6$

a) Sensitivity of arrangement algorithm to n_f

In the first simulation, sensitivity of the arrangement algorithm of Section 4-b1 to the number of frequency points (n_f) is discussed. For this purpose the probability of error (P_e) in the arrangement is derived for different values of n_f . In this context, for all Doppler values in the interval of $[0, PRF)$, we run the arrangement algorithm. If the arrangement is not completely correct for a desired Doppler value, an error of arrangement occurs in this Doppler. P_e is the probability of arrangement error in all Doppler values of $[0, PRF)$, which is calculated by averaging the overall Doppler values of this interval. As the performance of the arrangement algorithm depends on n_f , computation of P_e is repeated for different values of n_f . The result is shown in Fig. 7. It is shown that for $n_f \geq 105$, we have $P_e=0$, which means that the arrangement is ideally done for every Doppler value if we consider $nf \geq 105$.

b) Code selection

As mentioned in Section 5.a, a computer search is used to select proper transmitted signals for the ICA-based estimator. The procedure is described completely in Fig. 5. In this procedure, different sets of uniformly distributed and random binary signals are studied and the set which leads to the best separation performance in ICA estimation is determined. In our simulations, a set of $m \times L$ uniform signals (named $c1$) is utilized for the ICA-based estimator. A similar computer search is used for selecting a proper set of signals for the MF and ML estimators. In this procedure, different sets of uniform and random binary signals are studied. For each set of signals, the one having the closest beam pattern to the ideal beam pattern is selected. It is seen that compared to the ideal beam pattern, a set of uniformly distributed signals (named $c2$) has the closest beam pattern. So in our simulations, this set of $m \times L$ uniform signals ($c2$) is utilized for the MF and ML estimators.

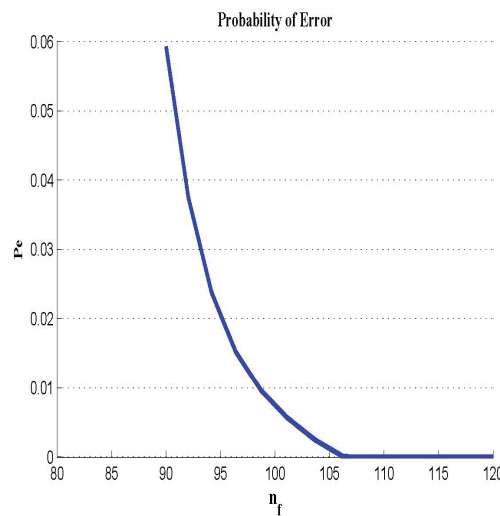
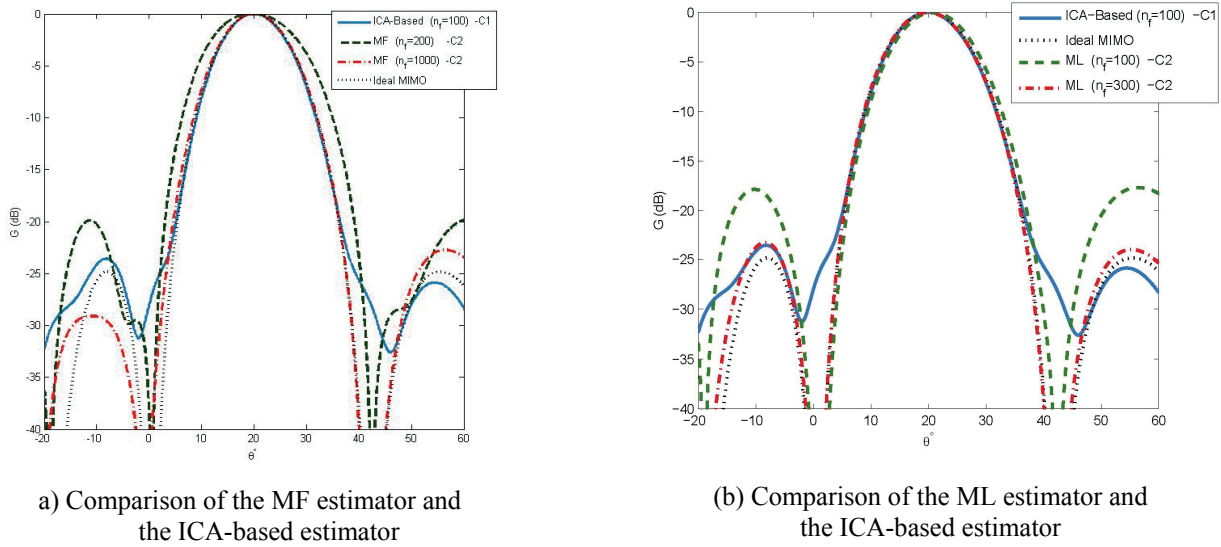


