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"Research Note"

THE INITIATION OF SEDIMENT MOTION IN FIXED BED CHANNELS*

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Abstract– The transport of sediment in open channels is a complex process, and the physics of this phenomenon have not been completely explored. The majority of research work on sediment transport has been concentrated on beds formed of the same mobile sediment and only a few researchers have been concerned with sediment motion over fixed bed. This paper reviews the state of the problem and focuses on some practical points. Sediment threshold experiments were conducted in the two types of V-shaped bottom channels. Sand and gravel particle movements were considered and the relationship between flow discharge and bed shear stress, as well as channel bed slope were found at the threshold condition. Some practical and design equations were found to be more appropriate. It may be found that the effect of cross sectional shape on sediment threshold in fixed bed channels should be examined.

Keywords – Sediment particles, incipient motion, fixed bed, V-shaped channel, boundary shear stress

1. INTRODUCTION

The transport of sediment in open channels is a complex process, and the physics of this two-phase phenomenon is not yet completely understood. In certain reaches of mountainous streams the bed configuration often changes from granular to rocky and vice versa, depending upon the flood conditions and the degree of sediment supply from upstream reaches. Similar features of sediment transport can also be observed in manmade channels such as surface drainage channels and sewer systems used for water intake whose beds are lined with concrete, steel, cast iron or pvc. The flow on rigid beds merely conveys all of the sediment supplied from upstream reaches in a free deposition condition as long as the rate of supply does not exceed a critical rate. On the basis of data analysis on sediment transport, some theories have paid attention to the physics of individual particle motions and others have relied on certain global principles or dimensional arguments. Recent studies have been undertaken in rigid smooth pipes with regard to sewer system design for a condition of free deposition. Particular emphasis has been put on the influence of rigid roughness and the effect of cross sectional shapes under the condition of non-deposition of non-cohesive sand and aggregate sediment particles.

The initiation of sediment motion has attracted a considerable amount of interest in the past because of its philosophically appealing nature, although in practice it is a difficult issue to deal with. This is because of a large number of degrees of freedom in two-phase flow with sediment, and also the highly variable parameters in natural situations. The mechanics of fluid flow interaction with granular solids has already been the subject of numerous textbooks, [1-5] and elsewhere, a literature survey indicates a limited experimental work on fixed beds [6-19].

2. LOOSE BOUNDARY CHANNELS AND MECHANISM OF INCIPIENT MOTION

The threshold condition of particles requires certain parameters to be considered, namely bed shear stress, critical velocity, cross sectional shape, fall velocity of individual particles, material size, and boundary

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conditions. When the value of the bed uplift velocity reaches a certain proportion of the fall velocity of particles, they can be lifted out of the bed by upward turbulent forces, and as a result some particles may go into suspension and the remainder move as bed load. It has long been established that the total bed shear stress is an important parameter to think about concerning the sediment threshold phenomenon. The resultant of grain and form shears establishes the skin friction and drag, respectively. Engelund & Hansen [1] described the significance of each of these parameters for sediment transport. Surface grains generally move under the influence of skin friction, and a corresponding Shields dimensionless shear parameter, τ^*, for a certain given flow depth, h, is

$$\tau^* = \frac{\tau'_b}{\rho(s-1)gd} \tag{1}$$

This equation is used to predict the initiation of motion and the magnitude of moving sediment concentration. For example, for a flat bed the form drag is absent and then $\tau'_b = \tau_b$. This is one of the key assumptions in the Shields approach. However a modified Shields diagram is useful for understanding the initiation of sediment particles. The following fundamental approaches may be applied for threshold and sediment transport both in loose and rigid boundary channels.

a) Critical velocity approach

The initiation of motion for sediment particles depends on critical bed velocity or critical mean velocity. A combination of the Shields criterion with Strickler's equation for Manning's n ($=0.04d^{1/6}$) d in mm, gives

$$\frac{U_c}{\sqrt{gd(s-1)}} = a\left(\frac{d}{R}\right)^{-1/6} \tag{2}$$

A comparison of dimensionless critical velocities for incipient motion versus d/R illustrate that for a given d/R, critical velocity for rigid bed channels, is lower than that of loose bed channels [21].

