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VIBRATION ANALYSIS OF ASYMMETRIC SHEAR WALL- FRAME STRUCTURES USING THE TRANSFER MATRIX METHOD*

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Abstract– Vibration analysis plays an important role in the structural design of tall buildings. In this study, a vibration analysis of asymmetric shear wall-frame structures is carried out by a transfer matrix method. The method assumes that walls and frames run in two orthogonal directions. The structural properties of the building may change in the proposed method. In this method the structure is idealized as an equivalent shear-flexure-torsion coupled beam in this method. The governing differential equations of equivalent shear- flexure-torsion coupled beam are formulated using the continuum approach and are posed in the form of a simple storey transfer matrix. By using the storey transfer matrices and point transfer matrices which take into account the inertial forces, the system transfer matrix is obtained. Natural frequencies can be calculated by applying the boundary conditions. At the end, a numerical example is presented to demonstrate the accuracy of the proposed method. The results of this example display the agreement between the proposed method and the other valid method given in the literature.

Keywords– Vibration, asymmetric, wall-frame, transfer matrix

1. INTRODUCTION

During the last three decades, many studies on the analysis of shear wall and frame structures have been carried out [1-46].

Ng and Kuang [15] considered the problem of triply coupled vibration of asymmetric structures. The governing equation of the natural vibration and its corresponding eigenvalue problem, which is a set of equations for flexural- shear vibrations in laterally orthogonal directions coupled with warping St. Venant torsional vibration are developed. By applying the Galerkin method, a generalized approximate approach is developed for the analysis of coupled vibration and for determining the natural frequencies and associated mode shapes of the structure triply coupled vibration.

Rafezy and Howson [42] proposed a global approach to the calculation of natural frequencies of doubly asymmetric, three dimensional, multi bay, and multi storey wall-frame structures. It was assumed that the primary frames and walls of the original structure ran in two original directions and that their properties may have varied in a step-wise fashion at one or more storey levels. The structure was therefore divided naturally into uniform segments according to changes in section properties.

A typical segment was then replaced by an equivalent shear-flexure-torsion coupled beam whose governing differential equations were formulated by using the continuum approach and being posed in the form of a dynamic member stiffness matrix. A method for a theoretical solution was proposed and a general solution to the eigenvalue equation of the problem was presented for determining the coupled natural frequencies and associated mode shapes based on the theory of differential equations.

Bozdogan and Ozturk [46] proposed the Transfer Matrix method for vibration analysis of asymmetric wall buildings.

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A method for the vibration analysis of non uniform asymmetric wall-frame structures is suggested in this study. The following assumptions are made in this study; the behavior of the material is linear elastic, small displacement theory is valid, P-delta effects are negligible, the flexural rigidity center at each floor is assumed to lie on a vertical line through the height of structures, the shear deformations of walls are negligible, the storey mass acts on the storey (floor) level, the frames are orthogonal, the dynamic coupling effect of the structure caused by the eccentricity between the center of shear rigidity and the center of flexural rigidity is ignored in the analysis and the floor system is rigid in its plane.

2. ANALYSIS

a) Physical model

Figure 1 shows a typical floor plan of asymmetric, three dimensional wall-frame structures [15]. If shear deformations in the wall and the axial deformations in columns and beams are ignored, wall-frame structures demonstrate the shear- flexure-torsion coupled beam behavior. The differential equation of this equivalent shear- flexure-torsion coupled beam can be initially written.

