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ESTIMATION OF FREIGHT O-D MATRIX USING WAYBILL DATA AND TRAFFIC COUNTS IN IRAN ROADS*

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Abstract– In this research, a realistic methodology, in order to estimate a true O-D matrix for freight movement in a designated region, is presented. This means, having had an initial O-D matrix (exp. Waybill data) and traffic count in many intercity roads, one can estimate an O-D matrix, having minimum distance from the initial matrix, and reproducing the observed traffic by assigning to the network.

In the first step, after conducting studies and field work, some special route choice patterns in an intercity network were recognized by heavy vehicle drivers. Using the Logit Formulation, some probable (pure stochastic) assignment models related to the condition of the case were developed. Also, due to existing errors in both information sources, waybill and traffic count information, after recognition and determination of errors amount and their precision, based on Analytical Hierarchy Process Method, reliability of the mentioned data sources was estimated. Hence, to approach the best model of O-D matrix estimation, an Entropy Maximization model with a composed objective function was calibrated. Thus, having a calibrated matrix estimation model and a developed a traffic assignment model as a sub-model, true freight O-D matrix could be estimate.

Keywords – O-D Matrix, estimation, traffic count, freight, entropy, pure stochastic assignment

1. INTRODUCTION

The Origin-Destination matrix, usually called the O-D Matrix, has a significant role in transportation analysis. This matrix indicates the number of trips or amount of freight which have been interchanged between zones in the specific region. In fact, this matrix indicates a demand pattern in that region.

Conventional methods for collecting O-D information like home or roadside interviews tend to be costly, labor intensive and time consuming for trip makers. This problem is even more serious in developing countries where rapid changes in land-use and population shorten the validation of the collected data. Thus, the need for developing low-cost methods to estimate the present and future O-D matrices is apparent.

Traffic counts (flow observations) provide direct information about the sum of all O-D pairs using the counted links. In fact, they can be seen as the result of combining a trip (or freight) matrix and a route choice pattern. This data source is very attractive because traffic counts are non-disruptive to travelers, generally available, relatively inexpensive to collect and their automatic collection is well advanced. The idea of estimating trip matrices from traffic counts deserves serious consideration.

2. PROBLEM DESCRIPTION

Having traffic count information of the network and a related route choice pattern, the problem of estimating the O-D matrix can be defined as follows:

"Find an O-D matrix, which when assigned to the network, reproduces the observed traffic counts."

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The most important stage for the estimation of transportation demand from traffic counts is to identify the route choice pattern of trip makers and the path followed by the trips from each origin to each destination. The variable P_{ij}^a is used to define the proportion of trips from zone *i* to zone *j* (T_{ij}) traveling through link a. Thus, the flow (V_a) in a particular link *a* is the summation of the contributions of all trips between zones to that link. Mathematically, it can be expressed as follows:

$$V_a = \sum_{ij} T_{ij} P_{ij}^a \quad , \quad P_{ij}^a = [0,1] \tag{1}$$

The variable P_{ij}^a can be obtained using various trip assignment techniques. The number of trips or amount of freight which have been interchanged between zones must be converted to the number of standard vehicles according to the V_a unit. In practice, the number of observed traffic counts is much less than the number of unknown T_{ij} 's. Therefore, it is impossible to determine a unique solution to the matrix estimation problem. In general, there will be more than one matrix which, when loaded onto the network, will reproduce (satisfy) the observed traffic counts. Thus, one may ask for the most "likely" or "best" O-D matrix causing the observed traffic counts.

Virtually all models for O-D matrix estimation use prior information on the O-D matrix. The prior information might be expressed in terms of a "target" or "initial" O-D matrix and/or the number of trips (amount of goods) attracted to/originated in different zones. In this case, the initial O-D matrix can be obtained by a sample survey or from an old (probable outdated) matrix.

This research, focused on estimating a true O-D matrix only in the case of a freight movement system carried by trucks using the national intercity road network in Iran.

a) Mathematical Specification of Problem

As a principle in most matrix estimation methods, the distance between the estimated O-D matrix and the initial O-D matrix is minimized subject to the flow constraints. Hence, the problem of finding the O-D matrix *g*, given that the initial O-D matrix $\hat{g}$ is stated as minimizing a function $F_1(g,\hat{g})$. Statistically, the observed set of traffic count data may also be assumed to be an observation of the "true" traffic count data to be estimated, related to and obtained as an assignment of the estimated O-D matrix. Also, for other reasons mentioned above, deviations between estimated counts and observed counts may be accepted (this conception is in contrast to the alternative assumption of exact reproduction of the observed traffic volumes). Hence, an O-D matrix which produces "small" differences between the estimated link flows v and the observed flows $\hat{v}$ is sought. This ambition can be expressed as a criterion $F_2(v,\hat{v})$ to be minimized, subject to the assigned constraints.

Although the underlying motivations and assumptions of the O-D estimation models are different, the related optimization problems can be expressed in the following general form [1]

$$\begin{aligned} \min \quad & F(g,v) = \gamma_1 F_1(g,\hat{g}) + \gamma_2 F_2(v,\hat{v}) \\ s.t. \quad & v = assign\,(g) \\ & v, g \geq 0 \end{aligned} \tag{2}$$

where $\hat{g}$ is the initial (target) O-D matrix and $\hat{v}$ is the observed traffic counts with F_1 and F_2 being some distance measures. The assignment of *g* to the transportation network is denoted *assign(g)*, leading to a split of the O-D flows (g_{ij}) over k-available routes with path flows (h_{ijk}).