Fig. 7. Probability of error in arrangement, for different frequency points

c) Comparison of beam patterns

In the next simulations, the performance of the presented ICA-based MIMO should be assessed. In this context, the beam pattern of the ICA-based MIMO, the ideal MIMO system (using the true array matrix) and the two presented MF and ML estimators are compared. In these simulations, a noise-free co-located MIMO system and a point target with $\theta_t=20^\circ$ and $f_d=0.1 \times PRF$ are considered. According to Fig. 7, to have ideal arrangement in ICA-based MIMO, the minimum number of frequency steps is $n_f=100$. For the MF estimator of (16) using $c2$ signal, the simulation result shows that poor estimation performance is achieved for $n_f=100$. So the beam pattern of the MF estimator for $n_f=200$ and $n_f=1000$ is shown in Fig. 8a and compared to the ICA-based estimator with $n_f=100$ and $c1$ signal. These beam patterns have been computed according to (36). In this stage, the features that are important for the comparison of beam patterns are the 3dB beamwidth of the beam pattern and the PSLL ratio of the beam pattern. It is shown that the ICA-based beam pattern is very close to the ideal MIMO. In addition, we can see that even by increasing the number of Doppler filters to $n_f=1000$, the performance of the MF estimator is less than the ICA-based estimator.

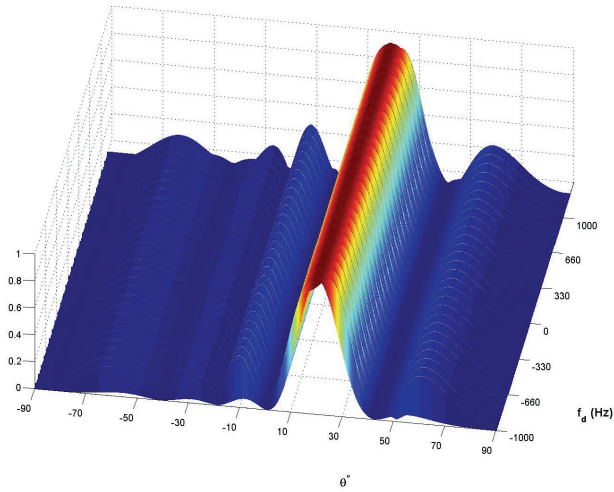
The beam pattern for the ML estimator of (14) using $c2$ signal, applying $n_f=100$ and $n_f=300$ is shown in Fig. 8b and is compared to the ICA-based estimator with $n_f=100$. It can be seen that increasing the number of Doppler filters improves the performance of the ML estimator. For $n_f=100$, the performance of the ICA-based estimator is better, but for $n_f=300$, the performance of the ML estimator is close to the ICA-based estimator with $n_f=100$.

Fig. 8. Absolute value of the beam pattern, considering $m=6$, $L=200$

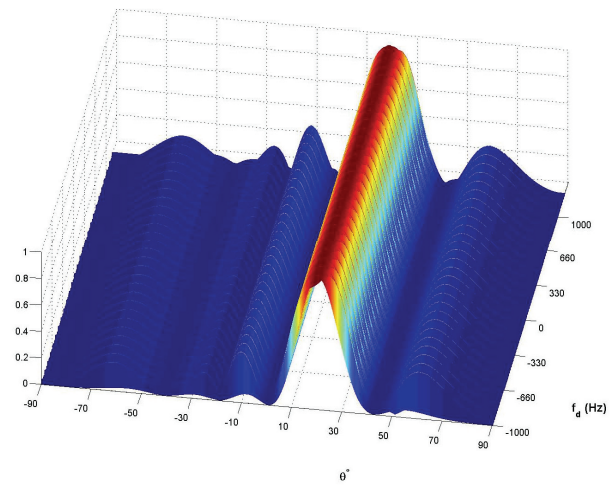
For a general comparison of these techniques, some features of the beam pattern should be checked in the complete span of Doppler values. First, the beam pattern of these techniques for all the frequencies in the interval of $[0, PRF)$ is computed. The cuts of these 3 dimensional beam patterns for the MF estimator with different frequency steps of $n_f=200$ and $n_f=1000$, and for the ML estimator with different frequency steps of $n_f=100$ and $n_f=300$ and also for the ICA-based technique with $n_f=100$, are shown in Fig. 9. In order to quantify the significant features of the estimated beam pattern, average overall frequency values in the interval of $[0, PRF)$ should be applied. Table 1 includes these quantified features of the beam pattern. In this table, an ideal co-located MIMO system is compared with the presented ICA-based MIMO and also the MF and ML estimators, with different n_f values. A comparison is done with an ideal phased array signaling as well. The three features that are measured in this table are the Root Mean Square Error (RMSE) of the estimated direction of target ($\hat{\theta}$), the average 3dB beamwidth of the beam pattern and the average PSLL ratio of the beam pattern. It is shown that the beam pattern features of the ICA-based techniques are much closer to the ideal MIMO compared to the other techniques. Although the beam pattern of the ML estimator in the case of $n_f=300$ has better PSLL, the greater beamwidth and less accuracy in DOA estimation are more significant in this technique. Unlike the MF estimator, the performance of the ML estimator improves by increasing n_f and, for example, applying $n_f=400$, the ML estimator has a slightly better performance compared to the ICA-based estimator with $n_f=100$.