b) Critical shear stress approach

To define the threshold condition, the equilibrium of a layer of sediment particles resting on the bed under the frictional drag of flow is often adopted as an alternate approach. Shields was the first to use the concept of friction velocity. He defined which physical parameters influenced sediment transport and gave a functional relationship

$$\tau_c^* = \frac{\tau_c}{\rho g(s-1)d} = F(\mathrm{Re}^*) = F\left(\frac{u_* d}{\nu}\right) \tag{3}$$

Shields (1936) conducted his experiments in two rectangular channels for the condition of the plane bed [22]. He then defined a critical dimensionless shear stress as values of the shear stress for zero sediment discharge. In fully turbulent flows ($Re_*>400$ and $d\geq 4.0$mm) the Shields equation may be written as

$$\tau_c = 0.056\rho g(s-1)d \tag{4}$$

Yalin [2] suggested a different combination of dimensionless parameters initially proposed by Shields, whereby the shear velocity is eliminated and only parameters of the fluid and sediment are retained

$$\frac{\tau_0}{\rho g(s-1)d} = F\left(D_{gr}\right) = F\left[\frac{(s-1)gd^3}{\nu^2}\right]^{1/3} \tag{5}$$

The advantage of Yalin's scheme is that the critical shear stress is evaluated without any iterative process.

c) Lift force approach

The incipient motion condition is reached when the upward force due to flow is just greater than the submerged weight of particles. Vanoni [23] stated that the earliest work on initiation of particles considered only the bed shear stress and completely ignored the lift force on particles, despite both analytical and experimental studies indicating its presence. Most treatments of forces on a single grain on the bed consider only drag, and the lift does not appear explicitly. The reason for this may be due to the difficulty of analysis in that the lift force is caused not only by the velocity gradient present in the flow (shear flow effect), but also by the spinning motion of the particle. This approach has not been widely used by engineers.

3. RIGID BOUNDARY CHANNELS AND MECHANISM OF INCIPIENT MOTION

According to Mayerle *et al*. [12], factors such as: hydraulic radius, channel cross-section, conveyance roughness, sediment concentration, grain size, and cohesivity influence sediment transport over rigid boundaries. The distributions of depth-averaged velocity, boundary shear stress on bed and wall, as well as local friction factors, all have a considerable impact on the transportation of particles. The threshold condition, in part full pipes studied by Craven [24] from which he concluded that to ensure no permanent deposition of sediments, the following relation should be satisfied as

$$\frac{Q}{D^2\sqrt{(s-1)gd}} \geq 2.50 \quad \text{(open channels);} \qquad \frac{U_c}{\sqrt{(s-1)gd}} \geq 3.18 \ \text{(part full pipe flow)} \tag{6}$$

An extensive investigation on rectangular free surface flumes and part full circular pipe channels was undertaken by Novak & Nalluri [10]. The critical velocity for incipient motion is

$$\frac{U_c}{\sqrt{gd(s-1)}} = a\left(\frac{d}{R}\right)^b \tag{7}$$

where a and b depend on the bed and whether there are single/touching particles. A plot of incipient motion equations illustrate that the equations for critical velocities on loose boundaries are shown in conjunction with the equations for incipient motion on rigid beds [10, 11]. It may also be seen that for small d/R values (close to 0.01) rigid boundary equations agree well with loose boundary equations [21]. For large values of d/R, this is not the case. A brief literature review on the matter is given for both circular and rectangular channels, [21].

4. ACKERS & WHITE TOTAL LOAD FUNCTION

By 1973 Ackers and White [25] proposed a total load sediment transport function. They used dimensional analysis, as well as some physical arguments for deriving the form of this function. The theory was developed on the basis of analysis for 1000 flume data carried out with near uniform sediment size ranging from $0.04<d(\text{mm})<4.00$, depths of flow up to 0.40m and Froude numbers up to 0.80. The function relates to the sediment transport, G_{gr}, in dimensionless form to the excess of a mobility parameter, F_{gr}, over its value at initial motion, A_{gr}. The basic formulation for a rectangular channel is given by

$$D_{gr} = \left(\frac{\gamma_s d^3}{\rho \nu^2}\right)^{1/3} = d\left(\frac{g(s-1)}{\nu^2}\right)^{1/3} \tag{8}$$

D_{gr} varies from 1.0 for fine sediments (d=*0.04* mm) to 60 for coarse sediments (d=2.50 mm).