If the target OD matrix is reliable and accurate, γ_1 should be more significant compared to γ_2, which would result in a *g* closed to $\hat{g}$. Then larger deviations between v and $\hat{v}$ would be accepted. If, on the other hand, the observed traffic counts are reliable compared to the information in $\hat{g}$, the magnitude of γ_2 should be larger than γ_1. The values of the weights (γ_i) are, thus, closely related to the conception of the modeling situation. In addition, the development and the solution of problem is highly related to the subject of v = *assign(g)*. In fact, approaching a real O-D matrix in a study area depends on the accuracy of the used traffic assignment model in comparison with the real route choice pattern in the study area.

b) Survey of Modeling Approaches

Many approaches for estimating or updating O-D trip tables from traffic counts have been suggested by researchers [2]. For instance, the equilibrium assignment approach estimates an O-D matrix that satisfies equilibrium assignment conditions and also is consistent with the observed link flows. This model is well suited to estimate an O-D trip matrix in a congested urban area. The equilibrium assignment approach requires observed link volumes (for all links), link impedance, a link performance function and initial trip table [3]. But, traffic modeling-based approaches including the "Entropy maximizing (Information minimizing)" model and combined models for traffic planning, estimate the most probable O-D matrix based on traffic counts under proportional assignment conditions. The estimation is consistent with the constraints having an entropy maximization problem. This model requires neither the traffic counts on all links in the network nor an initial O-D matrix. However there are modified models in which an available initial trip table can be used to increase accuracy. This approach is inaccurate if travel behavior is not well represented by the gravity model or similar formulations [3]. In this technique, if no prior matrix is available, it can be taken as a unity matrix ($\hat{g}_{ij} = 1$) [4].

3. DISCRETE CHOICE METHODS FOR MODELING TRAVEL DECISIONS

Among many potential discrete choice models that can be derived from the random parts of the utility functions, Logit based models are the most popular. These models are based on a probability distribution function of the maximum of a series of random variables introduced by Gumbel [5, 6]. The probability that a given individual *n* chooses alternative *i* within the choice set C_n is given by

$$\mathrm{P}(i \mid C_n) = \frac{e^{\mu V_{in}}}{\sum_{j \in C_n} e^{\mu V_{jn}}} \tag{3}$$

The IIA (Independence from Irrelevant Alternatives) property of Multinomial Logit Models is a limitation for some practical applications [5, 7]. The choice set generation in route choice modeling can be deterministic or stochastic, depending on the analyst's knowledge of the problem. In this way, many approaches have been proposed including the Dijkstra Algorithm to determine the path with the highest utility [8, 9], the k-shortest paths algorithm developed by Shier [9], Dial and Burrell methods to consider reasonable paths [10, 11], the Stochastic Assignment Model (SAM) proposed by Maher based on the Markov theory [12], and the Labeling Approach proposed by Ben-Akiva to determine paths meeting specific criteria (e.g., shortest paths, fastest paths, paths with least congestion, etc.) [7]. To make the choice set, an implicit probabilistic choice set generation model proposed by Cascetta and Papola [7] and engineering judgments to realize active routes between each origin-destination pair (in the limited cases).

4. MATRIX ESTIMATION USING ENTROPY MAXIMIZING METHOD

Entropy-maximizing techniques have been used as the model building tools in urban, regional and transportation planning for many years, particularly after the work of Wilson in 1970 [4].

The idea of estimating trip matrices from the traffic count using entropy maximizing was used by Willumsen [4]. This method uses the objective of maximizing the entropy of the trip matrix (as a non-linear objective function) to estimate the most likely trip matrix consistent with the observed traffic counts (as linear constraints). The original problem (without prior matrix) is written as

$$\max\ S(T_{ij}) = -\sum_{ij}(T_{ij}\,LogT_{ij} - T_{ij}) \tag{4}$$

Subject to:

$$\sum_{ij} T_{ij} P_{ij}^{a} - \hat{V}_a = 0 \tag{5a}$$

for each counted a, and

$$T_{ij} \geq 0 \tag{5b}$$

The use of Lagrangian methods with multipliers λ_a can permit the formal solution to this problem

$$L = -\sum_{ij}(T_{ij}\,LogT_{ij} - T_{ij}) - \sum_{a}\lambda_a\Big(\sum_{ij} T_{ij} P_{ij}^{a} - \hat{V}_a\Big) = 0 \tag{6}$$

$$\frac{\partial L}{\partial T_{ij}} = -LogT_{ij} - \sum_{a}\lambda_a P_{ij}^{a} = 0 \quad \Rightarrow \quad T_{ij} = \exp\Big(-\sum_{a}\lambda_a P_{ij}^{a}\Big) \tag{7}$$

As mentioned before, the availability of an old matrix or simply a matrix estimated from another study could be accommodated to some advantage. Let ***t*** be this prior (initial) matrix; the new objective function becomes

$$\max\ S_1\Big(\frac{T_{ij}}{t_{ij}}\Big) = -\sum_{ij}\Big(T_{ij}\,Log\frac{T_{ij}}{t_{ij}} - T_{ij} + t_{ij}\Big) \tag{8}$$

This objective function is also convex and the term $\boldsymbol{t_{ij}}$, as a constant, is only there for more accuracy. In fact, it will not be derived from the model. Using the same methodology and change of variables, the formal solution can be

$$T_{ij} = t_{ij}\exp\Big(-\sum_{a}\lambda_a P_{ij}^{a}\Big) \tag{9}$$

Note: in practical problems one can not hope to directly calculate the entropy value of all possible matrices. More importantly, reducing the number of counts increases the number of feasible trip matrices.