Table 1. Comparison of beam-pattern features considering $m = 6$ and $L = 200$

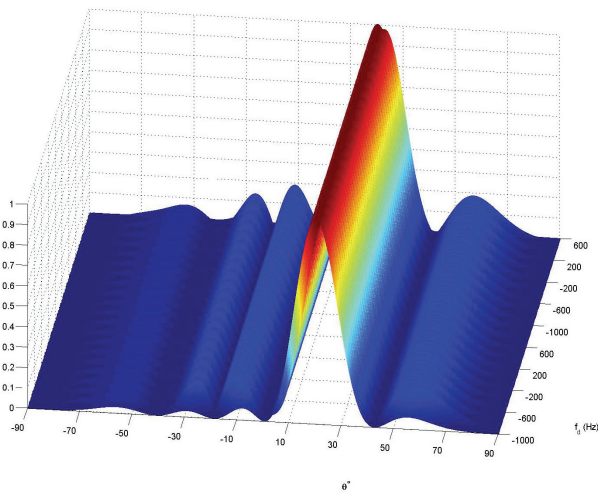
	RMSE ($\hat{\theta}$)	Av. BW	Av. PSLL (dB)
Ideal MIMO	0°	13.27°	24.85
ICA-Based	0.2°	13.3°	21.8
MF ($n_f=200$)	1.1°	13.9°	19.25
MF ($n_f=1000$)	0.81°	13.3°	21.9
ML ($n_f=100$)	0.6°	13.45°	21.8
ML ($n_f=300$)	0.25°	13.4°	22.1
ML ($n_f=400$)	0.12°	13.28°	22.5
Phased-Array	0°	18.47°	12.85



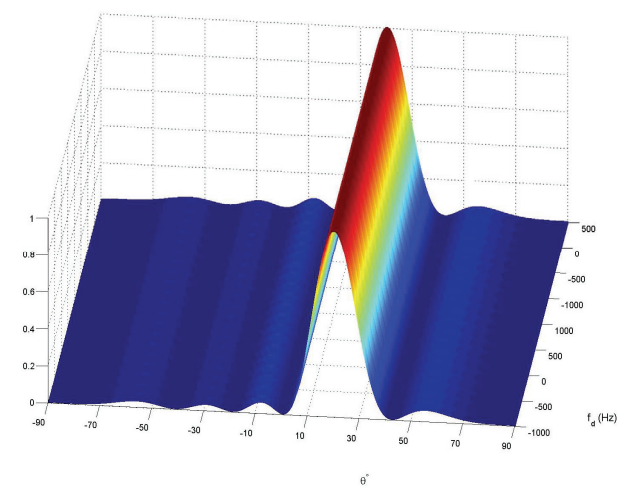
(a) MF estimator, considering $n_f=200$



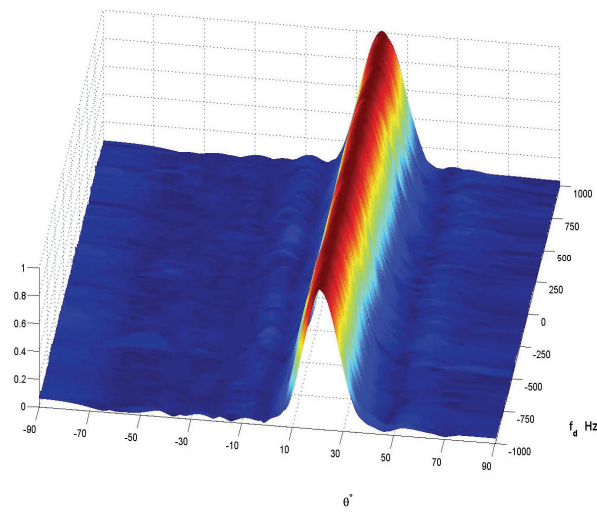
(b) MF estimator, considering $n_f=1000$



