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WAVE INDUCED UPLIFT FORCES ACTING ON HALF-BURIED SUBMARINE PIPELINE IN SANDY SEABED BY NUMERICAL METHODS*

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Abstract– Two dimensional finite element (FEM) and boundary integral equation methods (BIEM) are used to determine the uplift forces on a half-buried submarine pipeline in a sandy seabed. The main assumptions are potential flow theory for water, linear elastic stress-strain for soil, and Darcy's law for pore fluid flow. Dynamic pressures around the pipe and uplift forces are calculated based on BIEM and FEM. Also, dynamic uplift forces on a pipeline with variable burial depths are calculated. The results of the numerical approach are compared with some analytical formulations based on the potential flow theory and experimental data, and good agreement is shown with both analytical and experimental data. Parameters in this study have been discussed in relation to the uplift forces.

Keywords– Uplift force, numerical method, submarine pipeline, wave induced, sandy bed

1. INTRODUCTION

The search for oil under the seabed has led to extensive theoretical and experimental investigations in the field of offshore oil exploration and exploitation [1]. A submarine pipeline offers an efficient mode of transportation for oil and gas continuously from offshore to onshore. One of the simplest methods for transporting oil and gas from onshore to offshore is to lay pipe on the seabed. There are many thick under-deposit marine sediments located in several coastal areas and the scouring of foundations in such soft noncohesive sediments can be quite significant [2].

Scouring around the buried pipeline results in spanning and vibration and vortex induced loading. If the natural frequency of the pipe is closer to the frequency of the waves, then the pipeline vibration magnifies, resulting in failures of submarine pipelines. In brief, research regarding the wave induced uplift forces acting on half- buried submerged pipelines are discussed. All research is based on experimental and theoretical work.

Morison [3] derived equations based on the potential flow theory to determine vertical and horizontal forces acting on submerged pipelines. Shankar *et al*. [4] modified the Morison equation and obtained a better result. By checking his equation with experimental data, he found that the error of this method was less than 1%. McPherson [5] and Monkmeyer *et al*. [6] developed an analytical method based on potential flow theory and experimental coefficients.

Shankar *et al*., [7] used a boundary integral equation method based on Green's function theorem to determine uplift forces for pipe near the seabed and his advantage conformed to the experimental data. He did not consider soil property in his analysis. Talebbeydokhti and Pishvai [8] performed a dynamic analysis of submerged pipelines by finite element method. Magda [9] presented a finite element method

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for buried pipeline. He used potential flow theory to define pressure over the seabed, assumed a linear stress-strain relationship for soil, and Darcy's law for pore fluid. Most of the analytical [5, 6, 10] and numerical solutions [9, 11-13] to the uplift force for submarine pipelines buried in seabed sediments have the disadvantage in their limited practical applications due to their assumptions. Rao *et al*. [15] reported an experimental investigation to determine pressure around half-buried pipeline in sandy and clay seabeds for different burial depths, and plot the result in dimensionless diagrams that can be used for real conditions. Most investigations are based on experimental data and it is clear that laboratory and field conditions have some differences. Theoretical methods also have some differences with reality because of modeling soil behavior. In this paper, a finite element and boundary integral equation method is presented for the calculation of uplift forces acting on half-buried pipeline.

2. THEORETICAL DEVELOPMENT

A wave-induced excess pore pressure, cyclically generated in the vicinity of a buried submarine pipeline, constituted one of the main factors that has to be considered in the pipeline stability analysis**.** In a half-buried pipeline, at the section that is located in water, the fluid motion is idealized as linearized, as two-dimensional potential flow. An appropriate Green’s function was applied to model the fluid domain. The integration contours are discretized into small elements and the fluid velocity potential and its normal derivative are assumed to vary linearly in each element. This technique has been used in a variety of related problems by some investigators [10, 4]. The geometry of the problem is shown in Fig. 1.

