COST OPTIMIZATION OF A COMPOSITE FLOOR SYSTEM USING ANT COLONY SYSTEM^{*}

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Abstract– This paper presents an ant colony system model for cost optimization of a composite floor system based on the load and resistance factor design (LRFD) specification of the AISC. The model formulation includes the cost of concrete, steel beam, and shear studs. The objective function is considered as the cost of the structure, which is minimized subjected to serviceability and strength requirements. Examples of composite floor systems are presented to illustrate the performance of the present algorithm. A parametric study is also included to investigate the effects of beam spans and loadings on the cost optimization of composite beams.

Keywords– Ant colony system (ACS), cost optimization, composite floor systems, structural optimization, load and resistance factor design

1. INTRODUCTION

Composite floors are widely used in commercial multistory buildings because of their economy. In order to create a composite floor, a concrete slab is often mechanically connected to a hot-rolled steel section through shear connectors.

In practice, a composite beam is designed by a trial-and-error process to select the following parameters: (1) the concrete type expressed by its compressive strength and its unit weight, (2) the slab thickness, (3) the steel section size expressed by its cross-sectional area, and its steel grade expressed by its yield strength, and (4) the strength of the shear connectors expressed by its shear resistance, and the number of shear connectors provided.

The design of composite beams is complicated and highly iterative. Depending on the design parameters, a beam may be fully or partially composite. In the case of the LRFD design code [1], the plastic deformation has to be considered. A source of complexity is due to the fact that the location of the plastic neutral axis (PNA) may lie within the concrete slab, the flange of the steel beam, or the web of the steel beam. Since the value of a design parameter affects other design parameters, all design parameters cannot be found simultaneously.

Mathematical optimizations provide methodologies to automate the complicated design process. Moreover, one can achieve an optimum solution out of numerous solutions on the basis of a selected criterion such as the minimum weight or the minimum cost. In fact, most of the articles that have been published on the optimization of structural systems focus on the minimum weight design.

There are some articles published on the optimization of composite beams. Zahn [2] discussed the economies of the LRFD design code versus the AISC allowable stress design code in the design of composite beams through the weight comparison of some 2500 composite designs using A36 steel. The results indicated that the LRFD design code yielded a savings of 6-15% for span lengths ranging from 3 m

^{*}Received by the editors June 15, 2010; Accepted December 9, 2010.

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to 13.7 m. Lorenz [3] discussed the minimum cost design of composite beams based on the AISC-LRFD design code and argued that the real advantage of the AISC-LRFD concept could be realized in the minimum cost design. Bhatti [4] put the problem into a standard optimization formulation and solved the problem approximately using the Symbolic Algebra Mathematica [5]. His cost function, however, only includes the cost of the steel beams and the field-installed shear studs, neglecting the cost of the concrete. Long et al. [6] presented a non-linear programming based optimization of cable-staved bridges with composite superstructures and proposed a cost objective function which contained the costs of concrete, structural steel, reinforcement, cables and formwork. Kravanja and Šilih [7] introduced a non-linear programming optimization model for composite I beams. Kravanja and Šilih [8] also introduced a mixedinteger non-linear programming approach for cost optimization of composite I beams. Adeli and Kim [9] proposed a formulation for the cost optimization of composite beams based on the AISC LRFD specifications by including the costs of (a) concrete, (b) steel beam, and (c) shear studs. The problem was formulated as a mixed integer-discrete nonlinear programming problem and solved by the neural dynamics model of Adeli and Park. In addition, Kravanja and Šilih [10] performed an optimization based comparison between composite welded I beams and composite hollow-section trusses for a defined steel price of €0.4 kg and for fixed economical parameters. In their work, the cost objective function includes the costs of concrete, steel sections, reinforcement, shear studs, anti-corrosion paint, fire protection paint, sheet-steel cutting costs, and welding costs as well as the formwork costs. This objective function was also used by Klanšek and Kravanja [11] to compare different composite systems for a pre-defined imposed load and a fixed steel price. Kravanja et al. [12] presented a mixed-integer non-linear programming (MINLP) optimization approach to mechanical super-structures. Klanšek and Kravanja [13,14] presented the cost optimization, comparison, and competitiveness between three different composite floor systems: composite beams produced from duo-symmetrical welded I sections, composite trusses formed from rolled channel sections and composite trusses made from cold formed hollow sections. The optimization was performed by the non-linear programming (NLP) approach. The aim of the comparison was to define the spans and the loads, at which each of the presented composite structures showed its advantages. Comparative diagrams were displayed at the end of the paper for choosing the optimal type of a structure.