$$F_{gr} = \frac{u_*^n}{\sqrt{gd(s-1)}} \cdot \frac{U^{1-n}}{\left[\sqrt{32}\log(\alpha z_0/d)\right]^{1-n}} \tag{9}$$

n is varying between 0 and 1, and α is a parameter related to the roughness, estimated as 10-12;

$$G_{gr} = C_v \frac{z_0}{d}\left(\frac{u_*}{U}\right)^n \tag{10}$$

The following power relationship was evaluated among the three parameters:

$$G_{gr} = C[(F_{gr}/A_{gr}) - 1]^m \tag{11}$$

Table 1. gives the values of Ackers & White empirical parameters *n, A_{gr}, m* and *C* [25]

Table 1. The Ackers & White empirical parameters

Empirical parameters	Fine material	Coarse material
	$1 < D_{gr} < 60;\ 0.04 < d(\text{mm}) < 2.40$	$D_{gr} > 60;\ d(\text{mm}) > 2.40$
n	$n = 1.0 - 0.056 \log(D_{gr})$	$n = 0$
A_{gr}	$A_{gr} = (0.23 / \sqrt{G_{gr}}) + 0.14$	$A_{gr} = 0.17$
m	$m = (9.66 / D_{gr}) + 1.34$	$m = 1.50$
C	$\log C = 2.86 \log D_{gr} - (\log D_{gr})^2 - 3.53$	$C = 0.025$

5. EXPERIMENTAL APPARATUS AND PROCEDURE

The experiments were carried out under *uniform flow* condition in a 15m long glass-walled rigid tilting flume of 460mm wide × 380mm deep with a settling tank to trap sand washed down the channel. The experimental channels were built inside of the flume. Two types of channels with a V-shaped bottom cross section were examined. Firstly, the 300mm wide CIS units, 13.5m long, and a working cross section of 278mm wide × 76mm deep were tested. The second channel shape was built by using pvc panels to make a 14.5m long channel having 50mm crossfall. Figure 1 shows the experimental channel cross section. Individual bell-mouth shaped transition sections were made for each channel type and served to reduce separation and improve the development of the mean flow into the channels. An electromagnetic flow meter was also installed in the water supply line after the Venturi and was used to check discharges. A slatted tailgate weir was installed in the downstream end, hence allowing a greater reach of the channel to be employed for measurement in subcritical flows. The test section consisted of a 12m long zone, commencing at a distance of 1.25m from the channel entrance and 1.85m from the flume entrance. However for supercritical flows, an *S2* profile was developed and the test length was reduced to about 7m for some cases.

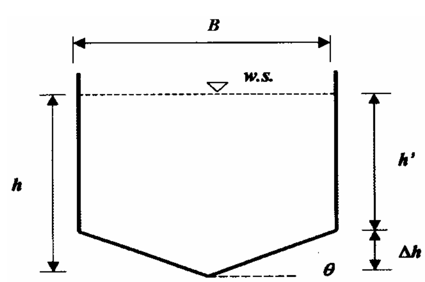

Fig. 1. Cross section of a V-shaped bottom channel or the CIS channel

In order to estimate the initiation of motion for sediment particles, several series of tests were carried out. Sieve analysis was undertaken and gave a sand material having d_{50}=0.87mm and a road/concrete aggregate material having d_{50}=7.72mm. Incipient motion of particles was investigated using seven channel target bed slopes of 0.1%, 0.2%, 0.4%, 0.5%, 0.6%, 0.9%, and 1.6%. For each fixed bed slope, a discharge was set and uniform flow was established. For each discharge, the motion of particles was then studied visually and sometimes by manual measurements. When the threshold condition occurred, the critical flow discharge and bed slope were recorded.

6. PRESENTATION OF EXPERIMENTAL RESULTS AND ANALYSIS

In the field of sediment transport, the threshold condition of particles is usually regarded as being very important. It is a phenomenon that is clearly observable and highly salient since it is characterized by the beginning of motion for bed load particles. In this case, when movement starts, particles begin to move at that moment when the shear stress has attained the limiting value at which the forces acting on the particles are balanced by the resistance due to the particles movement. The boundary condition between movement and deposition is referred to as the threshold condition.