(c) ML estimator, considering $n_f=100$



(d) ML estimator, considering $n_f=300$



(e) ICA-Based estimator, considering $n_f=100$

Fig. 9. Absolute value of beam patterns in a noise-free model, $m=6$, $L=200$

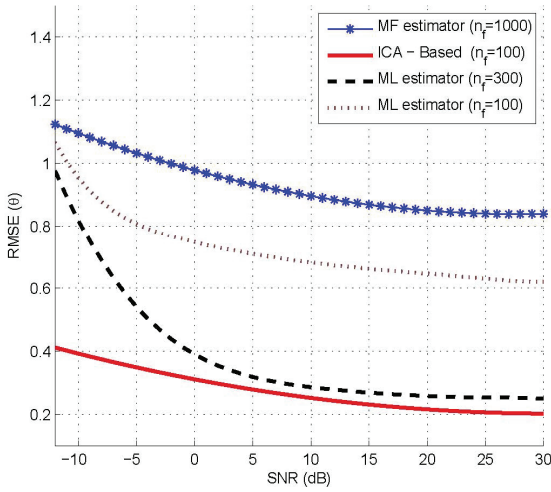


Fig. 10. RMSE of DOA estimation for different SNR values considering $m = 6$ and $L = 200$

d) Noise effect on DOA estimation

In this subsection, the performance of the ICA-based estimator should be derived by simulation for different values of Signal to Noise Ratio (SNR). In this simulation, a white Gaussian noise of $\mathcal{N}(0, \sigma^2 \mathbf{I})$ is considered for each receiver element and all receiver noise vectors are assumed to be independent. In this simulation, the results are calculated by averaging the overall Doppler values of $[0, PRF]$. In Fig. 10 it is shown that in a noisy model, the accuracy of the ICA-based estimator with the JADE algorithm is better than the other estimators. It is also shown that the accuracy of the MF estimator is very low compared with the ICA-based estimator (with $n_f = 100$). We can also see in Fig. 10 that in high SNR values, the performance of the ML estimator improves by increasing the n_f . But for $n_f \leq 300$ the RMSE of the ML estimator is more than the proposed ICA-based estimator. For low SNR conditions, the performance of the ICA-based estimator with $n_f=100$ is much better than MF and the ML estimators with large n_f .

e) Analysis of computational load

In this subsection, the computational complexity of all presented estimators is determined. According to [33, 34], the straightforward method of $n \times n$ matrix multiplication uses $O(n^3)$ operations. It can be shown that the production of $\mathbf{M}_{n \times p} \times \mathbf{Q}_{p \times \ell}$, will result in complexity of $O(np\ell)$. So, for the MF estimator of (16) and the subsequent estimator of (36) the order of complexity is:

$$O(n_f \times (Lm^2 + m) + n_\theta \times (m^2 + 2)) \quad (42)$$

where n_θ is the number of direction test points in the interval of $[0, \pi]$. For the two step ML estimator of (14) and the subsequent estimator of (36) the order of complexity is:

$$O(n_f \times (L^3 + 3L^2m + Lm^2 + m) + n_\theta \times (m^2 + 2)) \quad (43)$$

For the ICA-based estimator presented in Fig. 3, different blocks should be considered independently [21]. If we consider all possible values for L , the FFT block uses mL^2 complex operations. Wiener filter has a complexity of $O(m^2L + m^3)$ according to the structure described in Appendix II. For the JADE algorithm which is used as the ICA-estimator in this detection method, as described in [26], [27], the complexity is $O((k+2)m^3 + m^4 + 2m^2L)$. Here k is the number of iterations in Approximate Joint Diagonalization (AJD) step in JADE algorithm as described in [35] and simulation results show that k