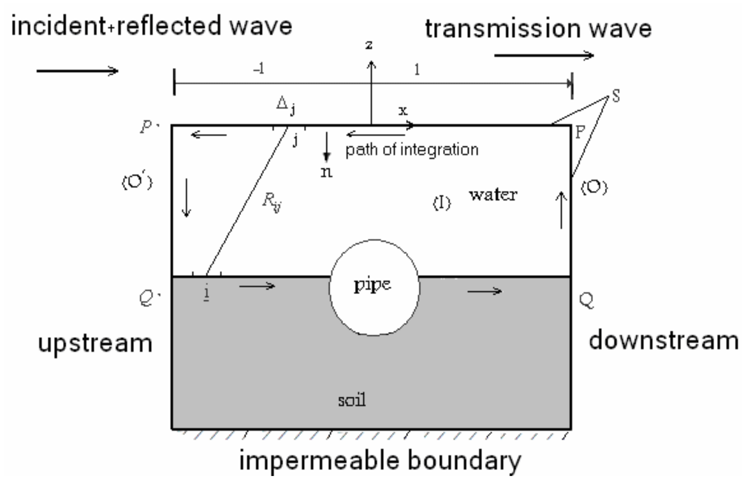

Fig.1. Definition sketch

Assuming an inviscid and incompressible fluid, the wave motion may be described in terms of a velocity potential $\phi(x,z;t) = \mathrm{Re}\left[\phi(x,z)e^{-i\omega t}\right]$ where ϕ is velocity potential, $\omega = \dfrac{2\pi}{T}$ and T is period of the wave and Re [] denotes the real part of the complex expression [7]. This potential function must satisfy the Laplace Equation everywhere in the region of flow:

$$\nabla^2\phi = 0 \tag{1}$$

This problem is subjected to the following boundary conditions on the quiescent fluid free-surface and the sea-bed, respectively,

$$\frac{\partial\phi}{\partial z} - \frac{\omega^2}{g}\phi = 0 \qquad \text{On } z = \text{-d} \tag{2}$$

$$\frac{\partial \phi}{\partial z} = 0 \qquad \text{On } z = 0 \tag{3}$$

In the above equation, the complex amplitude of the incident wave potential ϕ_I is defined by Eqn. 4 [7]:

$$\phi_I = \gamma_w H_w / 2 * \cosh(kz) / \cosh(kd) * \exp(ikx) \tag{4}$$

Where H_w is the wave height, $K = \frac{2\pi}{L_w}$, L_w is the wave length, γ_w is the unit weight of water, and x , z are horizontal and vertical coordinates. In region PQ and $P'Q'$ the value of potential flow is defined by Eq. 4.

The boundary integral equation can be expressed by Eq. 5:

$$\alpha \phi_p = -\frac{1}{\pi} \oint_s \phi_j \frac{\partial g}{\partial n} ds + \frac{1}{\pi} \oint_s g \frac{\partial \phi_j}{\partial n} \tag{5}$$

Where n is the inward normal vector on the boundary, α is the interior angle, and g is Green's function expressed by Eq. 6:

$$g_{pj} = \ln(R_{pj}) \tag{6}$$

$$R_{pj} = \sqrt{(x_j - x_p)^2 + (y_j - y_p)^2} \tag{7}$$

If the boundary is divided into N small facets, and the point p lies on S at $i = (x_i, z_i)$, it would lead to the Green's second identity for potential functions, given by Eqn. 8:

$$\phi_i = \sum_{j=1}^{N} \left[\bar{C}_{ij} \phi_j - C_{ij} \phi'_j \right] \tag{8}$$

That since point I is located on the boundary; C_{ij} and $\bar{C}_{ij}$ can be expressed on each facet by:

$$C_{ij} = -\frac{1}{\pi} \int_j \ln(R_{ij}) ds \tag{9}$$

$$\bar{C}_{ij} = -\frac{1}{\pi} \int_j \frac{\partial \ln(R_{ij})}{\partial n} ds \tag{10}$$

With a selection of N points on surface and bottom boundaries, Eq. (8) results in N simultaneous linear equations with the same number of unknowns and can be solved to yield all the unknown parameters. The hydrodynamic pressure p_j at any point j on the upper surface of the pipe and the surface of the soil is given as a function of time t as Eq. (11):

$$P_j(t) = \text{Re}\{\frac{1}{2} \rho g H_w (-i.\exp(i\omega t)) \phi_j\} \tag{11}$$