In recent years, structural optimization witnessed the emergence of novel and innovative design techniques. These stochastic search techniques make use of ideas taken from nature and do not suffer the discrepancies of mathematical programming based optimum design methods. The basic idea behind these techniques is to simulate the natural phenomena. The cost optimization problem for a composite floor was solved by Senouci and Al-Ansari [15] by using a genetic algorithm. Recently Kaveh and Shakouri performed cost optimization of a composite floor system via a modified harmony search algorithm [16]. In this paper, an ant colony system model is utilized for cost optimization of a composite floor system based on the LRFD specification of the AISC. In recent years the usage of meta-heuristic has been increased in many fields of engineering [17,18]. The formulation includes the cost of concrete, steel beam, and shear studs. The objective function is taken as the cost of the structure, which is minimized subjected to serviceability and strength requirements. Examples of composite floor systems are included to illustrate the performance of the present algorithm. A parametric study is also provided to investigate the effects of beam spans and loadings on the cost optimization of composite beams.

2. MODEL FORMULATION

In this section the primary purpose is to formulate an efficient optimization model that supports the cost minimization of composite beams. The present model is formulated in two major steps: (1) To determine the major decision variables affecting the design of composite beams; and (2) to formulate the objective function for composite beams and implement it in an efficient optimization model.

a) Decision variables

The present model is designed to consider all relevant decision variables that may have an impact on the cost optimization of composite floor system. These include for the concrete slab: (1) the compressive strength (f'_c), (2) the unit weight (γ_c), and (3) the thickness (t_c); for the steel section: (1) the yield strength (F_y), (2) the cross-sectional area (A_s), (3) the depth (h), (4) the web thickness (t_w), (5) the flange thickness (t_f), (6) the flange width (b_f), (7) the moment of inertia (I_s), (8) the plastic modulus (Z_s), and (9) steel beam spacing (d_0); and for the shear connectors: (a) the diameter (ASC) and (b) the number (N_s) of shear connectors. A schematic view of the composite floor variables and system are shown in Fig. 1.



Fig. 1. Schematic view of (a). variables of composite floor system, (b). a simple composite floor system

In order to reduce the complexity of the optimization model, the present model combines the decision variables related to the steel section into a single variable called a steel section decision variable. As an example, when an I-Shape section is selected as the steel section, h, t_w , t_f and b_f are available in the existing steel design codes, and the following variables can be obtained by the following formulas:

$$A_{s} = 2 \times b_{f} \times t_{f} + t_{w}(h - 2 \times t_{f})$$

$$I_{s} = \frac{t_{w}(h - 2t_{f})^{3}}{12} + 2\left(\frac{b_{f} \times t_{f}^{3}}{12} + (b_{f} \times t_{f})(\frac{h - t_{f}}{2})^{2}\right)$$

$$Z_{s} = (b_{f} \times t_{f})(h - t_{f}) + t_{w}(\frac{h}{2} - t_{f})^{2}$$
(1)

The yield strength, F_y , of the steel section is given and fixed at the onset of each design, and hence, the fourth decision variable (first steel section variable) is not considered in the present model.

A design alternative option which defines a complete design of a composite floor system includes the following decision variables:

 X_1 = concrete compressive strength,

 X_2 = concrete slab thickness,

 X_3 = steel section shape,

 X_4 = steel beam spacing,

 X_5 = shear connector diameter, and

 X_6 = number of shear connectors.

b) Optimization objectives

The present optimization model is formulated in order to provide the capability of cost optimization of composite floor systems. The model is also designed to quantify and measure the impact of various decision variables that affect the cost optimization of composite floor systems. It incorporates the following objective equation:

Minimize composite beam cost = $C_t = C_c + C_s + C_{sd}$

where C_c , C_s , and C_{sd} are the cost of concrete, steel beam, and shear connectors, respectively. The terms used in the objective equation are defined as follows:

$$C_c = \gamma_c LBt_c C'_c \tag{2}$$

$$C_s = G_s L(\overset{B}{/}_{d_0})C'_s \tag{3}$$

$$C_{sd} = N_s C'_{sd} \tag{4}$$

where L is the beam span (or the length of the floor), B is the width of the floor, G_s is the weight of the steel beam in length units, C'_c is the cost of concrete per unit volume, C'_s is the cost of the steel section per unit weight, and C'_{sd} is the cost of one shear connector including installation and material costs. These costs include labor payments and welding expense. The labor cost in a composite beam is almost permanent; therefore, it is not necessary to include it in the objective function.