One of the features of the Maximum Entropy Matrix Estimation Model, called the *ME2* model, is its multiplicative nature. This means that if a cell in the prior matrix is zero it will remain zero in the solution as well. This may be a source of problems if a cell in the prior matrix was zero by chance (i.e., because of sampling rate adopted in the study) instead of presenting an O-D pair with no trips at all. One pragmatic solution to this problem, for very sparse prior matrices, is to ‘seed’ the empty cells with a small value, for example 0.5 trips. The constraints, through the multi-proportional or other solution algorithm, will then ensure that some of these trips grow to one or more full trips, while others regain a zero value.

b) Limitations of ME 2

ME2, probably because of its simplicity, relative efficiency and ease of programming, has been widely implemented and used. However, this model has some known limitations. The main limitation of ME2 is that it considers the traffic counts as error-free observations on non-stochastic variables. In fact the model gives complete credence to the traffic counts and uses the prior matrix only to compensate the insufficiency of information in the estimation process. However, this may not be very appropriate in practice. For a start, one must acknowledge that traffic counts are certainly not error free. Apart from counting errors there is the problem of time variations (hourly, seasonal, etc.). One suggested solution to this problem is to use entropy-maximizing formalism with a composite objective (multi-objective) function.

5. ASSESSMENT OF UNCERTAINTY POTENTIALITIES IN MATRIX ESTIMATION

Data sources for estimating the number of trips between each O-D pair, which are assumed to be given, are information regarding a prior (e.g., out-dated) matrix, the number of trips (Vehicles) observed on the links and the usage proportion of the links (route choice pattern) for the trips in each O-D pair. Certainly, input data (mentioned above) is not error free and its reliability may vary. In the O-D matrix estimation process, the proportionate values are usually assumed to be fixed (i.e., without errors). Instead, the existing errors can be interpreted as an explanation of uncertainties in traffic count values and initial matrix [13].

Thus, there are two classes of information to be considered, the initial matrix (***t***), traffic count vector ($\hat{v}$) and also possibly some estimates of the magnitude of error in the input data.

In the original entropy-maximizing model, values of $\hat{v}$ are assumed to be fixed and the models produce a solution to the O-D matrix estimation problem that satisfies the equation $V_a = \sum_{ij} T_{ij} P_{ij}^a$ exactly. Although considerable uncertainty in $\hat{v}$ can be recognized, most of the models place a maximum belief in $\hat{v}$ and do not allow any solution with a deviation from these specified values ($\hat{v}$). In contrast, in these models, the values of ***t*** are defined as targets which can be interpreted as a minimum belief to assume an infinite uncertainty.

There are several models that account for the uncertainty in traffic count values. These models allow different beliefs in the two information classes ($\boldsymbol{t}, \hat{v}$) and produce solutions that do not necessarily reproduce the traffic count values exactly. Many researchers (e.g., Maher, Cascetta, Willumsen and etc.) presented some models to satisfy these criteria. The proposed model of Willumsen uses an entropy function as a distance measure based on multi-objective programming [4, 13].

a) Estimation of O-D matrix using multi-objective programming

To approach reality, the problem of estimating an O-D matrix using traffic counts can be viewed as a multi-objective problem. A typical problem with conflicting goals arises when one tries to satisfy both ***t*** and $\hat{v}$ types of targets. The different beliefs in the values of t and $\hat{v}$ correspond to different weights of the two terms of objective function, $F(\boldsymbol{t},\boldsymbol{T})$ and $F(\boldsymbol{v},\hat{v})$, respectively.

Multi-objective programming using weight factors is an old and efficient method. In this method, the problem is converted to a single objective program by considering different weights for each objective. A multi-objective problem can be written mathematically as below

$$\begin{aligned} &\max Z(x) = [Z_1(x), Z_2(x), ..., Z_P(x)] \\ &\text{St.} \\ &x \in F_d \end{aligned} \tag{10}$$

In this way, one may seek the optimal solution of this problem

$$\begin{aligned} &\max Z(x) = \sum_{K=1}^{P} W_K Z_K(x) \\ &\text{St.} \\ &x \in F_d \end{aligned} \tag{11}$$

where W_K can be neither zero nor negative. In fact, these weight factors are considered as scale parameters for each objective function. The problem of estimating an O-D matrix using traffic counts can be formulated as a two-objective programming model [13]

$$\min \begin{cases} \sum_{ij} (T_{ij} \log \frac{T_{ij}}{t_{ij}} - T_{ij} + t_{ij}) \\ \sum_{a} (V_a \log \frac{V_a}{\hat{v}_a} - V_a + \hat{v}_a) \end{cases} \tag{12}$$

St.