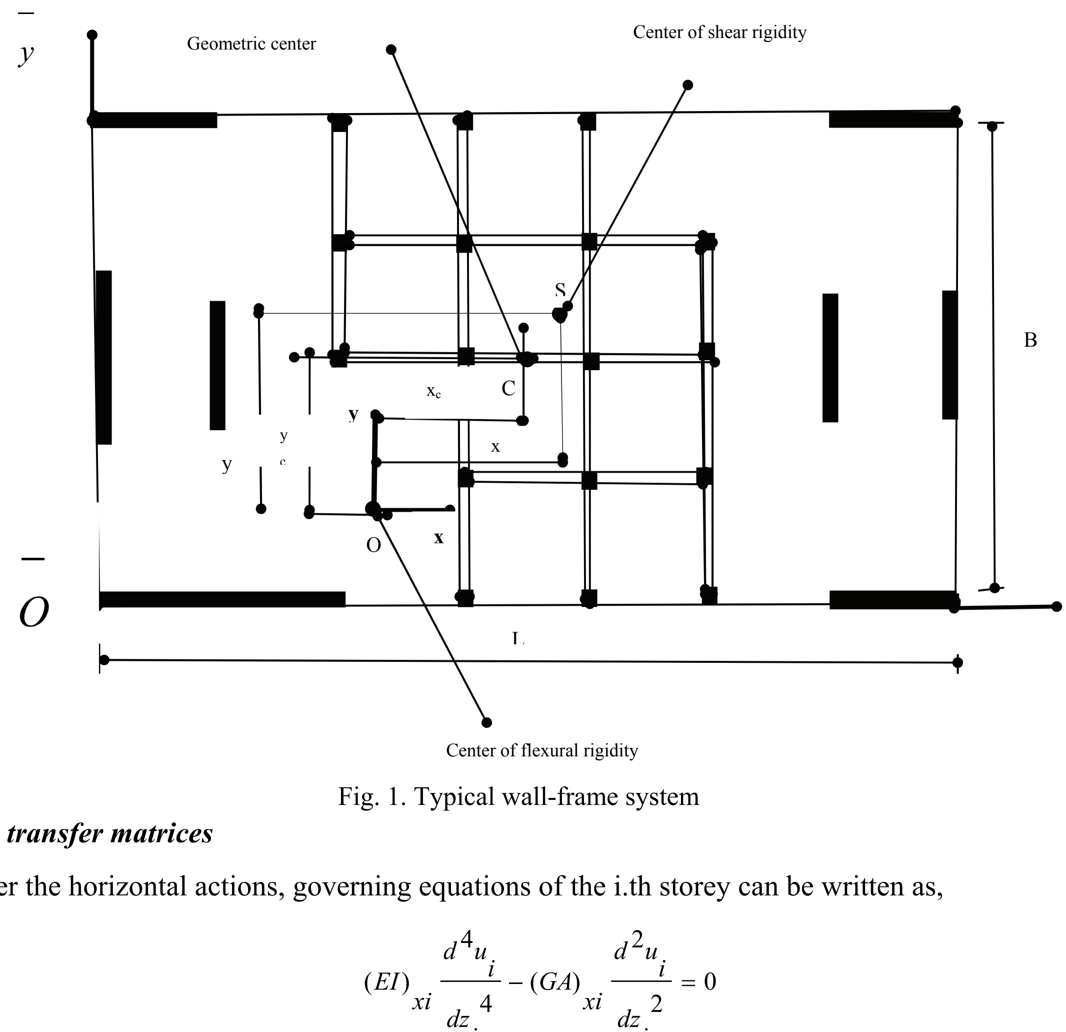

Fig. 1. Typical wall-frame system

b) Storey transfer matrices

Under the horizontal actions, governing equations of the i.th storey can be written as,

$$(EI)_{xi}\frac{d^4u_i}{dz_i^4}-(GA)_{xi}\frac{d^2u_i}{dz_i^2}=0 \tag{1}$$

$$(EI)_{yi}\frac{d^4v_i}{dz_i^4}-(GA)_{xi}\frac{d^2v_i}{dz_i^2}=0 \tag{2}$$

$$(EI)_{wi}\frac{d^4\theta_i}{dz_i^4}-(GJ)_i\frac{d^2\theta_i}{dz_i^2}=0 \tag{3}$$

where u_i and v_i are the lateral deflections of the flexural center, respectively, θ_i is the torsional rotation of the floor plan about flexural rigidity at the given height, and z_i is the vertical axis of each storey.