Under plane strain conditions, the following two Eqs. (12, 13), describe elastic deformations of the soil skeleton:

$$G\left(\frac{\partial^2 u_x}{\partial x^2} + \frac{\partial^2 u_x}{\partial z^2}\right) + \frac{G}{1-2\mu} \frac{\partial}{\partial x}\left(\frac{\partial u_x}{\partial x} + \frac{\partial u_z}{\partial z}\right) = \frac{\partial p}{\partial x} \tag{12}$$

$$G(\frac{\partial^2 u_z}{\partial x^2}+\frac{\partial^2 u_z}{\partial z^2})+\frac{G}{1-2\mu}\frac{\partial}{\partial z}(\frac{\partial u_x}{\partial x}+\frac{\partial u_z}{\partial z})=\frac{\partial p}{\partial z} \tag{13}$$

$$\beta' n\frac{\partial p}{\partial t}+\frac{\partial}{\partial t}(\frac{\partial u_x}{\partial x}+\frac{\partial u_z}{\partial z})=\frac{k}{\gamma_w}(\frac{\partial^2 p}{\partial x^2}+\frac{\partial^2 p}{\partial z^2}) \tag{14}$$

$$\beta' = \beta+\frac{1-S}{p_a} \tag{15}$$

$$p_a = p_{atm} + p_h \tag{16}$$

Equation (14) reflects the continuity principle incorporating Darcy's law of fluid through a porous medium. Where S is degree of saturation, u_x, u_z are horizontal and vertical displacement of soil, G is shear modulus of soil, μ is the Poisson ratio, K is the coefficient of the isotropic permeability, γ_w is the unit weight of the pore fluid, β is compressibility of water, p_{atm} is atmospheric pressure, p_a is static pressure, and n is the soil porosity. Using functions of time and space corresponding to a finite number of nodes, the pore pressure and displacements in the element are represented as follows:

$$u_x = \sum_{i=1}^{n_e} u_i \psi_i(x,z)e^{i\omega t} \tag{17}$$

$$u_z = \sum_{i=1}^{n_e} v_i \psi_i(x,z)e^{i\omega t} \tag{18}$$

$$p = \sum_{i=1}^{n_e} p_i \psi_i(x,z)e^{i\omega t} \tag{19}$$

Where ψ_i is the shape function of node i in each element. Using the Galerkin approach and the discretization equation, the finite element formulation of Eqs. (12) to (14) is derived as a matrix form:

$$[K]\{\phi\}=\{F\} \tag{20}$$

$$[K]=\begin{vmatrix} [k_{11}] & [k_{12}] & [k_{13}] \\ [k_{21}] & [k_{22}] & [k_{23}] \\ [k_{31}] & [k_{32}] & [k_{33}] \end{vmatrix} \tag{21}$$

$$\{\phi\}=\begin{Bmatrix} \{u\} \\ \{v\} \\ \{p\} \end{Bmatrix} \tag{22}$$

$$F=\begin{Bmatrix} F_1 \\ F_2 \\ F_3 \end{Bmatrix} \tag{23}$$

where v and u are the vertical and horizontal displacements of nodal points in the soil domain.

For each node, there exists three degrees of freedom: horizontal displacement, vertical displacement and pressure (u, v, and p). The domain of soil discretized by a four node element is shown in Fig. 2.

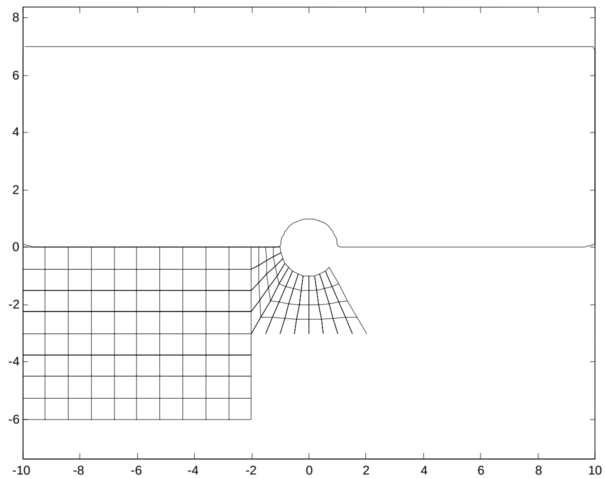

Fig. 2. Finite element mesh discretization of the

3. BOUNDARY CONDITIONS

1- At z = -h, boundary conditions assume impermeable boundary ($\frac{\partial p}{\partial z} = 0$) and $u_x = 0$ and $u_z = 0$

2- At $x = \pm L$ are assumed impermeable boundary ($\frac{\partial p}{\partial x} = 0$) and $u_x = 0$

3- On the pipe wall, boundary conditions are impermeable boundary ($\frac{\partial p}{\partial n} = 0$) and the pipe is fixed by anchors ($u_x = u_z = 0$)

4-On the seabed surface, pressures were already defined.