$$\sum_{ij} T_{ij} P_{ij}^{a} - V_a = \circ \quad , \quad a = 1,2,...,M \tag{13a}$$

$$T_{ij} \geq \circ \tag{13b}$$

The weights of the conflicting objective in (12), representing the relative beliefs in the two classes of information t and $\hat{v}$, may be expressed by introducing one weight factor for each of the objectives. By denoting the weight factors γ_1 and γ_2, a single objective model can be formulated as

$$\min \ \gamma_1 \sum_{ij} (T_{ij} \log \frac{T_{ij}}{t_{ij}} - T_{ij} + t_{ij}) + \gamma_2 \sum_{a} (V_a \log \frac{V_a}{\hat{v}_a} - V_a + \hat{v}_a) \tag{14}$$

This model is identical to the model formulated by Willumsen if $\gamma_1 = 1$ and $\gamma_2 = \gamma$. Then γ expresses the relative belief in the traffic counts $\hat{v}$ compared to the initial (target) matrix ***t***. If $\gamma_2 \to \infty$ for fixed γ_1 (or $\gamma_1 \to \circ$ for fixed γ_2), there is a maximal belief in the traffic counts, whereas if $\gamma_2 \to \circ$ for fixed γ_1 (or $\gamma_1 \to \infty$ for fixed γ_2), there is maximal belief in the target matrix. The factors can be scaled such that $\gamma_1 + \gamma_2 = 1$. Both γ_1 and γ_2 have to be predetermined by the user before Eq. (14) can be solved. A solution is said to be efficient (Pareto Optimal) if no other solution exists, such that at least one objective is strictly improved, while no other objective is worsened [13]. By using the Lagrangian method, the optimal solution can be expressed explicitly as

$$T_{ij} = t_{ij} \exp(1 - \frac{1}{\gamma_1} \sum_{a} \lambda_a P_{ij}^{a}) \tag{15}$$

$$V_a = \hat{v}_a \exp(\frac{\lambda_a}{\gamma_2}) \tag{16}$$

On the condition of the existence of disaggregated information in the input data regarding the problem, the totally disaggregated weighted entropy maximization model can be formulated as

$$\min \ \sum_{ij} \alpha_{ij} (T_{ij} \log \frac{T_{ij}}{t_{ij}} - T_{ij} + t_{ij}) + \sum_{a} \beta_a (V_a \log \frac{V_a}{\hat{v}_a} - V_a + \hat{v}_a) \tag{17}$$

Where α_{ij} is the weight factor corresponding to the target value in O-D pair *ij*, and β_a is the weight factor corresponding to traffic count observation *a*.

As a matter of fact, the weights are assumed to be defined as the inverse of some measure of uncertainty. This specification of weights can be made according to the planner's experience and judgment of the available data. If there is no uncertainty in a value, the measure of uncertainty equals zero and the corresponding weight factor approaches infinity, implying $T_{ij} = t_{ij}$ or $V_a = \hat{v}_a$.

6. DESIGNING ASSIGNMENT MODELS FOR TRUCK TRAFFIC IN IRAN INTERCITY NETWORK

The trip makers' behaviors in their relevant choices in the urban and intercity networks differ. Two reasons are recognized for this difference. The first is the existing difference between these two networks and the second is the different composition of the transportation vehicles. Due to this difference, the to-be-used model of simulation of the drivers' behaviors and traffic assignment in intercity networks will definitely differ from the one to be used in urban traffic assignment models.

After investigating different assignment models and the study of the amount of their usage in the simulation of the process of the traffic assignment in road networks in Iran, and according to the observations and the conducted samplings, one can come to the conclusion that none of the all-or-nothing or equilibrium models according to the current realities in the route choices in Iranian roads are being used. An all-or-nothing model does not have the possibility of considering an alternative route. Equilibrium models, due to relatively high traffic volume and the existing long differences between parallel (alternative) routes, are not a key to the solution of the problem.

a) Analysis of effective parameters in choices of truck drivers

The first step in interpreting a system is data and information collection. Because there has been no serious research related to the route choice patterns of heavy vehicle drivers, and also considering the determination of the share of existing routes in each of the origin-destination of the country, in the direction of the present study, according to the determined purposes, an (Stated Preference) SP (according to the drivers' interviews and explanations) was designed.

Having determined the purposes of the survey, one comes to the stage of designing the component of the questionnaire. In this stage, it was necessary to determine and recognize the country's origin-destination with more than one communication route. According to the results of this stage, we could ask the drivers about their choices on these roads. So, conducting expert analyses and consulting prominent experts and engineers of (Transportation and Terminals Organization) TTO, we listed as many of these routes as was possible. To this end, 16 O-D pairs were chosen with at least 2 active routes. After determining the suitable O-D pairs for this questioning and recognizing the active routes between each of them, the questionnaire was designed. In order to normalize the collected sample, 30 drivers were questioned. Consequently, regarding the fact that in each form there were 16 cases of questioning on the issue of route choice, and 30 drivers who were questioned, the total number of statistical cases (sample size) was 480.

b) The characteristics of the routes considered in the questioning process

After determining the origin-destination pairs with more than one active alternative route and also after distinguishing each of these routes, various engineering criteria were determined including the length of each route, the amount of slope, topographical features in regions and type of roads (according to physical features of cross sections in different segments). Furthermore, the level of service in routes (using existing information and capabilities of the GIS software) were calculated.