$(EI)_{xi}$ and $(EI)_{yi}$ are the equivalent flexural rigidity of the storey for wall structures in x and y directions and can be calculated as follows [15, 42]

$$(EI_{xi})=\sum_j EI_{xi,j} \qquad (EI_{yi})=\sum_j EI_{yi,j} \tag{4}$$

$(EI)_{wi}$ are the warping stiffness of i.th storey and can be calculated as follows [14];

$$(EI)_{wi}=\sum_j[(\bar{y}_j-\bar{y}_c)^2(EI)_{xi,j}+(\bar{x}_j-\bar{x}_c)^2(EI)_{yi,j}] \tag{5}$$

where $\bar{y}_j$ and $\bar{x}_j$ are the coordinates at the location of the center of flexural rigidity of the j-th bent at i-th storey in coordinate system ($\bar{y}$, $\bar{x}$).

$\bar{y}_c$ and $\bar{x}_c$ are the coordinate of flexural rigidity center and can be calculated as follows [15]

$$\bar{y}_c=\frac{\sum_j \bar{y}_j(EI)_{xj}}{\sum_j (EI)_{xj}} \tag{6}$$

$$\bar{x}_c=\frac{\sum_j \bar{x}_j(EI)_{yj}}{\sum_j (EI)_{yj}} \tag{7}$$

$(GA)_{xi}$ and $(GA)_{yi}$ are the equivalent shear rigidity of the storey for framework in x and y directions. For frame elements which consist of n columns and n-1 beams, $(GA)_i$ can be calculated as follows [47]

$$(GA)_i=\frac{12E}{h_i[1/\sum_1^n I_c/h_i+1/\sum_1^{n-1} I_g/l)]} \tag{8}$$

where $\sum I_c/h_i$ represents the sum of moments of inertia of the columns per unit height in i.th storey of frame j, and $\sum I_g/l$ represents the sum of moments of inertia of each beam per unit span across one floor of frame j .

$(GJ)_i$ are the St. Venant torsion stiffness of i.th storey and can be calculated as follows [15, 42]

$$(GJ)_i=\sum_j[(\bar{y}_j-\bar{y}_s)^2(GA)_{xj}+(\bar{x}_j-\bar{x}_s)^2(GA)_{yj}] \tag{9}$$

where $\bar{y}_j$ and $\bar{x}_j$ are the coordinates at the location of the center of flexural rigidity of the j-th bent at i-th storey in coordinate system (y , x).

When Eqs. (1-3) are solved with respect to the z_i, $u_i(z_i)$ and $v_i(z_i)$ and $\theta_i(z_i)$ can be obtained as follows

$$u_i(z_i) = c_1 + c_2 z_i + c_3 \cosh(\lambda_{xi} z_i) + c_4 \sinh(\lambda_{xi} z_i) \quad (10)$$

$$v_i(z_i) = c_5 + c_6 z_i + c_7 \cosh(\lambda_{yi} z_i) + c_8 \sinh(\lambda_{yi} z_i) \quad (11)$$

$$\theta_i(z_i) = c_9 + c_{10} z_i + c_{11} \cosh(\lambda_{\theta i} z_i) + c_{12} \sinh(\lambda_{\theta i} z_i) \quad (12)$$

where

$$\lambda_{xi} = \sqrt{\frac{(GA)_{xi}}{(EI)_{xi}}}, \qquad \lambda_{yi} = \sqrt{\frac{(GA)_{yi}}{(EI)_{yi}}} \text{ and } \lambda_{\theta i} = \sqrt{\frac{(GJ)_i}{(EI)_{wi}}} \quad (13)$$

$c_1, c_2, c_3, c_4, c_5, c_6, c_7, c_8, c_9, c_{10}, c_{11}, c_{12}$ are integral constants.