The use of the interface element is necessary in the pipe wall because of the relative displacement between the pipe and soil. Interface element is shown in Fig. 3.

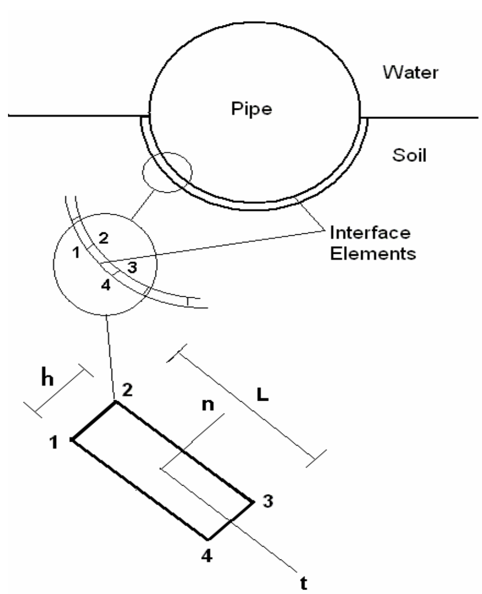

Fig. 3. Interface element sketch

In the interface element, stiffness is large in the normal direction and small in the tangetial direction. The difference in thickness can be expressed as follows:

$$\Delta u_{ni} = u_{ni}^{top} - u_{ni}^{bot} \tag{24}$$

$$u_{n}^{top} = \sum \psi_i u_{ni}^{top} \tag{25}$$

$$u_{n}^{bot} = \sum \psi_i u_{ni}^{bot} \tag{26}$$

$$\Delta u_n = \sum \psi_i u_{ni}^{top} - \sum \psi_i u_{ni}^{bot} \tag{27}$$

$$\varepsilon_n = \frac{\partial \Delta u_n}{\partial n} = \sum_{i=1}^{2} \frac{\partial \psi_i}{\partial n} u_{ni}^{top} - \sum_{i=1}^{2} \frac{\partial \psi_i}{\partial n} u_{ni}^{bot} \tag{28}$$

$$\varepsilon_t = \frac{\partial \Delta u_t}{\partial t} = \sum_{i=1}^{2} \frac{\partial \psi_i}{\partial t} u_{ti}^{top} - \sum_{i=1}^{2} \frac{\partial \psi_i}{\partial t} u_{ti}^{bot} \tag{29}$$

The above equation can be rewritten as a matrix form:

$$A = \begin{bmatrix} -\frac{\partial \psi_1}{\partial n} & 0 \\ 0 & -\frac{\partial \psi_1}{\partial t} \\ \frac{\partial \psi_2}{\partial n} & 0 \\ 0 & \frac{\partial \psi_2}{\partial t} \\ \frac{\partial \psi_3}{\partial n} & 0 \\ 0 & \frac{\partial \psi_3}{\partial t} \\ -\frac{\partial \psi_4}{\partial n} & 0 \\ 0 & -\frac{\partial \psi_4}{\partial t} \end{bmatrix} \tag{30}$$

$$B=AT \tag{31}$$

Where T is the rotation matrix. The strain energy of the interface element can be expressed as written below:

$$II = \frac{1}{2}\int_A \varepsilon^t C \varepsilon dA = \frac{1}{2}\int_A (Bu)^t C (Bu) dA = \frac{1}{2} u^t K_I u \tag{32}$$

Finally, the stiffness matrix of the interface element is concluded as follows:

$$[K] = \int B^T CBdA \tag{33}$$

$$C = \begin{bmatrix} k_n & 0 \\ 0 & k_t \end{bmatrix} \tag{34}$$

4. COMPUTER PROGRAM (UPLIFT)

All above processes can be calculated by a computer program called UPLIFT. The inputs of this program are:

1-Geometrical data consisting of the pipe diameter (D), burial depth (b), length of domains (L), height of water (h), soil layer thickness (H).