In the present research, 22 tests are carried out for two particles sizes in two different smooth channel shapes, as explained in the previous section. For the case of sand size particles, it was observed that after placing particles on the bed in any flow condition, all particles tend to move/slide towards the centerline part of the V-shaped channel. It was very difficult to see different modes of particle movements for the case of sand size. In contrast, for gravel size, various modes of particle movement (threshold condition) were observed closely and accurately.

An attempt has been made to relate the flow characteristics to the critical condition of sediment motion. It was found that a simple relationship between two parameters such as Re^*, and the dimensionless critical shear stress, τ_c^*, best describes the phenomenon. Figure 2 shows the variation of τ_c^* versus Re^*, for both sizes of bed materials considered. For the V-shaped bottom channel, the following equations are obtained

$$\tau_c^* = 7E-05\,\mathrm{Re}_*^2, \quad \text{for sand size of } d_{50} = 0.87\text{mm} \tag{12}$$

$$\tau_c^* = 1E-07\,\mathrm{Re}_*^2, \quad \text{for gravel size of } d_{50} = 7.72\text{mm} \tag{13}$$

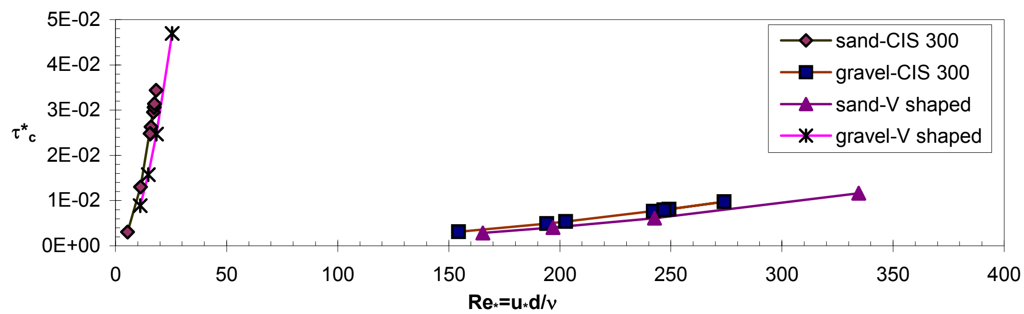

Fig. 2. Dimensionless shear stress, τ^*_c, versus Re_*

This Figure not only shows a discrepancy between two types of materials, but also shows a discrepancy between two channel types. This result describes the effect of material size as well as the channel cross-section on incipient motion of particles. The variation of flow discharge versus critical shear stress is shown plotted in Fig. 4. This Figure again confirms the above result, in order to get the threshold condition, the larger material size the larger flow discharge. Figures 2 & 3 are used for design purposes.

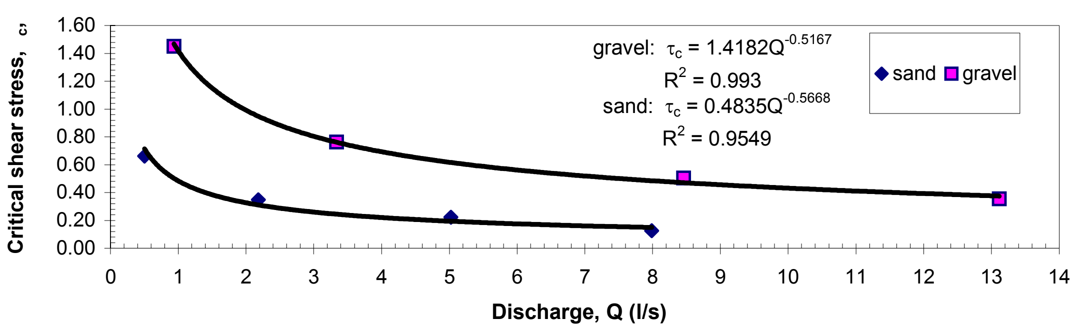

Fig. 3. Flow discharge versus critical shear stress (V-shaped channel)

The well known Shields [22] and Ackers & White [25] incipient motion equations were also examined. Although both of them were developed for wide alluvial channels, they sometimes give reasonable results in rigid boundary channels for design purposes. As a result, the experimental data and these two equations are compared in Fig. 4 for gravel materials. It can be seen that both the Shields criterion and the Ackers & White equation considerably overestimate the critical slope values results for sand particles. By contrast, it shows that there is close agreement with the data and both equations for large aggregate particles.