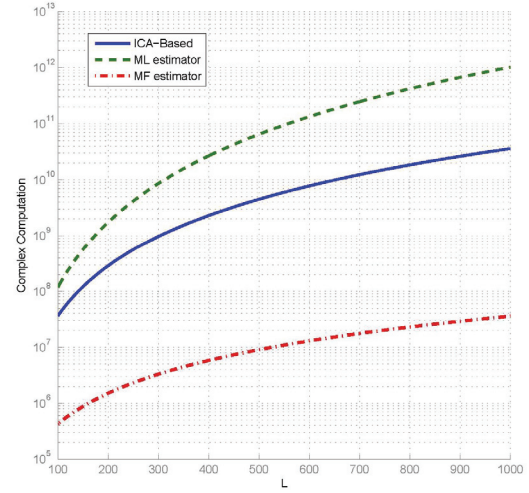


Fig. 11. The number of complex computations versus L

does not exceed 20 in our detection problem. Also, the complexity of arrangement and phase estimation is $O(n_f \times m^2 L^2 + 4mL)$. Finally, the complexity of (36) is $O(n_\theta \times (m^2 + 2))$. Eventually, the order of complexity of the ICA-based estimation algorithm is:

$$O(3m^2 L + (k + 3)m^3 + m^4 + n_f \times m^2 L^2 + n_\theta \times (m^2 + 2)) \quad (44)$$

For example, for a hypothetical system with parameters of $m=6$, $n_f=2L$, $n_\theta=1000$ and $k=20$, Fig. 11 compares the number of complex iterations for different values of L for all proposed estimators. It can be easily seen that for these parameters, the difference of complexity between the ML estimator and the ICA-based estimator is about 2 for small values of L , but this difference increases to an order of 10^2 for large values of L . Also, it can be easily seen that the complexity of the MF estimator is much less than the two other estimators.

7. CONCLUSION

In this paper, the problem of signaling in co-located MIMO radars is considered. The usual idea of co-located MIMO radars is based on the transmission of orthogonal pulse coded signals. In this paper the approach of transmitting a set of proper pulse-trains is proposed. To separate the received pulse trains, three different estimators, including a new approach based on the theory of Independent Component Analysis (ICA) are proposed. According to this approach, an appropriate signal design method is presented, based on the separation performance of the ICA algorithms; it is shown that independent random sequences are proper signals in this sense. It is also shown that the proposed ICA-based estimator is less sensitive to Doppler effect compared to the two other estimators that consist of a set of filter-banks. So, with an equal number of test frequencies, better beam-forming features and less error in DOA estimation is gained in the ICA-based estimator. It is also shown that the computational load of the ICA-based estimator is much less than the presented maximum likelihood estimator.

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APPENDIX I

DERIVATION OF ML ESTIMATOR

To solve the estimation problem of (11), we should reform it to a vector structure of:

$$\mathbf{v}_x = \mathbf{H}(f_d) \times \mathbf{v}_A(\theta, \phi) + \mathbf{v}_N \quad (\text{A1.1})$$

where

$$\begin{aligned} \mathbf{v}_x &= \text{Vec}(\mathbf{X}^T) & \mathbf{v}_N &= \text{Vec}(\mathbf{N}^T) \\ \mathbf{v}_A(\theta, \phi) &= \text{Vec}(\mathbf{A}^T(\theta, \phi)) & \mathbf{H}(f_d) &= \mathbf{I}_m \otimes \tilde{\mathbf{S}}^T(f_d) \end{aligned} \quad (\text{A1.2})$$

In this equation $\otimes$ denotes the Kronecker matrix product, $\text{Vec}(\cdot)$ denotes the vectorization of a desired matrix and $\mathbf{v}_N$ is the white Gaussian noise vector that is $N(0, \sigma_n^2 \mathbf{I}_{L \times m})$. Now, the ML estimator can be formed as:

$$\hat{f}_d, \hat{\theta} = \text{Arg} \underset{f_d, \theta}{\text{Max}} f_{\mathbf{v}_x}(f_d, \theta) \quad (\text{A1.3})$$

We can simplify (A.3) by replacing the formulation of complex Gaussian PDF as:

$$\hat{f}_d, \hat{\mathbf{v}}_A(\theta) = \text{Arg} \underset{f_d}{\text{Max}} \left\{ \underset{\theta}{\text{Max}} \left\{ \exp \left[-\frac{1}{\sigma_n^2} (\mathbf{v}_x - \mathbf{H}(f_d) \mathbf{v}_A(\theta))^H (\mathbf{v}_x - \mathbf{H}(f_d) \mathbf{v}_A(\theta)) \right] \right\} \right\} \quad (\text{A1.4})$$