By using equations (10), (11) and (12), the rotation angles in x and y directions (u_i',v_i') , the rate of twist (θ_i'), bending Moments in x and y directions (M_{xi}, M_{yi}) and bi-moment (M_{wi}), shear forces in x and y directions (V_{xi}, V_{yi}) and torque (T_i) for i.th storey can be obtained as follows;

$$u_i'(z_i) = c_2 + c_3 \lambda_{xi} \sinh(\lambda_{xi} z_i) + c_4 \lambda_{xi} \cosh(\lambda_{xi} z_i) \quad (14)$$

$$v_i'(z_i) = c_6 + c_7 \lambda_{yi} \sinh(\lambda_{yi} z_i) + c_8 \lambda_{yi} \cosh(\lambda_{yi} z_i) \quad (15)$$

$$\theta_i'(z_i) = c_{10} + c_{11} \lambda_{\theta i} \sinh(\lambda_{\theta i} z_i) + c_{12} \lambda_{\theta i} \cosh(\lambda_{\theta i} z_i) \quad (16)$$

$$M_{xi}(z_i) = (EI)_{xi} \frac{d^2 u_i}{dz_i^2} = (EI)_{xi}[c_3 \lambda_{xi}^2 \cosh(\lambda_{xi} z_i) + c_4 \lambda_{xi}^2 \sinh(\lambda_{xi} z_i)] \quad (17)$$

$$M_{yi}(z_i) = (EI)_{yi} \frac{d^2 v_i}{dz_i^2} = (EI)_{yi}[c_7 \lambda_{yi}^2 \cosh(\lambda_{yi} z_i) + c_8 \lambda_{yi}^2 \sinh(\lambda_{yi} z_i)] \quad (18)$$

$$M_{wi}(z_i) = (EI)_{wi} \frac{d^2 \theta_i}{dz_i^2} = (EI)_{wi}[c_{11} \lambda_{\theta i}^2 \cosh(\lambda_{\theta i} z_i) + c_{12} \lambda_{\theta i}^2 \sinh(\lambda_{\theta i} z_i)] \quad (19)$$

$$V_{xi}(z_i) = (EI)_{xi} \frac{d^3 u_i}{dz_i^3} - (GA)_{xi} \frac{du_i}{dz_i} = -c_2 \quad (20)$$

$$V_{yi}(z_i) = (EI)_{yi} \frac{d^3 v_i}{dz_i^3} - (GA)_{yi} \frac{dv_i}{dz_i} = -c_6 \quad (21)$$

$$T_i(z_i) = (EI)_{wi} \frac{d^3 \theta_i}{dz_i^3} - (GJ)_{xi} \frac{d\theta_i}{dz_i} = -c_{10} \quad (22)$$

Equation (23) show the matrix form of the Eqs. (10)-(12) and (14)-(22):

$$\begin{bmatrix} u_i(z_i) \\ v_i(z_i) \\ \theta_i(z_i) \\ u_i'(z_i) \\ v_i'(z_i) \\ \theta_i'(z_i) \\ M_{xi}(z_i) \\ M_{yi}(z_i) \\ M_{wi}(z_i) \\ V_{xi}(z_i) \\ V_{yi}(z_i) \\ T_i(z_i) \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \\ c_7 \\ c_8 \\ c_9 \\ c_{10} \\ c_{11} \\ c_{12} \end{bmatrix} = A_i(z_i) \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \\ c_7 \\ c_8 \\ c_9 \\ c_{10} \\ c_{11} \\ c_{12} \end{bmatrix} \tag{23}$$

