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Research Article

Application of Modified Chebyshev Points to Decrease Residual Stress Noises in Hole-Drilling Method

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ABSTRACT

The uncertainty on the values of the measured residual stresses in hole-drilling integral method, the most widely used technique for measuring residual stress, will be called hereinafter as residual stress noises, which initiate high sensitivity of stress to strain measurement errors due to ill-conditioning of inverse integral equations. This study aimed to investigate the use of Chebyshev points to decrease residual stress noises in the hole drilling method. The Chebyshev points were extended to hole-drilling increments from surface to specified interior depth. The bending of an aluminum beam was used to validate the extended method, and the results were compared with the standard reduction noises technique, Tikhonov-Morozov, and optimum steps technique. The result obtained from the extended method was shown to lead not only an accurate determination of de-noised residual stress, but also a simple calculation procedure in comparison with the Tikhonov-Morozov and optimal steps method. The results indicate that the use of modified Chebyshev points decreased the mean absolute error of residual stress to 8.17 MPa from 14 MPa in Tikhonov-Morozov and 10.52 MPa in optimum steps methods.

1. Introduction

The impacts of residual stresses on the integrity, properties, and performance of materials are of significant concern for most industrial applications. The residual stresses are produced because of non-uniform plastic deformation or phase change in the fabrication process [1]. The crucial issue about them is that they play an essential role in the strength of materials and combine with in-service stresses. This combination affects the fatigue life of components, dimensional instability and causes stress corrosion cracking initiation [2]. For the components to avoid failure and to maintain dimensional stability, the amount of the residual stresses should be determined beforehand and reduced as much as possible [3]. To this end, there has been extensive research regarding the selection of a measurement technique, which offers a significant increase in measurement accuracy of residual stress and reducing stress-evaluated uncertainty.

The hole-drilling method, introduced in 1932 by Mathar [4], is the most widely appropriate method for measuring residual stress due to the fertility and rich

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potential of this technique [1-3]. In this method, residual stresses are measured near surfaces, and it has been considered the only standardized method [5]. It involves applying a unique rosette strain gauge in the desired position and drilling a hole in small shallow depth increments [6]. Hence, the stresses in the hole location can be evaluated by some inverse mathematical equations and specific calibration data. These mathematical equations are based on the linear elastic principle and typically carry out the integral method [7-9]. However, in this method, while the number of increments increase, the measurement errors will accumulate. Besides, by increasing the distance from measurement surface, the relieved strain becomes insensitive to deep interior stresses due to the Saint Venant's principle. Due to these facts, the associated matrix of inverse mathematical integral equations comes to be ill-conditioned. So, the impact of measurement errors on residual stress results significantly grows, and the profile of residual stress becomes noisy.

Numerous investigations have established that the residual stress noises in the hole drilling method depend on the maximum depth value and the total number of increments [8-10]. As early as 1994, Vangi [11] introduced the concept of limiting the number of hole depth steps to a small number, just four or five, or choosing hole depth steps that initially are small and then increasing them in size to reduce the sensitivity to strain data errors. However, limiting the amount of strain data available for calculation have been largely unsuccessful in illustrating the trend in data more clearly. Zuccarello [10] revealed that by optimizing the step distribution, one could minimize the influence of experimental errors on computed stress. Using optimal steps, less error amplifications were reported with lower sensitivity to the measurement errors than equally value steps or constant ones [10]. Although some details have been lost in the related literature [10], it appears to be clear that finding the optimum step is based on a numerical try and error procedure by utilizing the finite element or boundary element method.