To develop the considered assignment models for heavy vehicles (trucks) in the intercity road network, which are sub-models for the mentioned model of commodity O-D matrix estimation, the personal characteristics of the drivers were ignored and analyses were conducted according to the principle that considering the clarity of engineering information for every active route between each O-D pair, one can find the possibility of different route choices by drivers.

c) Presenting calibrated models for the truck traffic assignment

In this study, according to the developed probability-based algorithm and also due to the fact that the perception of drivers is different for effective factors in route choice, using the theory of the utility and Logit model, some models based on probability were calibrated. In terms of assignment models, these models are placed in the category of proportional models and a sub-division of pure stochastic models.
Regarding the collected information and drivers' conditions in theirs choices of intercity roads and routes (according to the selected origin-destination pairs), 3 types of probability models were calibrated. It seems that the choice set of route selection, while the drivers of trucks have the chance of selection, is restricted to just one of 3 following categories:
First group: Calibration when the choices of drivers are two active principle roads (except toll freeways).
Second group: Calibration when the drivers' choices include two routes: 1. usual highways (including expressways, major and minor roads) and 2. toll freeways.
Third group: Calibration when the drivers' choice includes 3 independent routes (except toll freeways).
Due to limitations of SPSS software, only the software GAUSS was used to calibrate the second and third group of models. In GAUSS software, it is possible to define different utility functions for each alternative according to appropriate variables for each of the choices.
Due to the limitations existing with an IIA (Independence of Irrelative Alternatives) problem in Logit models, the scope of practicality in all of the suggested models is in the conditions that the alternative routes do not have a considerable correlation with each other.
I. Presented models for the first group (two non-freeway alternatives)
- *Calibrated models with SPSS*

Many models were calibrated and checked with this software, but only six models were recognized as valid models. According to Table (1), three variables have been used in these models. These variables are defined as the difference of the related characteristics of each alternative. The amount of *LL(0)* in these six models is -831.8.

Table 1. Difference measures used in calibrated models with SPSS

Variable	**X_1**	**X_2**	**X_3**
Difference measure	Total length of two routes	Mountainous length of two routes	Length of minor class in two routes

Among six calibrated models, the last model is the best one because the amount of $-LL(\beta$ in this model is the 479.6, which is the minimum, and "Measure of Significance" of all variables are zero. This model is presented as below

$$P(\text{Selection of first route}) = \; P_1 = \frac{1}{1+\exp(-0.789-0.004X_1-0.06X_2-0.018X_3)} \tag{18}$$

- *Calibrated models with GAUSS*

Among models calibrated in the group with GAUSS software, 3 models were found to be valid. -LL(0) in every three models is 831.8. These models consist of the following variables:
X_1^i *:* Total length of route *i (km).*
X_2^i : Mountainous length of route *i (km).*
X_3^i *:* Length of route *i with type of minor highway (km).*
X_4^i *:* Length of route *i* with level-of-service D or E *(km).*

First model:

$$\begin{aligned} U_{r1} &= \underset{t=-2.14}{-0.0033\ X_1^1} \underset{t=-5.46}{-\ 0.0215\ X_2^1} \\ U_{r2} &= \underset{t=-4.10}{-0.0002\,(X_1^2)^{1.6}} \underset{t=-2.9}{-\ 0.0215\ X_2^2} \end{aligned} \tag{19}$$

- $LL(\hat{\beta})$ =626.1. Hence, this model is relatively valid and credible.

Second model:

$$U_{ri} = \underset{t=-3.4}{-0.013\ X_1^i} \underset{t=-11.2}{-\ 0.042\ X_2^i} \underset{t=-13.5}{-\ 0.016\ X_3^i} \tag{20}$$

In this function, *i* will be 1 or 2 (for defining two alternatives). - $LL(\hat{\beta})$ *=663.5*

Third model:

$$U_{ri} = \underset{t=-2.66}{-0.012\ X_1^i} \underset{t=-11.1}{-\ 0.043\ X_2^i} \underset{t=-11}{-\ 0.018\ X_3^i} \underset{t=-1.6}{-\ 0.003\ X_4^i} \tag{21}$$

i will be 1 or 2 (for defining two alternatives) and amount of - $LL(\hat{\beta})$ is 662.3.

II. Presented models for the second group (one of two alternatives is freeway)

In this group, the choice set of truck drivers consists of two alternatives. One of these alternative routes is a toll-freeway and the other is a principle road.

$$\begin{aligned} U_f &= \underset{t=-6.25}{-0.0094\ X_f} \\ U_{pr} &= \underset{t=-4.55}{-0.0117\ L_{pr}} \end{aligned} \tag{22a}$$

$$X_f = \exp(0.55 * \frac{T_f}{L_f}) \tag{22b}$$

where,

L_f : Length of freeway (km).

T_f : Average toll cost for all kinds of trucks (Tomans).

L_{pr} : Length of principle road (km).

X_f : Dummy variable for defining toll cost per unit of length for freeway.

U_f : Utility function of freeway.

U_{pr} : Utility function of principle road.

According to the log-likelihood function that is presented below, the model is credible and accredited.

$$\begin{cases} -LL(0) = 138.63 \\ -LL(\hat{\beta}) = 88.67 \end{cases}$$

III. Presented models for the third group (three non-freeway alternatives)

In this group, the choice set of truck drivers consists of three alternatives. None of them is a toll-freeway. In this case, 3 models were selected as the best choice, but according to the conditions of real routes (used for modeling), the length of the longest route must not exceed a 15% longer length than that of the shortest route. In all three presented models, the sum of LL(0) is 219.7.