The minimization of the objective function is subjected to the constraints prescribed by the AISC– LRFD specifications [1]. These constraints are described briefly in the following section.

c) Design constraints

1. Flexural strength constraints: The ultimate bending moment must be less than or equal to the nominal flexural strength multiplied by the resistance factor ($\varphi = 0.9$). Two cases must be considered. First, the moment capacity of the non-composite steel section (excluding the concrete strength) must be checked to make sure that the steel section can support its own weight, the weight of the wet concrete, and the temporary loads such as construction loads. This constraint is expressed as

$$M_{u-noncomposite} \le 0.90M_{n-noncomposite} \tag{5}$$

where $M_{u-noncomposite}$ is the ultimate factored moment due to the wet concrete weight, the temporary loads, and the own weight of the steel section, and $M_{n-noncomposite}$ is the nominal moment capacity of the steel section.

Second, the moment capacity of the composite section must be checked to make sure that the composite section can support all dead and live loads, as defined by the following constraint:

$$M_{u-composite} \le 0.85M_{n-composite} \tag{6}$$

where $M_{u-composite}$ is the factored moment due to dead and live loads, and $M_{n-composite}$ is the moment capacity of the composite beam.

2. Deflection constraint: The deflection of a composite beam depends on whether it is shored or not during the construction phase. The unshored construction is less labor-intensive and faster than the shored construction, and hence, it is often the preferred method of construction. For unshored composite beams, the deflection of the composite beam due to live loads, Δ_{IL} , is given by [1]:

$$\Delta_{LL} = \frac{5W_{LL}L^4}{384E_s I_{LB}} \le C_1 L \tag{7}$$

where W_{LL} is the service live load per unit length of the beam, E_s is the modulus of elasticity of the steel section, I_{LB} is the lower bound moment of inertia, and C_1 is a coefficient ranging from $\frac{1}{300}$ to $\frac{1}{360}$ for building structures or $\frac{1}{500}$ to $\frac{1}{900}$ for highway bridges.

3. Shear stud spacing constraints: AISC–LRFD defines the minimum center-to-center spacing of shear connectors, p, not to be less than six times the diameter, φ , of the shear connector, and the maximum center-to-center spacing not to be greater than eight times the total slab thickness, t_c , i.e.

$$p \ge 6\varphi$$
 (8)

$$p \le 8t_c \tag{9}$$

3. ANT COLONY SYSTEM FOR COST OPTIMIZATION OF COMPOSITE FLOOR SYSTEMS

A meta-heuristic algorithm based on the ants' behavior was developed in the early 1990s by Dorigo and Gambardella [19]. This algorithm was called ant colony optimization (ACO) because it was motivated by the social behavior of ants. Ant colony system is a variation of the ACO, which has proven to behave more robustly and provide far better results for certain problems.

The building blocks of these algorithms are cooperative agents called ants. These agents have simple capabilities, which make their behavior similar to real ants. Real ants are capable of finding the shortest path from food source to their nest or vice versa by smelling pheromones which are chemical substances they leave on the ground while walking. Each ant probabilistically prefers to follow a direction rich in pheromone. Since pheromones do evaporate and lose strength over time, the final result is that more ants tend to pass over the shortest path and this path is visited more often as the amount of pheromone being laid increases. As an illustrative example, consider the sketch shown in Fig. 2. The number of dashed lines in Fig. 2c is approximately proportional to the amount of pheromone deposited by ants.

In order to apply the ACS algorithm to a specific problem, it is necessary to represent it as a set of different paths for ants to travel. In this problem, different feasible variables selection is supposed as a tour for an ant to travel, therefore the cooperative ant agents search to find the best set of decision variables, resulting in minimal objective function.



Fig. 2. Ant technique to find an optimum solution.