2- Mesh input consisting of the number of elements in the horizontal and vertical directions in the soil and the number of elements near the pipe.

3- Wave input consisting of wave length (L_w) and wave height (H_w).

4- Soil property consisting of the degree of saturation (S), porosity (n), permeability (K) and shear modulus (G).

Outputs of this computer are dynamic pressure distribution around the pipe and dynamic uplift forces.

5. RESULTS AND DISCUSSION

To verify the results of the computer program, two parts must be checked:

1-Checking the BIEM method:

a) In the absence of the pipe, there is a straight surface and on this, the results of the program and the potential flow theory should conform. Results of surface pressures on soil when the wave length is 30m, wave height is 0.5 m, and the phase of the wave is zero are shown in Fig. 4.

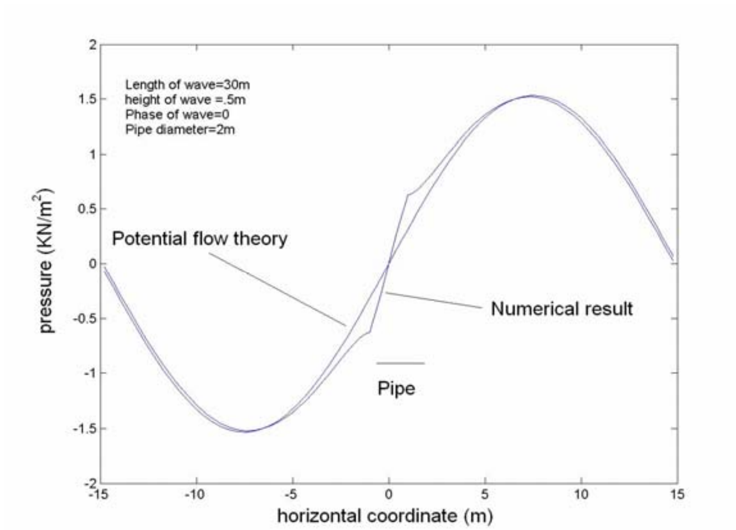

Fig. 4. Comparison between numerical and potential flow theory results

b) Far away from the half-buried pipeline pressure distribution on the soil surface must agree with the potential flow theory. The results of the numerical and potential flow theory when the wave length is 30 m, wave height is 0.5 m, pipe diameter is 2 m, burial depth is 1 m and the phase of the wave is zero are shown in Fig. 5

2- Checking FEM by SIGMA/W software:

Pressure distribution in the absence of the pipe must agree with other software such as SIGMA/W. The results of pressure distribution on the impermeable boundary are compared with the results of the educational edition of SIGMA/W as shown in Figs. 6 and 7.

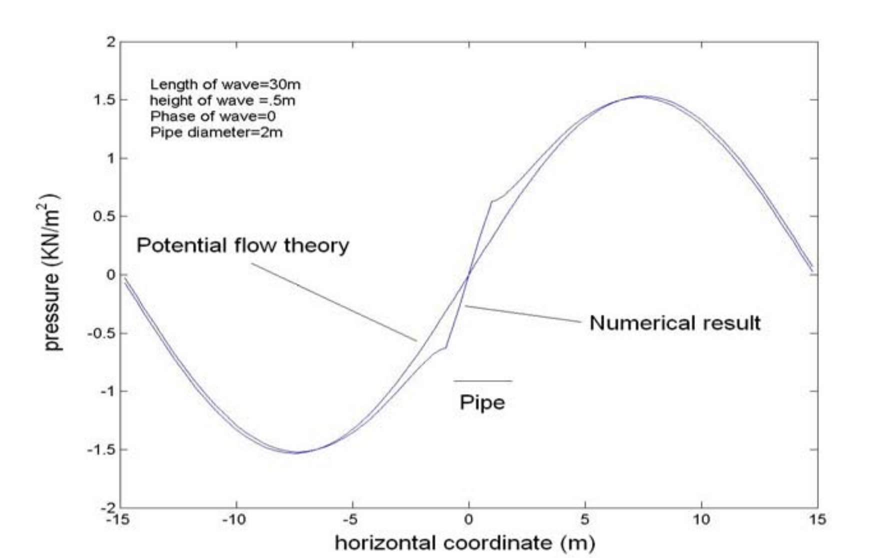

Fig. 5. Comparison between numerical and potential flow theory for half-buried pipeline