By using the Tikhonov regularization method, it became possible to use data rich strain sets measured at many small hole-drilling steps and for the regularized calculations to extract the stress results without substantial sensitivity to noise [12]. However, the regularization parameter is an essential element in stabilizing reliable results since excessive regularization causes the function of stress solution to remain unclear. On the other hand, low regularization causes calculated stress to remain very close to the original ill-posed problem [12]. Thus, it is difficult to make a convincing decision to choose the appropriate regularization parameter that balances these two tendencies, minimizing extra smoothing while removing most noises. Schajer [12, 13] used the Morozov Discrepancy Principle [14, 15] to determine the value of the regularization parameter in the hole-drilling method that combines the integral method with the Tikhonov-Morozov regularization, and was recommended in ASTM E837-13a standard [5]. Jun and Korsunsky [16] proposed an inverse semi-empirical approach to residual stress analysis, called the eigenstrain reconstruction method. Faghidian [17] developed a regularized inverse eigenstrain method for reconstruction of residual stress field solution by Tikhonov-Morozov and the gradient iterative method. Wang et al. [18] used the generalized cross-validation (GCV) method to determine the regularization parameter to estimate the completed surface residual stresses by using a Fourier series bivariate polynomial as an Airy stress function. Naskar and Banerjee [19] proposed and numerically explored a linear inverse problem to reconstruct full field residual stress while the regularization term was selected based on a modified L-curve approach described in [20]. Liu et al. [21] proposed a termination criterion in Tikhonov regularization for the hole-drilling method based on restricting the strain misfit values to 1% of the corresponding measured strains to determine the appropriate value of the regularization parameter. Unfortunately, these methods do not always guarantee to find suitable regularization parameters in discrete ill-posed problems due to their limitation and weakness.

By performing the Monte Carlo uncertainty examination, Peral et al. [22] confirmed that the hole depth is the most important uncertainty source in the hole drilling method. The objective of this paper is to seek a method to reduce the effect of the measurement noise and relate it to depth increments, which improve the benefits of the Tikhonov regularization and the optimum step methods, without the necessity of complicated and time-consuming calculations. Consequently, in this paper, a novel method, which implements the procedure of using Chebyshev points, is first reported, and then compared with the Tikhonov-Morozov and optimized step techniques by de-noising the residual stress of a four-point bent beam.

2. Theory

2.1. The hole-drilling integral method

The integral method proposes the through depth profile of three in-plane stress components. Theoretically, in the integral method, a relation between residual stress and the relieving strain can be represented in the form of inverse matrix equation as [23]:

$$AX = b \tag{1}$$

Eq. (1) is a linear system of equations that widely appears in science and engineering. We can consider b as the vector of the recorded elastic strains after each increment, X as the vector of stress components through the hole depth, and A as the matrix that relates to the stress components of the relaxed strains. In hole-drilling, generally, strain is measured in 3 directions, for three-element strain gauge rosette $(\varepsilon_1, \varepsilon_2, \varepsilon_3)$ along the rosette axis $(0^{\circ}, 90^{\circ}, 135^{\circ})$. The residual stress can be interpreted in three in-plane components $(\sigma_x, \sigma_y, \tau_{xy})$. If the elements of Eq. (1) are written in terms of the transformed stress and strain variables of the Eqs. (2) and (3), then Eq. (4) is achieved as [7-9]:

$$P = \frac{\sigma_x + \sigma_y}{2}, Q = \frac{\sigma_x - \sigma_y}{2}, T = \tau_{xy} \tag{2}$$

$$p = \frac{(\varepsilon_1 + \varepsilon_3)}{2}, q = \frac{\varepsilon_1 - \varepsilon_3}{2}, t = \frac{\varepsilon_1 + \varepsilon_3 - 2\varepsilon_2}{2} \tag{3}$$

$$\left[\frac{1+\vartheta}{E}\right]\bar{A}P = p, \left[\frac{1}{E}\right]\bar{B}Q = q, [\frac{1}{E}]\bar{B}T = t \tag{4}$$

In Eqs. (2) to (4), E and ϑ denote Young's modulus and Poisson's ratio, respectively. P is hydrostatic stress, Q is 45° shear stress, T is x-y shear stress, and p , q, t are corresponding strains, respectively [8]. $\bar{A}$ and $\bar{B}$ are lower triangular calibration coefficient matrices. By calculating the transformed stress, the principal stresses and principal direction, β, have been calculated using [7-9]:

$$\sigma_{max,min} = P \pm \sqrt{Q^2 + T^2} \tag{5}$$

$$\beta = \arctan\left(\frac{T}{Q}\right) \tag{6}$$

2.2. Extended Chebyshev points method

Pafnuty Chebyshev originally introduced the Chebyshev polynomials of the first kind in 1853 [26] as:

$$T_n(x) = \cos[n \arccos x] \qquad n = 0,1,2,\ldots, x \in [-1,1] \tag{7}$$

In 1859 [27], Pafnuty Chebyshev conducted the best approximation procedure by using the zeros of these polynomials that are called Chebyshev points of the first kind, Chebyshev nodes, or, more formally, Chebyshev–Gauss points [28]. These zeroes can be obtained by Eq. (8) [29]:

$$x_{k+1} = \cos\left(\frac{2k+1}{2n}\pi\right) \qquad k = 0,1,\ldots,n-1 \tag{8}$$

These points' projections onto the horizontal or vertical axis are symmetric about the real axes. These points are beneficial in many areas of numerical analysis, such as function approximation [29]. Suppose $f_{(x)}$ is a continuous function on [-1, 1] as:

$$f_{(x)} = P_x + \frac{f^{n+1}(\xi(x))}{(n+1)!}(x-x_0)(x-x_1)\ldots(x-x_n) \quad \text{for some } \xi(x) \in [-1,1] \tag{9}$$

f is the function that we are approximating in the interval $[-1,1]$ with the use of interpolation points, $\vec{x} = (x_0, x_1, \ldots., x_n)$, and interpolant P_x. It is proved that by selecting Chebyshev points as interpolation points, the maximum value of product term or $|(x-x_0)(x-x_1)\dots(x-x_n)|$ will effectively minimize allowing the establishing of the best approximation of f [28, 29]. In the hole-drilling method, there is no information about residual stress function or *P*, and the integral methods must be defined for the approximation of functions on specified hole depth increments [7-9]. Surprisingly however, if we have a free choice of hole depth increments, it is not necessarily a good idea to select them in equally spaced form.

In the present paper, a method to construct residual stress approximations in the integral method which were used to reduce interpolation error and then keep adequate resolution of final solution was conducted. A simple idea for improving the accuracy of approximate solutions of residual stress function is to use the Chebyshev points as interpolation points or hole drilling increments [28, 29]. Firstly, the Chebyshev points x_k in the interval [-1, 1] were changed to $\bar{x}_k$ in the interval [0, h], by Eq. (10) as:

$$\bar{x}_k = \frac{1}{2}[(h)\times x_k + h] \quad (10)$$

After that, the intervals were split into *n* subintervals of size $\Delta(h)_k$ as hole-drilling increments:

$$\Delta(h)_k = \bar{x}_{k+1} - \bar{x}_k \qquad k = 1, \dots, n \quad (11)$$

where each increment equals to the differences between the two consecutive points at an interval from the surface up to the desired interior depth. In this case, the desired depth was selected to be 2.6 mm according to the experimental procedure reported in reference [10]. In addition, the number of increments selected equals to 10. It should, however, be noted that in 10 Chebyshev increments up to 2.6 mm, the first and last increments have a small value near zero. Due to this, it is impractical to have appropriate and corrected measured strains at these intervals with such small amounts, which would probably result in a lot of noise. In order to overcome this limitation, we extended the value of first and last increments by increasing the number of Chebyshev points and using ten midpoints as the hole-drilling steps to promote computational stability. Fig. 1 represents physical interpretation of this procedure. Because there was no prediction about how many Chebyshev points are more satisfying to reduce residual stress noises, four groups of the number of Chebyshev points were determined, consisting of 18, 20, 22, and 24 points, and 10 of the midpoints were selected as depth increments. Selecting these values was based on having reasonable lengths of near-surface and interior increments. Table 1 shows various step distributions in 10 increments obtained for Chebyshev points with different numbers, optimized steps as well as constant equal steps made with the Tikhonov-Morozov regularization procedure that is used in this study. After determining the value of each step in different methods, the corresponding strain data has been derived from Fig. 6 in reference [10] by interpolating curves.