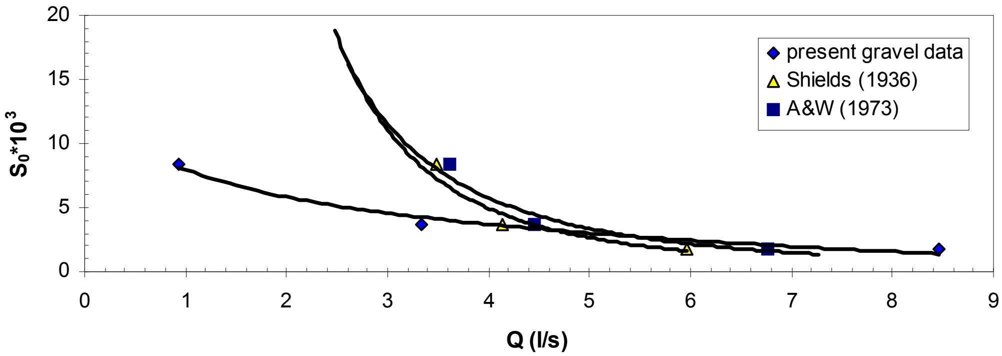

Fig. 4. A comparison of gravel motion with Shields and Ackers & White equations

A comparison of results illustrates that the obtained results for the present channel confirms the use of Shields and Ackers & White for large size materials, as well as at higher flow discharges [21]. This result has not exactly been obtained for the CIS 300mm channel. This confirms the effect of cross sectional shape affecting on the initiation of particles motion in open channels. The last comparison also shows that for the case of wide channels, the results for large particles and for the case of narrow channels results for small particles, give the best fit with those two equations [19]. In order to present an equation similar to Eq. (7), the variation of particle Froude number, *Fr(p)*, is plotted against d_{50}/R. A power law relationship gives the coefficient values of *a*=0.0052 and *b*=-1.7465 for sand and *a*=0.106 and *b*=-1.7079 for gravel materials.

6. CONCLUDING REMARKS

From the foregoing analysis, the following conclusions are drawn:

1. The critical velocity, shear stress and particle Froude number for particles resting on the bed are substantially lower than for particles of the same size forming a movable bed. Thus the Shields criterion gives an overestimate value for rigid bed channels.

2. For a given flow discharge, Q, channel bed slope, S_0, or critical shear stress at the threshold condition gives a high flow depth. However since the sediment particles slide to the centerline of the channel, the sediment carrying capacity of a V-shaped bottom channel is greater than that of a rectangular channel, for example.

3. The results from τ_c^* =F*(Re*)* for both channels show that critical shear stress increases as the particle diameter and roughness increases as more energy has to be used to overcome the higher friction between the particles and the bed roughness (see Fig. 2).

4. In a situation where particles are touching each other, there would be a greater friction between them. This friction increases with the increase in the number of particles and tends to bind the particles together. Hence a higher value of shear stress or velocity will be required to dislodge the larger particles and move them rather than the small ones.

NOMENCLATURE

A	transverse cross-sectional area	D_{gr}	dimensionless grain size parameter
B	top width of the channel	F_{gr}	sediment mobility $\left(=\sqrt{\tau*}\right)$
C	Chezy coefficient; a constant	g	acceleration due to the gravity
d_x	sediment size such that x% is finer	G_{gr}	dimensionless sediment transport

h	flow depth	U_c	critical velocity in the flow direction
P	wetted perimeter	γ	specific weight
Q	flow discharge	ν	kinematic viscosity of fluid
R	hydraulic radius ($=A/P$)	ρ	fluid density
Re_*	particles Reynolds number	τ^*	Shields mobility number
s	sediment relative density	τ_b	bed shear stress
S_0	channel bed slope	τ'_b	bed shear stress due to grain
u_*	shear/friction velocity in the boundary		