The maximization over $\mathbf{v}_A(\theta)$ is derived by applying a derivative to the inner argument, and we have:

$$\begin{aligned} 2 \times \mathbf{v}_x^H \mathbf{H} - 2 \times \mathbf{v}_A^H (\mathbf{H}^H \mathbf{H}) &= 0 \quad \Rightarrow \hat{\mathbf{v}}_A^H(\theta) = \mathbf{v}_x^H \mathbf{H} (\mathbf{H}^H \mathbf{H})^{-1} \\ \Rightarrow \hat{\mathbf{A}}(\theta) &= \mathbf{X} \tilde{\mathbf{S}}^H (\tilde{\mathbf{S}} \tilde{\mathbf{S}}^H)^{-1} \end{aligned} \quad (\text{A1.5})$$

Replacing $\hat{\mathbf{v}}_A^H(\theta)$ in (A1.4), the Doppler estimator can be simplified to:

$$\begin{aligned} \hat{f}_d &= \text{Arg} \underset{f_d}{\text{Max}} \mathbf{v}_x^H \mathbf{H} (\mathbf{H}^H \mathbf{H})^{-1} \mathbf{H}^H \mathbf{v}_x \quad \Rightarrow \hat{f}_d = \text{Arg} \underset{f_d}{\text{Max}} \left\{ \text{tr} \left(\mathbf{X} \tilde{\mathbf{S}}^H (\tilde{\mathbf{S}} \tilde{\mathbf{S}}^H)^{-1} \tilde{\mathbf{S}} \mathbf{X}^H \right) \right\} \\ \Rightarrow \hat{f}_d &= \text{Arg} \underset{f_d}{\text{Max}} \left\| \mathbf{X} \tilde{\mathbf{S}}^H (\tilde{\mathbf{S}} \tilde{\mathbf{S}}^H)^{-1} \tilde{\mathbf{S}} \right\|^2 \end{aligned} \quad (\text{A1.6})$$

APPENDIX II

WIENER FILTER

Considering the described model in Section 2, the received signal of (11) is formed of L independent samples (pulses) of $m \times 1$ receiver vector. For each pulse, the vector of m received signal can be formulated as:

$$\mathbf{x} = \mathbf{A} \tilde{\mathbf{s}} + \mathbf{n} = \mathbf{q} + \mathbf{n} \quad (\text{A2.1})$$

where $\mathbf{n}$ is the vector of independent and identically distributed (iid) noise variables. Considering a white Gaussian distribution as $\mathcal{N}(0, \sigma_n^2 \mathbf{I})$, we can form the Wiener filter [36] or equivalently Linear Mean Square estimator of $\mathbf{q}$ as:

$$\mathbf{F} = \mathbf{R}_x^{-1} \mathbf{R}_{qx} \quad (\text{A2.2})$$

where $\mathbf{R}_x$ is the correlation matrix of received data and $\mathbf{R}_{qx}$ is the cross-correlation matrix of data and signal. By definition we have:

$$\begin{aligned} \mathbf{R}_x &= E[\mathbf{x}\mathbf{x}^H] \\ \mathbf{R}_{qx} &= E[\mathbf{x}\mathbf{q}^H] = E[\mathbf{q}\mathbf{q}^H] = \mathbf{R}_q \end{aligned} \quad (\text{A2.3})$$

But as only L samples of each variable are available in this model, the estimates of these parameters are:

$$\begin{aligned} \hat{\mathbf{R}}_x &= \mathbf{X}\mathbf{X}^H / L \\ \hat{\mathbf{R}}_q &= \mathbf{X}\mathbf{X}^H / L - \sigma_n^2 \mathbf{I}_m \end{aligned} \quad (\text{A2.4})$$

By applying $\hat{\mathbf{F}} = \hat{\mathbf{R}}_x^{-1} \hat{\mathbf{R}}_q$ as a pre-multiply operation to the signal matrix, we have:

$$\hat{\mathbf{X}} = \hat{\mathbf{F}}\mathbf{X} = \mathbf{A}\tilde{\mathbf{S}} + \mathbf{E} \quad (\text{A2.5})$$

where $\mathbf{E}$ is the error of estimation and is given by:

$$\mathbf{E} = \hat{\mathbf{X}} - \mathbf{A}\tilde{\mathbf{S}} \quad (\text{A2.6})$$