3. Finite Element Simulation

The obtained hole-drilling calibration coefficients for 0.2 mm increments length, 3.1 mm hole diameter, and 5.12 mm mean gauge radius that was used in this study is not directly provided in ASTM-E837-13A [5]. Therefore, in this paper, the calibration matrixes $\bar{A}$ and $\bar{B}$ were evaluated from three-dimensional finite element modeling (FEM) of the hole-drilling process. The same procedure for evaluating calibration coefficients

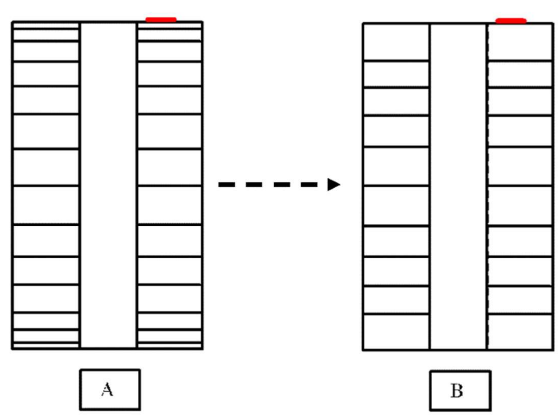

Fig. 1. Physical interpretation of Chebyshev points into hole-drilling steps, A: implementing 14 Chebyshev points as hole drilling step, and B: modifying Chebyshev points to 10 with removing two early and final steps.

Table 1. Dimension of each step for different de-noising methods

Step type	Δh_1	Δh_2	Δh_3	Δh_4	Δh_5	Δh_6	Δh_7	Δh_8	Δh_9	Δh_{10}
18 Chebyshev Points	0.38	0.17	0.19	0.21	0.22	0.22	0.22	0.21	0.19	0.54
20 Chebyshev Points	0.45	0.16	0.18	0.19	0.20	0.20	0.20	0.19	0.18	0.61
22 Chebyshev Points	0.52	0.15	0.16	0.17	0.18	0.18	0.18	0.17	0.16	0.67
24 Chebyshev Points	0.57	0.14	0.15	0.16	0.16	0.17	0.16	0.16	0.15	0.72
Tikhonov-Morozov Regularization	0.26	0.26	0.26	0.26	0.26	0.26	0.26	0.26	0.26	0.26
Optimum step [10]	0.16	0.11	0.12	0.13	0.14	0.15	0.18	0.25	0.37	0.91

has already been used by many others [7-9, 10].

It is worth mentioning that an initial simulation of other standardized calibration coefficients of ASTM-E837-13A [5] has been performed to validate the procedure of FEM analysis to confirm whether the simulation was functionally correct. To this aim, these calibration coefficients obtained from numerical analysis were compared with those conducted by Schajer's polynomial [36]. This continuous polynomial provides calibration coefficients of ASTM-E837-13A [5] with average assurance within 1% [36].

The simulation was conducted utilizing a commercial FEM software [30]. To enable less computational economy, only one quarter of the model was simulated due to symmetry in geometry, boundary condition, and loading. The model's geometry was based on the specimen in the experimental procedure reported in reference [10]. In the experimental procedure, a bending moment of 132.5 N.m at four-points bending beam using a hydraulic testing machine was applied on an aluminum beam with a length of 400 mm, a thickness of 5 mm and width of 100 mm with Young's modulus of 100900 MPa, elastic-plastic modulus equals to 700 MPa, Poisson's ratio of 0.31 and yield strength of 211 MPa [10].

The spatial resolution of the FEM mesh was such that the element sizes are 0.05 mm for inner hole surface, 0.2 mm for gauge area and 0.1 mm between gauge and the hole alongside 16 elements in the hoop direction. A schematic of the developed mesh is shown in Fig. 2. The models with finer mesh were examined, but the results were practically unaffected at any point of the mode. Additionally, the relevant displacement and rotation constraint along the two planes of symmetry and complete fixity at the end of the beam (i.e., all displacement and rotations are set to zero there) were imposed in the case of boundary condition followed by an experimental procedure. For the three intermediate restraint setups, displacements of bottom nodes in z-direction were suppressed as well (see Fig. 3).

The simulation of drilling procedures was performed in the middle region of the specimen between the two loading points as reported in the experimental procedure of reference [10], using depth increments of 0.1 mm until a depth of 2.6 mm was achieved. The distance between the two inner and outer loading noses of the fixture was 250 mm, and 400 mm, respectively in line with reference [10]. In simulation, a rosette with a dimension of 5.12 mm mean gauge radius and 3.1 mm hole diameter was modeled following the experimental procedure of reference [10].