REFERENCES

1. Engelund, F. & Hansen, E. (1972). A Monograph on sediment transport in alluvial streams. Teknisk Vorlag, Copenhagen.
2. Yalin, M. S. (1992). *River Mechanics*. Pergamon Press, Oxford, UK.
3. Garde, R. J. & Ranga Raju, K. G. (1985). *Mechanics of sediment transportation and alluvial stream problems*. Wiley Eastern Limited, New Delhi, India.
4. Chang, H. H. (1988). *Fluvial Processes in River Engineering*. John Wiley, New York.
5. Graf, W. H. (1998). *Fluvial hydraulics*. John Wiley & Sons Ltd., New York.
6. Pedroli, R. (1963). Bed load transportation in channels with fixed and smooth inverts. Mitteillung des Eidg. Amtes fur Wasserwirtschaft, Dienst Exemplar, No. 43, Bern, Switzerland.
7. Yang, C. T. (1973). Incipient motion and sediment transport. *J. Hydraulic Div., ASCE, 99*(HY10), October.
8. May, R. W. P. (1982). Sediment transport in sewers. Hydraulic Research Station, HR, Report IT 222, UK.
9. Ojo, S. I. A. (1980). Incipient motion of grouped particles on fixed beds. Symposium on River Eng. and its Interaction with Hydrological and Hydraulic Research, IAHR, Yugoslavia.
10. Novak, P & Nalluri, C. (1975). Sediment transport in smooth fixed bed channels. *J. Hydraulics Div., ASCE, 101*(HY9), September.
11. Novak, P. & Nalluri, C. (1984). Incipient motion of sediment particles over fixed beds. *J. Hydr. Research, IAHR, 22*(3).
12. Mayerle, R., Nalluri, C. & Novak, P. (1991). Sediment transport in rigid bed conveyances. *J. of Hydr. Research, IAHR, 29*(4).
13. Loveless, J. H. (1991). Sediment transport in rigid boundary channels with particular reference to the condition of incipient motion deposition. *PhD Thesis*, University of London.
14. Simons, D. B. & Sentürk, F. (1992). *Sediment Transport Technology*. Water Resource Publications, Colorado.
15. Nalluri, C. & Kithsiri, M. M. A. U. (1992). Extended data on sediment transport in rigid bed rectangular channels. *J. Hydr. Research, IAHR, 30*(6), January.
16. Ackers, P. (1993). Sediment transport in open channels: Ackers and White update. *Proc. Instn. Civ. Engrs. Wat., Marit. & Energy, 1993, 101, Dec*., 247-249, Water Board, Technical Note 619.
17. Butler, D., May R. W. P. & Ackers, J. C. (1996a & b). Sediment transport in sewers part 1 & 2: background. *Proc. Inst. Civ. Engrs Wat, Marit. & Energy*, paper 10956 & 10957, June.
18. Okabe, T., Anase, Y. & Yamashita, H. (1996). A formula for bed load capacity of flow on smooth and fixed beds. *J. of Hydroscience and Hydraulic Engineering, JSCE, 14*(1), June, 25-35.
19. Mohammadi, M. & Knight, D. W. (1999). Threshold for sediment motion in V-shaped channel. *Proceeding, XXVIII IAHR Congress*, 22-27 August, Graz, Austria.
20. Keshavarzy, A. & Ball, J. (1999). An application of image processing in the study of sediment motion. *J. of Hydraulic Research, 37*, 559-576.
21. Mohammadi, M. (1998). Resistance to flow and the influence of boundary shear stress on sediment transport in rigid boundary channels. PhD Thesis, School of Civil Engineering, The University of Birmingham, England.
22. Shields, A. (1936). Anwendung der anelichkeismechanik und der turbulenze forschung auf die geschiebebewegung (Application of similarity principles and turbulence research to bed load movement). Mitteilungen 26, preussischen

versuchsanstalt fur wasserbau and schiffbau, Berlin, (Translated to English by W. P. Ott and van Uchelen, California, Institution of Technology, Pasadena, CA.

23. Vanoni, V. A. (1975). *Sedimentation engineering*, M. & R. No. 54, ASCE, New York.
24. Craven, J. P. (1953). The transportation of sand in part-full pipe flows. *Proc. of the Fifth Hydraulics Conference, Bulletin 34, State University of Iowa Studies in Engineering*, Iowa State University, Iowa.
25. Ackers, P. & White, W. R. (1973). Sediment transport: new approach and analysis. *J. Hydraulics Div., ASCE, 99*(HY11), November.