First, m artificial ants are initially positioned on m decision variables as primary selected variables, and then ACS algorithm is applied as follows:

An ant k chooses the r^{th} decision variable by applying the rule of the following equation:

$$j = \begin{cases} \arg\max_{u \in L_k(r)} (\tau_{ru} \,\eta_{ru}^{\beta}) & \text{if } q \leq q_0 \\ J & \text{otherwise} \end{cases}$$
(10)

Where q is a random number uniformly distributed in [0..1], q_0 is a parameter $0 \le q_0 \le 1$, and J is a random variable selected according to the probability distribution given in the following equation:

$$P_{rs}^{k} = \begin{cases} \frac{\tau_{rs}\eta_{rs}^{\beta}}{\sum\limits_{u \in L_{k}(r)} \tau_{ru}\eta_{ru}^{\beta}} & \text{if } S \in L_{k}(r) \\ 0 & \text{otherwise} \end{cases}$$
(11)

 $L_k(r)$ is the set of variables that remain to be chosen by ant k as the r^{th} variable, and τ_{rs} is the amount of pheromone deposited on the variable number s as a candidate for being the rth variable. It is assumed that there is an equal amount of pheromone τ_0 , deposited initially on each decision variable. η_{rs} is the corresponding heuristic value which remains constant throughout the iterations and, unlike pheromone, amount is not modified. Moreover, β is a parameter for controlling the relative importance between τ and η .

After an ant chooses one number as a decision variable, the local updating rule on that chosen variable is performed in order to shuffle the solution and prevent focusing on a specific solution. The local updating rule modifies the amount of pheromone by

$$\tau_{rs} \leftarrow (1 - \xi)\tau_{rs} + \xi\tau_0 \tag{12}$$

where $0 < \xi < 1$ is a parameter for adjusting the pheromone previously deposited on τ_{rs} .

Once all the ants complete their own tours, the pheromone will be updated for all the variables according to the global updating rule. This pheromone updating is intended to allocate a greater amount of pheromone to shorter tours. The rule is given by the following equation:

$$\tau_{rs} \leftarrow (1 - \rho)\tau_{rs} + \rho \Delta \tau_{rs} \tag{13}$$

where

$$\Delta \tau_{rs} = \begin{cases} (D_{gb})^{-1} & \text{if } (r,s) \in \text{global best tour} \\ 0 & \text{otherwise} \end{cases}$$
(14)

where D_{gb} is the amount of objective function for best tour and $0 < \rho < 1$ is the pheromone decay parameter. The best ant tries to find the minimal objective function.

The ant colony optimization initiates the design process by selecting random values for the steel beam section and spacing, compressive strength and thickness of slab concrete, number and distance of shear connectors. The algorithm tries to find the best value for each design variable to minimize the objective function.

After this brief introduction of ant colony optimization, we should mention how this problem is solved by this meta-heuristic algorithm. First, ants select ASC and N_s randomly, because the selection of these variables is independent from others. Then, t_c is selected satisfying the following condition:

$$t_c \ge \max(\frac{L}{4N_s}, t_{c_{\min}})$$
(15)

Now, d_0 is the next choice of each ant. In this step, some of the values of d_0 cannot be selected by some ants, because d_0 must be chosen such that the compatibility between t_c and d_0 holds.

Each ant selects steel beam section in this step by satisfying the constraint $M_{u-noncomposite} \leq 0.90M_{n-noncomposite}$ and after this selection, compressive strength of concrete slab should be selected by satisfying the following two conditions:

1) $M_{u-composite} \leq 0.85 M_{n-composite}$.

2) The maximum moment produced in concrete slab should be smaller than its moment capacity.

4. NUMERICAL RESULTS

Here we have considered a single span floor unit, which is repeated in the structure to cover a ceiling. Such a span behaves independently, and once we optimize the problem for one span the result will correspond to the entire ceiling. This process can be repeated for spans of different dimensions. Furthermore, the following parameters values are considered in the proposed ACS algorithm: $\beta = 2$, $\xi = 0.1$, $\rho = 0.1$ and $q_0 = 0.5$. Table 1 lists a number of possible values for the six decision variables. The cost values C'_c, C'_s , and C'_{sd} are selected to be 50 $\$/m^3$, 1 \$/kg, and 0.5 \$/stud, respectively.

a) Example

The considered composite I-beam floor system is 6 m long, subjected to the combined effects of the self-weight and the imposed dead load of 2.94 $kN/m^2 (300 kg/m^2)$ and imposed live load of 1.96 $kN/m^2 (200 kg/m^2)$; the width of the floor is 8 m. The overall height of studs is 50 mm.

The output consists of the following:

Steel beam spacing = 210 cm, Steel beam size = INP 200, Concrete slab thickness = 8 cm, Concrete slab compressive strength = 19613.3 kN/m^2 ((200 kg/cm^2), Diameter of shear connectors = 12 mm, and Number of shear connectors = 18.