First model:

The utility function of this model has a generic form as follows:

$$U_i = \underset{t=-3.33}{-0.876\ X_i} + \underset{t=5.74}{0.85\ Y_i} \tag{23a}$$

Consisting of two dummy variables as below

$$X_i = \ln(1 + L^i_{Roll} + 2L^i_{Moun})$$
$$Y_i = \frac{1}{1 + L^i_{Minor}} \tag{23b}$$

Where,

L^i_{Roll} : Length of roll conditions in route *i* (km).

L^i_{Moun} : Length of mountainous conditions in route *i* (km).

L^i_{Minor} : Length of minor roads in route *i* (km).

U_i : Utility function for Alternative *i* (km).

Where, *i* is 1,2 or 3 for defining three alternatives. $-LL(\hat{\beta}) = 192.9$.

Second model:

The utility function of this model also has a generic form which can be written

$$U_i = \underset{t=-2.58}{-0.0038\ X_i} \underset{t=-3.98}{-0.0028\,L^i_{Minor}} \tag{24a}$$

$$X_i = L^i_{Roll} + 2L^i_{Moun} \tag{24b}$$

Where, *i* is also 1,2 or 3 for defining three alternatives. $-LL(\hat{\beta}) = 198.3$.

Third model:

The utility function of this model also has a generic form which can be written:

$$U_i = \underset{t=-2.8}{-0.0043\ X_i} + \underset{t=3.99}{0.0013\,L^i_{Expw}} \tag{25a}$$

$$X_i = L^i_{Roll} + 2L^i_{Moun} \tag{25b}$$

Where,

L^i_{Expw} : Length of expressway road in route i (km).

Where, *i* is also 1,2 or 3 for defining three alternatives. $-LL(\hat{\beta}) = 198.5$.

The first model is more accredited and credible because of having the minimum amount of $-LL(\hat{\beta})$.

7. CALIBRATION OF ENTROPY-MAXIMIZATION MODEL WITH COMPOSITE OBJECTIVE FUNCTION

In the section of intercity transportation in Iran, one could use two categories of information to estimate the real and correct origin-destination matrix of freight movement. These two information sources are registered as waybill data and traffic counts data. The obtained O-D matrix from waybill data is only representing the freight movement pattern in the country. Due to errors existing in the waybill source, this matrix could be different from the real commodity movement matrix in the country. So, the purpose of this research is to estimate the real O-D matrix; consider the waybill data (in the form of a matrix of the commodity movement in the under-study area) as an initial (prior) matrix and also traffic count data (according to performance limitations). In this process, developed assignment models for heavy traffic are being used to load each estimate matrix on the road network.

a) Evaluation of the information source of waybill

The information taken from the waybill is one the most valuable collected and kept by the TTO. This information, which in fact represents the freight demand pattern in the country, has been utilized in different studies. Referring to this information source, two groups of errors may be considered: the first, group errors which may exist in the process of collecting registered waybill data in the country and saving this information in the relative databases, and the second, group errors which may exist due to not matching the waybill information with real information of commodity transportation. The reason for the second error is not getting the waybill by drivers of the vehicles (trucks). This issue is much more frequent in the pick-ups which carry commodities.

b) Evaluation of the information source of traffic counts

Relating to the survey on traffic counts, two kinds of errors are to be considered. The first kind of relevant errors in traffic counting consists of human errors and systematic errors. This means that in addition to the errors due to human error the selection of the station location, the arrangement of the survey agents and also the survey instructions have significant and considerable influence on the amount of type 1 errors. The second type of errors may exist in traffic count data due to the considered time span for traffic counting and its inefficiency. According to the standard method mentioned in the valid references, to estimate average annual daily traffic (AADT), one must conduct the traffic counting during the whole year and/or on the usual days of the year. One can get to AADT for each considered segment after averaging all the results.

c) Estimation of utility of traffic counts and waybill information

Under the circumstance in which there is an uncertainty about each information source; initial matrix and traffic counts, if one could understand the related errors to each of these sources (or on the other hand, the amount of their utility), it is possible to estimate a matrix with the following considerations:

1. The distance between estimated and initial matrix is appropriate with the relevant errors in the initial matrix.
2. The difference between estimated traffic volumes (which are from assigning estimated matrix on the road network) and observed volumes are also appropriate with the accuracy of traffic counts.

Because of the above reasons, it seems that the best and ideal method for solving the problem is to use the entropy method with a composite objective function (multi-objective function). It is similar to a model presented by Willumsen. In this method, aggregated weight factors λ_1 and λ_2 or disaggregated weight factors α_{ij} and β_a possess a high significance in the calibration of models. However, using the disaggregated weight factors seems to be almost impossible. Because of problems induced by disaggregated information, to calibrate the multi-purpose entropy function, aggregated weight factors have been used. These coefficients, which are considered as parameters representing credit measure, are defined as the utility criteria for each of these two information sources.

Due to the fact that in the condition of information shortage for determining weight factors, using expert-judgment could be useful. In this problem, after conducting interviews with many transportation managers and engineers, and using logics of the AHP method, two weight factors, λ_1 and λ_2 were estimated.

In this research, a hierarchy system was designed with the purpose of "correct estimation of origin-destination matrix of freight movements". Two criteria of "systematic errors (including human errors, in survey designing errors and errors in data collection and also data entry process)" and "covering errors (including errors due to not getting the waybill by the drivers and the errors due to the time span and considering small time span for traffic counting in comparison with standard situation)" were also considered. This system has two choices of "information source of the waybill" and "information source of traffic counts". The system has been presented in Fig. 1.