Where A_{11}, A_{12},, A_{21}, A_{22} are the sub matrices of A and are defined as

$$A_{11}(z_i) = \begin{bmatrix} 1 & z_i & \cosh(\lambda_{xi} z_i) & \sinh(\lambda_{xi} z_i) & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & z_i \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & \lambda_{xi} \sinh(\lambda_{xi} z_i) & \lambda_{xi} \cosh(\lambda_{xi} z_i) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{24}$$

$$A_{12}(z_i) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ \cosh(\lambda_{yi} z_i) & \sinh(\lambda_{yi} z_i) & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & z_i & \cosh(\lambda_{\theta i} z_i) & \sinh(\lambda_{\theta i} z_i) \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \lambda_{yi} \cosh(\lambda_{yi} z_i) & \lambda_{yi} \sinh(\lambda_{yi} z_i) & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & \lambda_{\theta i} \cosh(\lambda_{\theta i} z_i) & \lambda_{\theta i} \sinh(\lambda_{\theta i} z_i) \end{bmatrix} \tag{25}$$

$$A_{21}(z_i) = \begin{bmatrix} 0 & 0 & (EI)_{xi} \lambda_{xi}^2 \cosh(\lambda_{xi} z_i) & (EI)_{xi} \lambda_{xi}^2 \sinh(\lambda_{xi} z_i) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{26}$$

$$A_{22}(z_i) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ (EI)_{yi} \lambda_{yi}^2 \cosh(\lambda_{yi} z_i) & (EI)_{yi} \lambda_{yi}^2 \sinh(\lambda_{yi} z_i) & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & (EI)_{\theta i} \lambda_{\theta i}^2 \cosh(\lambda_{\theta i} z_i) & (EI)_{\theta i} \lambda_{\theta i}^2 \sinh(\lambda_{\theta i} z_i) \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \end{bmatrix} \tag{27}$$

At the starting point of the storey for z_i=0, Eq. (23) can be written as;

$$\begin{bmatrix} u_i(0) \\ v_i(0) \\ \theta_i(0) \\ u_i'(0) \\ v_i'(0) \\ \theta_i'(0) \\ M_{xi}(0) \\ M_{yi}(0) \\ M_{wi}(0) \\ V_{xi}(0) \\ V_{yi}(0) \\ T_i(0) \end{bmatrix} = A_i(0) \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \\ c_7 \\ c_8 \\ c_9 \\ c_{10} \\ c_{11} \\ c_{12} \end{bmatrix} \tag{28}$$

When vector c is taken out from formula (28) and substituted in Eq. (23), then Eq. (29) would be obtained.

$$\begin{bmatrix} u_i(z_i) \\ v_i(z_i) \\ \theta_i(z_i) \\ u_i'(z_i) \\ v_i'(z_i) \\ \theta_i'(z_i) \\ M_{xi}(z_i) \\ M_{yi}(z_i) \\ M_{wi}(z_i) \\ V_{xi}(z_i) \\ V_{yi}(z_i) \\ T_i(z_i) \end{bmatrix} = A_i(z_i)A_i(0)^{-1} \begin{bmatrix} u_i(0) \\ v_i(0) \\ \theta_i(0) \\ u_i'(0) \\ v_i'(0) \\ \theta_i'(0) \\ M_{xi}(0) \\ M_{yi}(0) \\ M_{wi}(0) \\ V_{xi}(0) \\ V_{yi}(0) \\ T_i(0) \end{bmatrix} = T_i(z_i) \begin{bmatrix} u_i(0) \\ v_i(0) \\ \theta_i(0) \\ u_i'(0) \\ v_i'(0) \\ \theta_i'(0) \\ M_{xi}(0) \\ M_{yi}(0) \\ M_{wi}(0) \\ V_{xi}(0) \\ V_{yi}(0) \\ T_i(0) \end{bmatrix} \tag{29}$$

T_i represents the storey transfer matrix for $z=h_i$ in Eq. (29).