The history of design for this example is shown in Fig. 3.

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$f'c(kg/cm^2)$	I – Shape	$t_c(cm)$	$d_0(cm)$	ASC(cm)	N_s
200	INP120	8	50	1.3	10
250	INP140	9	60	1.6	12
300	INP160	10	70	2.0	14
350	INP180	11	80	2.2	16
400	INP200	12	90		18
	INP220	13	100		20
	INP240	14	110		22
	INP260	15	120		24
	INP280	16	130		26
	INP300	17	140		28
	IPE120	18	150		30
	IPE140	19	160		32
	IPE160	20	170		34
	IPE180		180		36
	IPE200		190		38
	IPE220		200		40
	IPE240		210		42
	IPE270		220		44
	IPE300		230		46
	IPB120		240		48
	IPB140		250		50
	IPB160		260		
	IPB180		270		
	IPB200		280		
	IPB220		290		
	IPB240		300		
	IPB260				
	IPB280				
	IPB300				

Table 1. Design variable range values



Fig. 3. Design history for Example 1

5. PARAMETRIC STUDY

A parametric study is also performed to investigate the effects of beam spans and loadings on the cost optimization of composite beams. Table 2 summarizes the design results obtained in the case study using the present model. As expected, the steel section size increases with both the beam span and acting loads as to satisfy the strength and the deflection constraints. Similarly, the size and the number of studs increase with both the beam span and the loadings as to satisfy force and moment equilibrium. Table 3 summarizes the second-order polynomial fits between the beam costs and the spans, which can be used to get an initial estimation of the total cost under a given span length or a given loading combination.

Figure 4 shows the curves representing the variations between the total costs and the floor spans under three different loadings. One can obtain these second-order polynomial fits among the beam costs and dead load, live load and span length. The equation will have the following pattern:

$$Cost = a_1 DL^2 + a_2 LL^2 + a_3 L^2 + a_4 DL \times LL + a_5 DL \times L + a_6 LL \times L + a_7 DL + a_8 LL + a_9 L + a_9$$

where the coefficients must be calculated by nonlinear three-dimensional regression.

Cc/Cs	Csd/Cs	В	L	DL	LL	Ι	tc	fc	ASC	Ns	d0
		(m)	(m)	(kg/m^2)	(kg/m^2)	section	(cm)	(kg/cm^2)	(cm)	(number)	(cm)
			4		100	IPE160	8	200	1.3	12	230
		8	6	200		IPE220	8	200	1.3	18	230
			8			IPE300	8	200	1.3	24	250
50 0.6		8	4	300	200	IPE160	8	200	1.3	12	210
	0.6		6			IPE220	8	200	1.3	18	210
			8			IPE270	8	200	1.6	24	210
		4 8 6	4			INP140	8	200	1.6	12	180
			400	300	IPE200	8	200	1.6	18	170	
			8			IPE270	8	200	1.6	24	190

Table 2. Parametric study results

Table 3. Polynomial best-fit equation

Dead load (kg/m^2)	Live load (kg/m^2)	Floor span (m)	Floor total cost(\$)	Coefficient of determination (R^2)
200	100	L	$22.756L^2 - 19.352L + 91.47$	0.9989
300	200	L	$21.878L^2 - 0.5865L + 51.449$	0.9964
400	300	L	$28.069L^2 - 57.009L + 190.2$	0.9984

6. CONCLUDING REMARKS

An efficient optimization model is suggested to perform the cost optimization of composite beams. The composite floor system consists of a reinforced concrete slab and steel I-beams. The proposed model enables structural designers to generate and evaluate optimal/near-optimal design solutions. To accomplish this, the model incorporates (1) a design module that performs the design of composite beams (LRFD-AISC rules); (2) a cost module that computes the total cost of composite beams; and (3) an optimization module that searches for and identifies optimal/near-optimal design alternatives. Substantial cost savings can be achieved by using the present model. The main aim of this paper has been to present a simple and efficient algorithm that can be used in practical engineering problems. Such a simple approach can be utilized in some other engineering design problems such as cost optimization of bridges, arch-dams and retaining walls to reduce the cost of the construction.



Fig. 4. Optimal composite floor design total costs.

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Acknowledgement: The first author is grateful to the Iran National Science Foundation for the support.

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