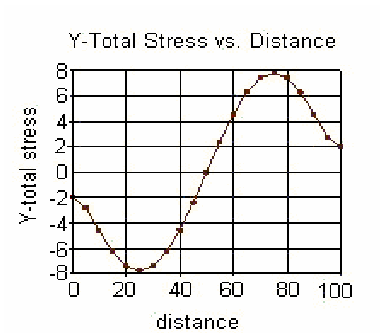

Fig. 6. Result of SIGMA/W program for pressure at bottom of soil layer
(distance in meter and stress in KN/m^2)

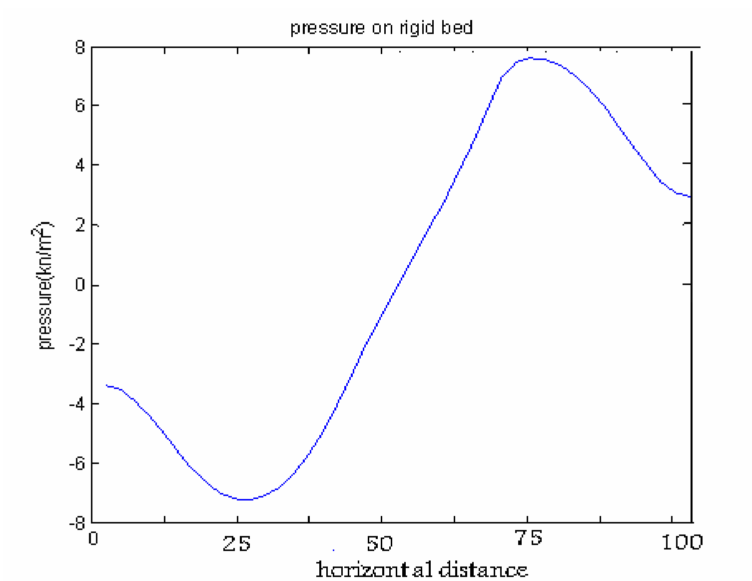

Fig. 7. Result of this program for pressure at bottom soil layer

The finite element results show good agreement with the output of SIGMA/W program.

6. DISCUSSION ABOUT PARAMETERS AFFECTING UPLIFT FORCE

a- Effect of wavelength is shown in Fig. 8 and 9.

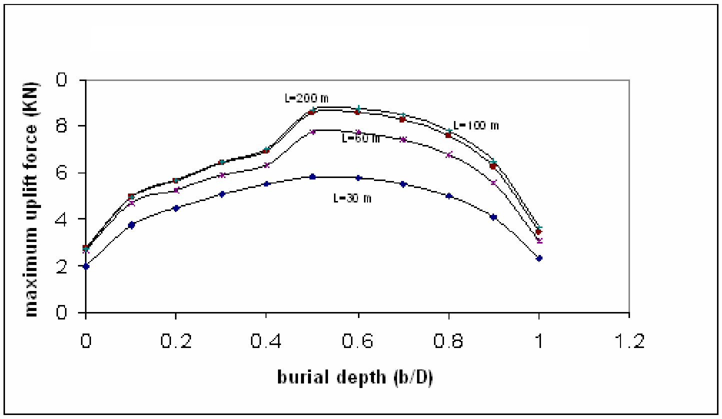

Fig. 8. Effect of wavelength on max. uplift force for different burial depths

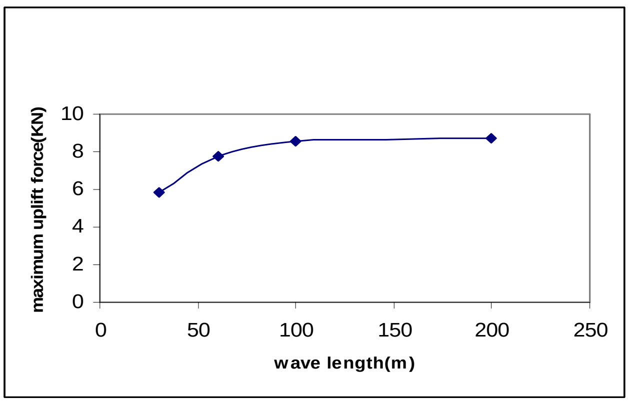

Fig. 9. Effect of wavelength on uplift force AT b/D=0.5

Figures 8 and 9 show that maximum uplift force occurs at b/D=0.5, and by increasing the wavelength, maximum uplift force is limited to a maximum value. This conclusion was already shown by investigators such as Lennon [11] and Magda [9].

b- Effect of permeability is shown in Fig. 10.