The storey transfer matrices obtained from Eq. (29) can be used for the dynamic analysis of the asymmetric- plane frame. Therefore, when considering the inertial forces in the storey levels, the relationship between the ith and the (i+1)th stories can be shown by the following matrix equation;

$$\begin{bmatrix} u_i(h_i) \\ v_i(h_i) \\ \theta_i(h_i) \\ u_i'(h_i) \\ v_i'(h_i) \\ \theta_i'(h_i) \\ M_{xi}(h_i) \\ M_{yi}(h_i) \\ M_{wi}(h_i) \\ V_{xi}(h_i) \\ V_{yi}(h_i) \\ T_i(h_i) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ \omega^2 m_i & 0 & -\omega^2 m_i y_c & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & \omega^2 m_i & \omega^2 m_i x_c & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ -\omega^2 m_i y_c & \omega^2 m_i x_c & \omega^2 m_i r_m^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} T_i \begin{bmatrix} u_i(0) \\ v_i(0) \\ \theta_i(0) \\ u_i'(0) \\ v_i'(0) \\ \theta_i'(0) \\ M_{xi}(0) \\ M_{yi}(0) \\ M_{wi}(0) \\ V_{xi}(0) \\ V_{yi}(0) \\ T_i(0) \end{bmatrix} = T_{di}^* \begin{bmatrix} u_i(0) \\ v_i(0) \\ \theta_i(0) \\ u_i'(0) \\ v_i'(0) \\ \theta_i'(0) \\ M_{xi}(0) \\ M_{yi}(0) \\ M_{wi}(0) \\ V_{xi}(0) \\ V_{yi}(0) \\ T_i(0) \end{bmatrix} \tag{30}$$

where m_i is the mass of the ith storey and ω are the natural frequencies of the system and r_m^2 is the inertial radius of gyration, and can be calculated as [15, 42]:

$$r_m^2 = \frac{L^2 + B^2}{12} + y_c^2 + x_c^2 \tag{31}$$

y_c and x_c are the dimensions of the location of the geometric center and can be calculated as follows;

$$y_c = \bar{y}_c - \bar{y}_o \tag{32}$$

$$x_c = \bar{x}_c - \bar{x}_o \tag{33}$$

where the coordinate ($\bar{y}_c$, $\bar{x}_c$) is the location of the geometric center C in the coordinate system (y, x).

Dynamic transfer matrix can be shown as T_{di}.

$$T_{di} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ \omega^2 m_i & 0 & -\omega^2 m_i y_c & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & \omega^2 m_i & \omega^2 m_i x_c & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ -\omega^2 m_i y_c & \omega^2 m_i x_c & \omega^2 m_i r_m^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} T_i \tag{34}$$

The displacements - internal forces relationships from the base and to the top of the structure-can be found as follows;

$$\begin{bmatrix} u_{top} \\ v_{top} \\ \theta_{top} \\ u'_{top} \\ v'_{top} \\ \theta'_{top} \\ M_{xtop} \\ M_{ytop} \\ M_{wtop} \\ V_{xtop} \\ V_{ytop} \\ T_{top} \end{bmatrix} = T_{dn} * T_{d(n-1)} * \ldots T_{di} * \ldots T_{d2} * T_{dl} \begin{bmatrix} u_{base} \\ v_{base} \\ \theta_{base} \\ u'_{base} \\ v'_{base} \\ \theta'_{base} \\ M_{xbase} \\ M_{ybase} \\ M_{wbase} \\ V_{xbase} \\ V_{ybase} \\ T_{base} \end{bmatrix} \tag{35}$$

The boundary conditions of the equiavalent beam are;

1) $u_{base}=0$ 2) $v_{base}=0$ 3) $\theta_{base}=0$ 4) $u'_{base}=0$ 5) $v'_{base}=0$ 6) $\theta'_{base}=0$

7) $M_{xtop}=0$ 8) $M_{ytop}=0$ 9) $M_{wtop}=0$ 10) $V_{xtop}=0$ 11) $V_{ytop}=0$ 12) $T_{top}=0$