To determine the a_{ij} components of $\bar{A}$, a unit pressure ($P = \sigma_x = \sigma_y = \sigma$) was applied to the inside of the surface in layer j during hole-drilling in step i . The pressure in other layers is set to be zero where $j \leq i$. Then, the radial displacement of the initial and final points of the central axes of the strain gauge on the surface was measured, and $\bar{A}$ was obtained by Eq. (12) as [31]:

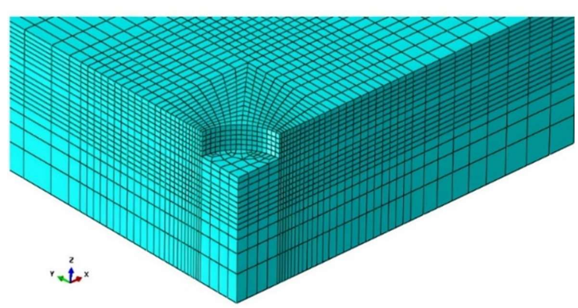

Fig. 2. Schematics of spatial resolution of mesh used in the model of hole-drilling method.

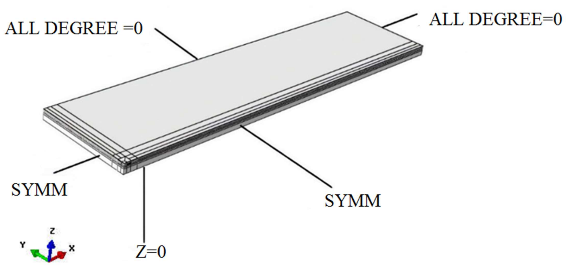

Fig. 3. The boundary condition applied on the simulation model.

$$\bar{A} = \frac{E}{1+\vartheta} \times \frac{U_{2(x)} - U_{1(x)} + U_{2(y)} - U_{1(y)}}{GL} \quad (12)$$

where GL refers to the gauge length, $U_{2(x)}$ refers to the displacement of the final point of strain gauge aligned in the x-direction, and $U_{1(x)}$ refers to the displacement of the initial point as well as for $U_{2(y)}$ and $U_{1(y)}$ in y-direction. Similarly, the b_{ij} of $\bar{B}$ was determined by applying the shear stress field ($\sigma = P \times \cos(2\theta), \tau = -P \times \sin(2\theta)$) inside the surface of the hole in each direction as Eq. (13) [31]:

$$\bar{B} = \frac{E}{1} \times \frac{U_{2(x)} - U_{1(x)} - (U_{2(y)} - U_{1(y)})}{GL} \quad (13)$$

In the interpretation of $\bar{A}$ or $\bar{B}$ matrices, it is necessary to drive their components using the repetitive solution. This procedure was conducted in ABAQUS scripting interface by removing elements with the model change tool, applying specified load inside of the hole, and calculating strain in the surface. An algorithm was written, and a series of routines were coded in Python as shown in Fig. 4. So, $\bar{A}$ and $\bar{B}$ coefficients were obtained in each step. It must be noted that the gauge wide effect was neglected in simulation because it was recognized that it does not have a significant effect on the results. However, if there is a tendency to embed gauge wide effect, some of the authors consider it in their simulations [32-35].

Initial validation of FEM analysis was performed to confirm whether the simulation was functionally corrected. The number and the length of each step are assumed 10 and 0.2 mm, respectively. the dimension of hole was based on Table 5 in ASTM-E837-13A [5]. A

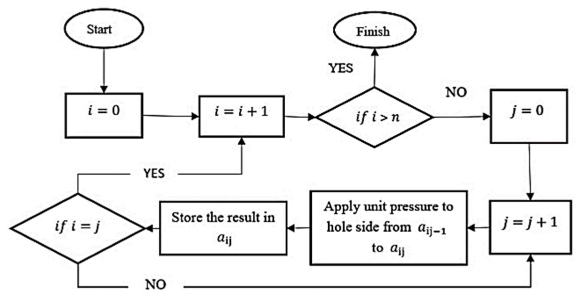

Fig. 4. Calculation procedure of components of the a_{ij} matrix.