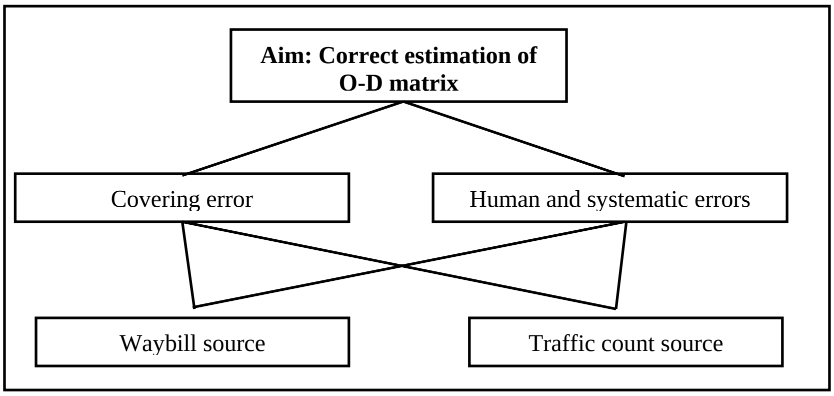

Fig. 1. The hierarchy process in determining the utility of each of the information sources

The considered ranking system has a scale of 0-20 and for 5 scores of 0, 5, 10, 15 and 20, some definitions for interviewees were presented matching this ranking system (Table 2). In this regard some of the professional experts in the levels of management and transportation engineering were interviewed.

Table 2. Defined ranking system

Expert-judgement	Score
Absolutely preferable (without any error or considerable difference)	20
Highly preferable (with accepted error and 90% accuracy)	15
Intermediate preferable (with high error and 50% accuracy)	10
Low preferable (with very high error level)	5
Not preferable (absolutely not accurate)	0

The results obtained from this survey indicate that the approaches are different from each other. These ideas must be framed into 4 averaged numbers (for the utility of the waybill and traffic count information regarding the two error sources). Hence, in each condition, the geometric mean of the obtained scores was applied. It is worth mentioning that the geometric means of some numbers or weights are obtained as below

$$W_{ij} = (W_{ij}^1 \times W_{ij}^2 \times \ldots \times W_{ij}^n)^{1/n} \tag{26}$$

where W_{ij} is the weight of the choice *i* with regard to criterion *j*, and W_{ij}^n is this weight related to each individual n. Accordingly, the geometric mean of the under-investigation measures including the utility of traffic counts regarding systematic errors, the utility of traffic counts regarding covering error, the utility of waybill information in terms of systematic errors and finally, the utility of the waybill in terms of the covering error was calculated according to the expert-judgment of knowing agents. The amounts of these utilities and their normalized amounts have been presented in Table 3.

Table 3. Geometric mean of weights of each of information sources

	Geometric mean of weights		Sum	Normalized weights		Sum
	Systematic utility	Cover utility		Systematic utility	Cover utility	
Traffic count	13.081	10.629	23.710	0.552	0.448	1
Waybill	15.229	12.309	27.539	0.553	0.447	1

The normalized calculated weights are the weights of each of the information sources in terms of two kinds of utilities (or in the reverse form, uncertainty). But in the AHP model, the criteria themselves must be compared with each other. In other words, the question is "Is it correct to suppose equal weight and its effect for each criterion?"

Because there is no particular and clear answer to this question, and also because of the complexity of the problem, its pair comparison matrix between these two error sources was considered as shown below

	Systematic error	Covering error
Systematic error	1	α
Covering error	$1/\alpha$	1

In order to calculate each of these error sources, after discovering the geometric mean from the elements of the previous matrix, the obtained phrases were normalized. The results are presented as below:

Criteria	Geometric mean	Normalized weights
Systematic error	$(\alpha\times 1)^{\frac{1}{2}}=\sqrt{\alpha}$	$\dfrac{\alpha}{1+\alpha}$
Covering error	$(1\times\dfrac{1}{\alpha})^{\frac{1}{2}}=\dfrac{1}{\sqrt{\alpha}}$	$\dfrac{1}{1+\alpha}$
Total	$\sqrt{\alpha}+\dfrac{1}{\sqrt{\alpha}}$	*1*

Hence, with the determination of weight of each information source in terms of the two error sources and also considering the effect of each of these error sources (their weight in the problem), the coefficients λ_1 and λ_2 (the aggregated utility of waybill and traffic counts) are presented as follows:

$$\gamma_1=0.553(\frac{\alpha}{1+\alpha})+0.447(\frac{1}{1+\alpha}) \qquad \& \qquad \gamma_2=0.553(\frac{\alpha}{1+\alpha})+0.448(\frac{1}{1+\alpha})$$

Since the weights 0.533 and 0.447 for waybill information and weights 0.552 and 0.448 for traffic count information are very close to each other, the coefficients λ_1 and λ_2 can be assumed to be equal 0.5. This means that, according to expert-judgments, the amount of reliability to each of these two information sources is equal and is almost 50%. This way the presented model, which is based on the method of the entropy-maximization with a composite objective function is calibrated as follows:

$$\min\ 0.5\sum_{ij}(T_{ij}\log\frac{T_{ij}}{t_{ij}}-T_{ij}+t_{ij})+0.5\sum_{a}(V_a\log\frac{V_a}{\hat{v}_a}-V_a+\hat{v}_a) \tag{27}$$

$St.$

$$\sum T_{ij}P_{ij}^{a}-V_a=\circ$$
$$T_{ij}\geq\circ$$

where,

T_{ij} : The estimated trip interchange between origin *i* and destination *j*.

t_{ij} : The trip interchange between origin *i* and destination *j* in the initial matrix.