When boundary conditions are considered in equation (35) for the nontrivial solution of $t = T_{dn} T_{dn-1} T_{dn-2} ...... T_{d1}$, Eq. (36) can be attained;

$$f = \begin{bmatrix} t(7,7) & t(7,8) & t(7,9) & t(7,10) & t(7,11) & t(7,12) \\ t(8,7) & t(8,8) & t(8,9) & t(8,10) & t(8,11) & t(8,12) \\ t(9,7) & t(9,8) & t(9,9) & t(9,10) & t(9,11) & t(9,12) \\ t(10,7) & t(10,8) & t(10,9) & t(10,10) & t(10,11) & t(10,12) \\ t(11,7) & t(11,8) & t(11,9) & t(11,10) & t(11,11) & t(11,12) \\ t(12,7) & t(12,8) & t(12,9) & t(12,10) & t(12,11) & t(12,12) \end{bmatrix} \quad (36)$$

The values of ω, which set the determinant to zero, are natural frequencies of the asymmetric wall building.

3. PROCESS OF COMPUTATION

The process of the computation for the transfer matrix method is presented below:
1. The equivalent rigidities of each storey are calculated by using the geometric and the material properties of the structure.
2. Storey transfer matrices are calculated for each storey by using the equivalent rigidities.
3. System transfer matrix (Eq. (35)) is obtained with the help of storey transfer matrices and inertia forces effecting the storey levels with the procedure specified in section 2.
4) The nontrivial equation is obtained by using Eq. (36) as a result of the application of the boundary conditions.
5) The angular frequencies and relevant periods are found with the help of a method obtained from numerical analysis.
6) The modes are found with the help of the angular frequency and the Eq. (30).
7) The effective mass ratio and participation factor are found by using the modes.
8) With the help of the acceleration and the displacement spectrums, obtained from an earthquake record or design spectrum from codes, the displacement and internal forces are found by using the effective mass and the participation factor.

4. NUMERICAL EXAMPLE

A numerical example has been solved by a program written in MATLAB to verify the proposed method in this part of the study. The results are then compared with those given in the literature.

Example 1. A typical asymmetric wall-frame structure (Fig 1) is analyzed as an example. The structure has 30 storeys with total height H=90 m, and floor dimensions L=42 m and B=24 m. The structure consists of eight walls 0.25-m thick and the multibent frames, an elastic modulus $E=20*10^6$ kN/m^2 and the density of floor slabs $\rho=2.350$ kg/m^3. The structural properties are given in Table 1. The natural frequencies calculated by this method are compared with the results in reference [15]. The results are presented in Table 2, Figs. 2-4.

Table 1. Structural property of asymmetric wall-frame structures

Structural properties	
$(EI)_x$	$990.70*10^6$ kNm^2
$(EI)_y$	$574.53*10^6$ kNm^2
$(EI)_w$	$264.22*10^9$ kNm^4
$(GA)_x$	$274.29*10^3$ kN
$(GA)_y$	$297.14*10^3$ kN
(GJ)	$43.54*10^6$ kNm^2
x_c	7.81 m
y_c	7.63 m
m	355.41 kNsn^2/m
r_m	17.726 m

Table 2. Comparison of natural frequencies in Example 1

	Proposed method			Ng and Kuang [15]			ETABS [15]		
Mode	ω_1	ω_2	ω_3	ω_1	ω_2	ω_3	ω_1	ω_2	ω_3
1	1.128	1.540	2.362	1.163	1.587	2.437	1.197	1.539	2.299
2	5.611	7.405	11.944	5.799	7.655	12.348	5.898	7.313	11.642
3	15.037	19.892	29.003	15.317	20.265	33.108	14.775	19.455	31.350