comparison between the calculated coefficients of simulation results and the values obtained by the Schajer's polynomial [36] are shown in Figs. 5 and 6 for $\bar{A}$ and $\bar{B}$ calibration coefficients. This comparison was performed to ensure the obtained calibration coefficients as accurate and reliable. As can be seen, the differences are in the order of a few percentage points up to 6.57% for $\bar{A}$ and 7.59% for $\bar{B}$ which are consistent with simulation results obtained in previous studies [35, 37] and 2-D and 3-D simulations [38,39]. In these studies, it was shown that the differences between the calibration coefficients obtained from simulation with standard reference [5] variate between 3% to 5%, depending on the geometry of strain gauge, hole radius and the type of simulation 2-D or 3-D or boundary conditions. Therefore, in the presence of corresponding measured strains and calibration coefficients, the transformed stresses were calculated using Eqs. (2) to (4) in different methods. Consequently, principal residual stresses were calculated easily utilizing Eq. (5).

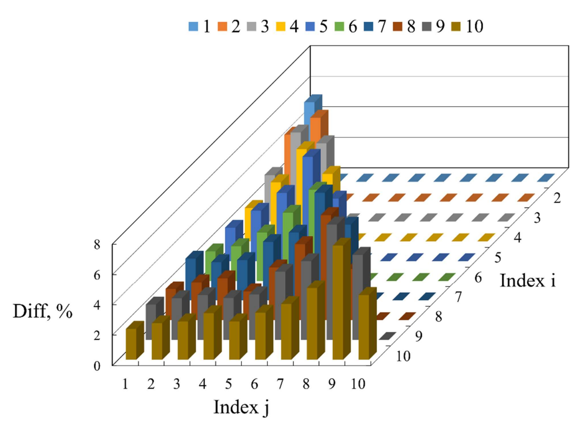

Fig. 5. Percentage of relative differences between coefficients of $\bar{A}$ obtained from simulation and Schajer's polynomial.

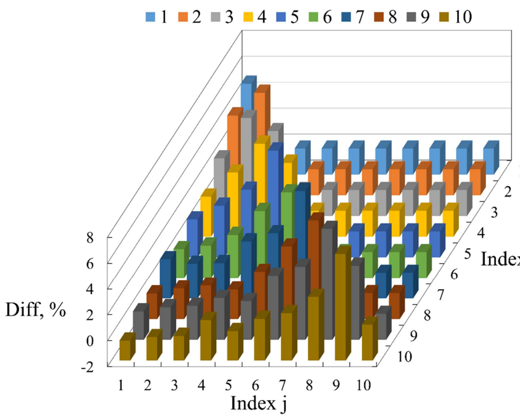

Fig. 6. Percentage of relative differences between coefficients of $\bar{B}$ obtained from simulation and Schajer's polynomial.

4. Results and Discussion

Fig. 7 to 10 show the comparison between actual maximum principal residual stresses with those calculated by the optimum step, the proposed Chebyshev points with different numbers and the Tikhonov-Morozov regularization methods. The residual stresses close to the specimen surface are compressive. The magnitude of the compressive residual stresses reduced by increasing depth so that at a depth of 0.8 mm the residual stresses changed from compressive to tensile and then decreased to zero at a depth of 2.6 mm.

Table 2 shows the absolute error in each increment as well as mean absolute error (MAE) in each method, separately. From Fig. 7, it can be seen that in early increments, the 18 Chebyshev points proposed approach has relatively lower noises than the Tikhonov-Morozov and optimum step method. However, in deeper thickness, the results of 18 Chebyshev points proposed method were not consistent with actual stress, while the optimum step and Tikhonov-Morozov regularization methods managed to diminish the errors and provided stabilized results better than the 18 Chebyshev points method. This is consistent with the results obtained in Table 2, where the MAE of the stress estimation by the modified 18 Chebyshev points, Tikhonov-Morozov, and optimum step methods, in comparison with the actual stress are equal to 17.67 MPa, 14 MPa and 10.52 MPa, respectively. From Table 1 and 2 it can be concluded that the final increments of 18 Chebyshev points are not big