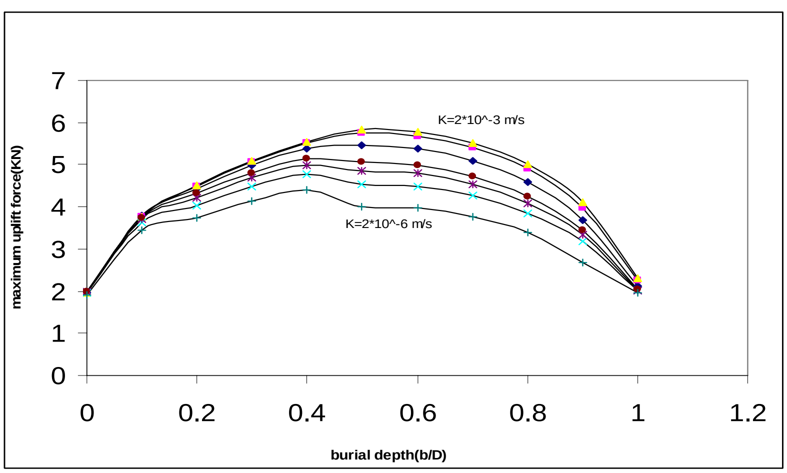

Fig. 10. Effect of permeability on max. uplift force

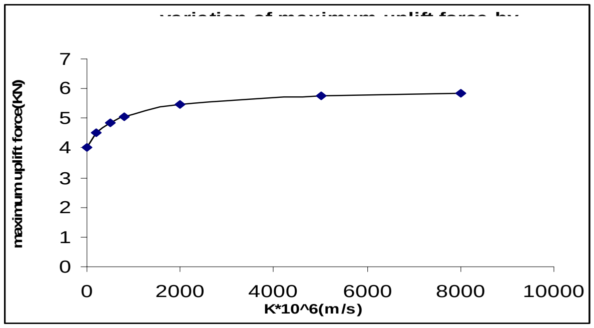

Fig. 11. Effect of permeability on max. uplift forces at b/D=0.5

As can be seen from these figures, by decreasing the permeability, the maximum uplift force decreases, and the maximum uplift occurs on b/D=0.5

c- Degree of saturation: degree of saturation affects the compressibility of fluid and its effect is shown in Fig. 12.

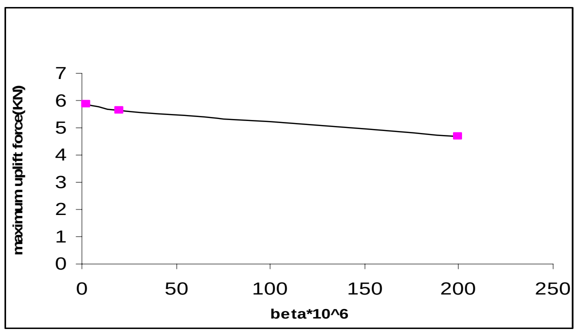

Fig. 12. Effect of compressibility of fluid on maximum uplift force for b/D=0.5

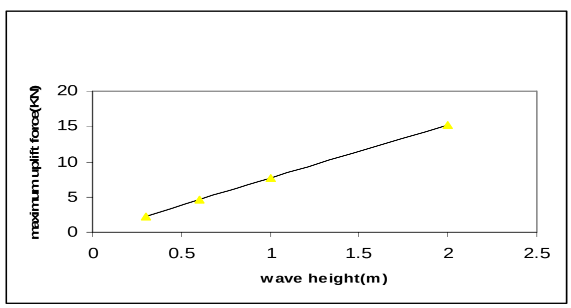

Fig. 13. Effect of wave height on maximum uplift forces at b/D=0.5

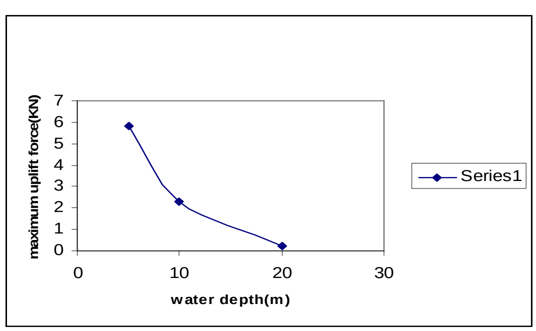

Fig. 14. Variation of maximum uplift forces by water depth AT b/D=0.5

7. CONCLUSION

1- The finite element and the boundary integral equation method are capable in calculating dynamic uplift forces on half buried submerged pipeline successfully.