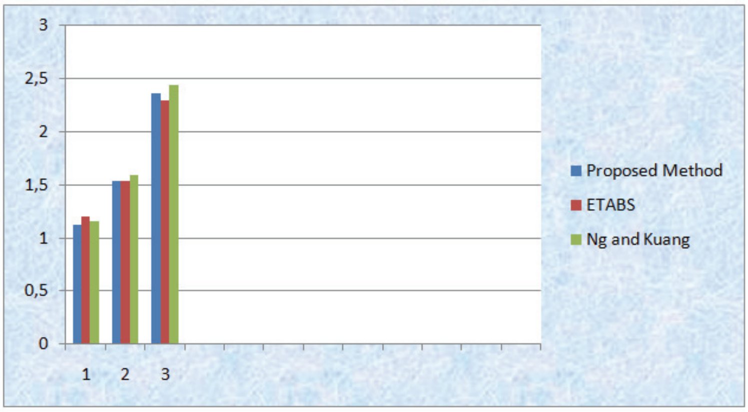

Fig. 2. Comparison of natural frequencies of the first mode (s^{-1})

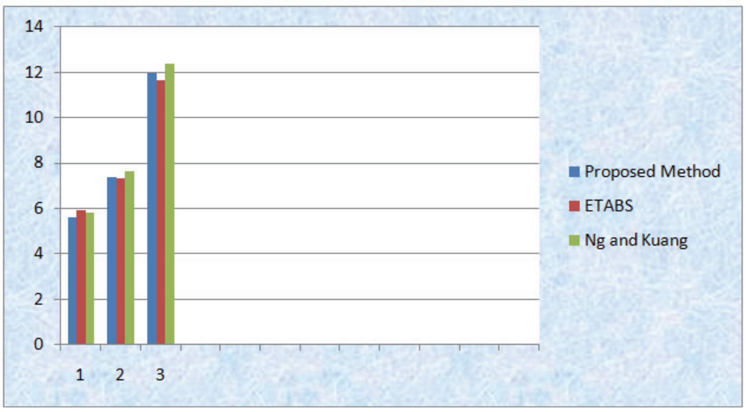

Fig. 3. Comparison of natural frequencies of the second mode (s^{-1})

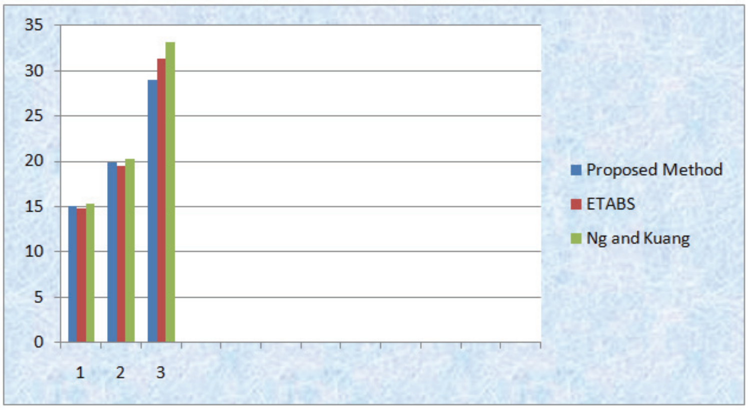

Fig. 4. Comparison of natural frequencies of the third mode (s^{-1})

5. CONCLUSION

In this study, a vibration analysis of asymmetric shear wall-frame structures is carried out by transfer matrix method. The whole structure is assumed to be an equivalent shear- flexure-torsion coupled beam in this method. The governing differential equations of equivalent beam are formulated using the continuum approach and are posed in the form of the simple storey transfer matrix. By using the storey transfer matrices and the point transfer matrices which take into account the inertial forces, the system transfer matrix is obtained. Natural frequencies can be calculated by applying the boundary conditions. At the end of the study, to verify the present method a numerical example has been solved by a program written in MATLAB. The results are compared with the results of the literature. The comparison which is given in Table 2 shows that the results obtained from the proposed method are in close agreement with the solution developed in the literature. In the proposed method the structural properties of the building are alterable and different numerical examples can also be solved. The proposed method is simple and accurate enough to be used both at the concept design stage and for final analyses.

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