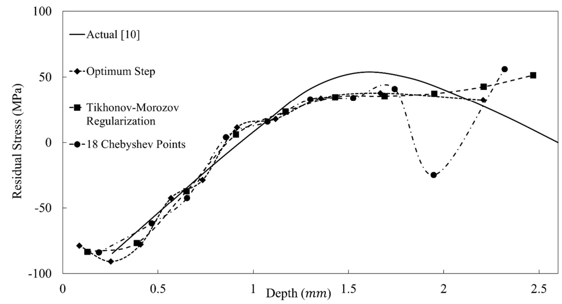

Fig. 7. Comparison of maximum principal residual stresses between 18 Chebyshev points, Tikhonov-Morozov regularization and optimum steps method with actual values versus hole depth.

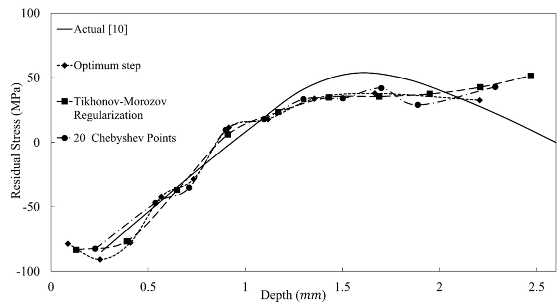

Fig. 8. Comparison of maximum principal residual stresses between 20 Chebyshev points, Tikhonov-Morozov regularization and optimum steps method with actual values versus hole depth.

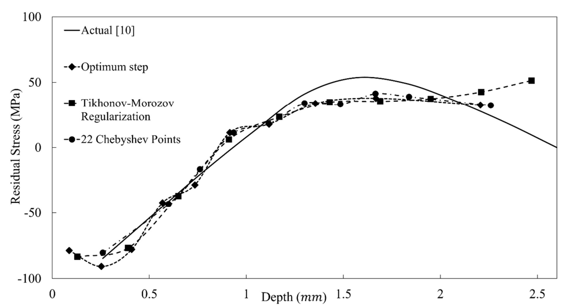

Fig. 9. Comparison of maximum principal residual stresses between 22 Chebyshev points, Tikhonov-Morozov regularization and optimum steps method with actual values versus hole depth.

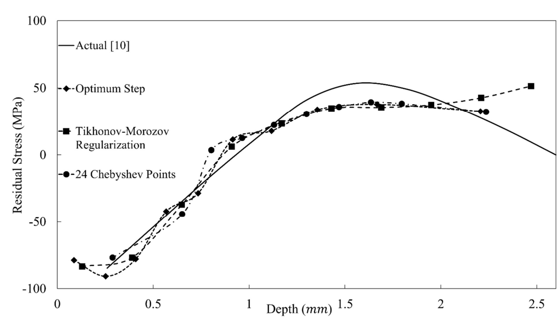

Fig. 10. Comparison of maximum principal residual stresses between 24 Chebyshev points, Tikhonov-Morozov regularization and optimum steps method with actual values versus hole depth.

Table 2. Absolute error corresponding to different methods (MPa)

Step Number	1	2	3	4	5	
Tikhonov-Morozov Regularization	16.78	9.48	1.63	9.37	4.69	
18 Chebyshev Points	9.27	4.06	7.07	13.26	0.41	
20 Chebyshev Points	6.82	2.23	8.27	14.13	1.14	
22 Chebyshev Points	4.17	1.49	3.82	11.77	3.16	
24 Chebyshev Points	3.81	8.96	18.94	9.20	1.10	
Optimum steps	26.83	5.07	13.12	3.46	4.40	
Step Number	**6**	**7**	**8**	**9**	**10**	**MAE**
Tikhonov-Morozov Regularization	17.10	17.88	5.64	15.62	41.77	14.00
18 Chebyshev Points	8.09	19.32	11.39	67.79	36.03	17.67
20 Chebyshev Points	7.73	19.18	11.28	17.21	20.80	10.88
22 Chebyshev Points	7.13	19.94	12.21	9.44	8.527	8.17
24 Chebyshev Points	10.56	17.15	14.17	12.09	6.77	10.28
Optimum step	14.19	3.81	13.39	15.73	5.21	10.52

enough to reduce the uncertainty of the calculated residual stress to a minimum value. Therefore, the 18 Chebyshev points method slightly deteriorates the reference stresses for the interior depths, although the difference between the 18 Chebyshev points and reference values was within the small error magnitude at the early and middle steps.