2- Critical burial depth was found to be around b/D=0.5 and in this depth we have the maximum uplift force relative to the wave propagation. Also the minimum uplift force occur at b/D=1.

3- Parameters such as wavelength, permeability and water depth have a logarithmic effect on uplift forces; and degree of saturation and wave height have a linear effect on uplift forces, therefore maximum uplift forces can be expressed by Eqn. 35:

$$U_{\max} = P_0 * \{\alpha_1 + \alpha_2 H_w + \alpha_3 b^2 + \alpha_4 \log(L_w) + \alpha_5 \log(K) + \alpha_6 \beta + \alpha_7 \log(h_w)\} \tag{35}$$

4- This procedure can be used for dynamic loading on the pipe for the submerged pipeline design.

REFERENCES

1. Geo, F. P., Gu, X. Y. & Jeng, D. S. (2003). Physical modeling of untrenched submarine pipeline instability. *J. Ocean Eng*., Vol. 30.
2. Vijaya Kumar, A., Neelamani, S. & Narasimha Rao, S. (2003), Wave pressures and uplift forces on and scour around submarine pipeline in clayed soil. *Ocean Eng*., Vol.30.
3. Morison, J. R., Johanson, J. W. & Schaff, S. A. (1950). The force exerted by surface wave on pile. *Trans. ASME*, Vol. 189.
4. Shankar, N. J., Subbian, K. & Cheong, H. F. (1989). Inertia dominated forces on submarine pipeline near seabed. *J. of Hydraulic Research*, Vol. 27.
5. Macpherson, H. (1978). Wave forces on pipelines buried in permeable seabed. *Journal of the Waterway, Port, Coastal and Ocean Eng., ASCE*, Vol. 104, pp. 407-419.
6. Monkmeyer, P. L., Mantovani, V. & Vincent, H. (1983). Wave-induced seepage effect on a buried pipeline. *Proc. Of the Coastal Structure, 83 Conf*., pp. 519-531.
7. Shankar, N. J. & Subbian, K. (1991). Regular and random wave pressures on and around large diameter submarine pipeline near ocean. *J. of Hydraulic Research*, Vol. 29.
8. Talebbeydokhti, N. & Pishvai, S. A. (1995), Dynamic analysis of submerged pipelines by finite element method. *Iranian Journal of Science & Technology, Transaction B: Engineering*, Vol. 19, No. 1, pp. 39-66.
9. Magda, W. (1996). Wave-induces forces on submarine buried pipeline. *Computer & Geotecnic*s, pp. 47-73.
10. McDougal, W. G., Davidson, S. H., Monkmeyer, P. L. & Sollit, (1988). Wave-induced forces on buried pipeline. *J.Water Way,Port,Coastal and Ocean Eng., ASCE 114*.
11. Lennon, G. P. (1985). Wave-induced forces on submarine pipeline, *J. Waterway, port, Coastal and Ocean Eng. ASCE*, pp. 511-524.
12. Magda, W. (1997). Wave-induced uplift force on a submarine pipeline buried in a comprehensive sea bed. *J. Ocean Eng*., Vol. 24, pp. 551-576.
13. Cheng, A. H. D. & Liu, P. L. (1986). Seepage force on a pipeline buried in a poroelastic seabed under wave loading. *Applied Ocean Research 8*, pp. 22-32.
14. Magda, W. (2000). Wave-induced cyclic pore-pressure perturbation effect in hydrodynamic uplift force acting on submarine pipeline buried in seabed sediment. *Ocean Eng.*, Vol. 39.
15. Narasimha Rao, A., Neelamani, S. & Vijaya Kumar, S. (2003). Wave pressures and uplift forces on and scour around submarine pipeline in clayed soil, *J. of ocean Eng*., Vol. 30.