V_a : The estimated traffic volume in arc *a*.

$\hat{v}_a$: The observed traffic volume in arc *a*.

P_{ij}^{a} : The proportion of arc *a* due to trip interchange between origin *i* and destination *j*.

Because of the equality which exists in the weights, the objective function can be simplified as shown below (under same constraints)

$$\min \sum_{ij} (T_{ij} \log \frac{T_{ij}}{t_{ij}} - T_{ij} + t_{ij}) + \sum_{a} (V_a \log \frac{V_a}{\hat{v}_a} - V_a + \hat{v}_a) \quad (28)$$

8. METHODOLOGY OF ESTIMATING REAL FREIGHT MATRICES

With the availability of the initial information on the problem, the estimation of the correct (real) O-D matrix in the case of freight movements in rural areas can be conducted by applying the following steps:

Step 1. Preparation of the information of initial O-D matrix: To start with, one could use the following formulas for converting the weight of annual transported freight between each two regions to the number of trucks departing from these two regions [14]:

$$N = \frac{W}{\rho_{avg} \sum_{i=1}^{n} p_i v_i} \quad (29)$$

$$N = \frac{1.3 * W}{\sum_{i=1}^{n} p_i W_i (1 - p_{ei})} \quad (30)$$

N : The total number of trucks (all kinds) which carry the weight W from one region to another region.
W : The weight of annual transported freight between a pair of origin-destinations in kilograms.
ρ_{avg} : The average density of transported freight ($202.68 \, kg/m^3$ or $12.5 \, lb/ft^3$).
p_i : The average percent of kind i trucks departing in the network from n kinds.
v_i : The average volume (capacity) of kind i trucks (m^3).
W_i : The average weight of loaded trucks kind i in kg (average weight of trucks kind i plus the average weight of load)
p_{ei} : The average of percentage of vacant trucks from kind i.

Step 2. Preparation of the road network and traffic count information: In order to simulate the network and save its related information, one needs to make use of a GIS environment and its techniques. In this environment, the database (e.g., DBF files) and graphical system (e.g., links and nodes) are interactive and simultaneous. Furthermore, some powerful analytical toolboxes and presentation instruments in the technical software of the issue such as Arcview and Mapinfo are considered.

Step 3. Estimating the share of each arc in the freight interchange flow based on the assignment models: Since the intercity road network is usually uncongested, and due to the independence of the share of each network segment from the traffic volume of other segments, P_{ij}^{a} values in the model of O-D estimation is measurable exogenously and independent from the estimation process of O-D matrix estimation. The calibrated assignment models can be considered as the base for this calculation. In this way, using the extension of Network Analyst in Arcview software, one could determine the shortest path between each O-D pair. Also, in the case of the presence of other active alternative routes rather than the existing shortest path, with the determination of the required characteristics of models for each of the routes, shares from trip exchange flow could be determined by using developed assignment models.

This share, which must be presented as a percentage, will prove valid for each of the arcs composing the route. Hence, in this method, after assigning the flow to the routes, the flow is assigned to the arcs (segments). The limitation of the presented probable models which originate from the Logit formulation is that the alternative routes must not have a high correlation, and there should be a possibility of defining these two routes independently.

Step 4. Estimation of the correct O-D matrix using the obtained information: The is numerous initial information for starting the process of estimation of O-D matrix closest-to-reality; including waybill information, the information related to the conditions of network arcs (segments) and observed traffic volumes. It is also necessary to calculate the share of each route, and consequently the share of each arc from trip exchanges (in the case of freight movement). Hence, using the calibrated model based on the nonlinear programming method of entropy-maximization with the composite objective function, one can embark on this important issue. Although the amounts of parameters λ_1 and λ_2 in this model were estimated to be equal as 0.5, one could choose different amounts of λ_1 and λ_2 for solving the model and comparing the results. For solving this nonlinear model, software such as LINGO could be used.

This methodology was performed with the existing information of the Kurdistan province road network and waybill information for this province as a case study by using ARCVIEW software for GIS working, FOXPRO software for keeping databases and programming and LINGO software for mathematical non-linear programming. The obtained results indicate a good and close-to-realistic estimation of a freight O-D matrix for Kurdistan province in Iran.

9. CONCLUSION

According to existing information of waybill and traffic counts in the Iran Transportation and Terminal Organization, a process was designed to update the existing waybill matrix (to estimate a true O-D matrix for freight movement) in the case of truck activities. After conducting some initial studies, due to the special route choice pattern by truck drivers in intercity networks, using the Logit formulation, 13 probable (pure stochastic) assignment models related to the condition of a choice set for truck drivers in an intercity network were developed. In the next step, existing errors in information sources of waybill and traffic counts were considered and the related uncertainties were estimated for each one by an AHP method. Thus, to approach the best model of O-D matrix estimation, an entropy maximization model with a composed objective function was calibrated. In this way, using the GIS environment, calibrated assignment models and also a calibrated entropy function, the methodology of updating the waybill matrix is presented.

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