By increasing Chebyshev points to 20, the obtained stress data in Fig. 8 indicates a more proper fitting with the actual stress. It can be observed from Fig. 8 that there was only minimal noise enhancement in deeper thickness in comparison with 18 points (Fig. 7), and the MAE reduces to 10.88 MPa from 17.67 MPa. Although it can be seen from Table 2 that the value of MAE in 20 Chebyshev points method is approximately similar to the other methods, inspection of Fig. 8 indicates that there remained a slight amplification of noises in deeper thickness. It can be seen from Table 1 and 2 that the MAE in steps 8 to 10 of 20 Chebyshev points method has been amplified due to the small size of these steps. To eliminate drawback of this limitation and find the optimum number of Chebyshev points in order to minimize the noises, the Chebyshev points have been increased to 22. The results were depicted in Fig. 9. As can be seen, it is evident that the 22 Chebyshev points result obtained are in excellent agreement with actual results, and MAE decreased to 8.17 MPa in comparison with 10.52 MPa for the optimized step and 14 MPa for regularized steps. In particular, using 22 points of Chebyshev no longer has the high error sensitivity in interior and near-surface depths, where the sensitivity to measurement errors is high. It is seen that the 22 Chebyshev points method tends to give prediction that is in exceptionally good agreement with the trend of the actual four-points bending stress, especially up to 0.75 mm, fitted well with the actual values contrary to the Tikhonov-Morozov regularization and the optimum step methods. However, only in the middle range, 22 Chebyshev points have negligible noise due to the small value of each increment in comparison with other methods as can be seen from Table 1. These noises are tiny, but for assurance of probable existence of the better condition of Chebyshev points, we increased the number to 24. Fig. 10 illustrates the findings of 24 Chebyshev points method and compares it with other methods. Comparing Figs. 9 and 10 shows that the error sensitivity developed in near-surface steps after increasing the number of Chebyshev points to 24 and MAE increased to 10.28 MPa. By increasing the number of Chebyshev points from 22 to 24, the results will slightly deteriorate in the second and third steps. However, the magnitude of the residual stress noises in the first and last steps were lower than other methods.

5. Conclusion

Results from three methods of de-nosing residual stress were presented and the results illustrated that in virtually all cases of different numbers of the Chebyshev points technique can accurately reduce the residual stress noises with a simple calculation method. In addition, it was found that by optimizing the number of Chebyshev points, the minimum value of residual stress noises can be reachable in comparison with other methods. This finding confirmed that the modified Chebyshev points method used as hole-drilling increments tends to produce more simple calculation and more precisely obtained residual stress in line with those obtained from four-points bending in comparison with cases where the Tikhonov-Morozov regularization and optimum step methods were used.

In addition, the improvements noted in this study were unrelated to the geometry of hole or gauge, type of calibration coefficient, the shape of residual stress function, and regularization parameter, unlike the Tikhonov-Morozov and optimum step method. This study, therefore, indicates that this method not only decreased the amount of the residual stress error amplification, but also succeeded in diminishing the value of uncertainty. Our result provides compelling evidence for a simple approach to calculate residual stress with such noises and suggests that this approach appears to be effective in another semi destructive or destructive residual stress calculation method that may contain uncertainties, such as ring-core, slitting and deep hole-drilling. However, some limitations are worth-nothing. Although the modified Chebyshev method can reduce noises of results, there is a specific optimum number of points that significantly stabilize the results. Future study should, therefore, include follow-up work designed to evaluate whether the highly accurate number of Chebyshev points are needed and also whether they continue to be used to improve another semi destructive residual stress measurement errors.

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Conflict of Interests

On behalf of all authors, the corresponding author states that there is no conflict of interest.

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