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Hotelling Location Model, Triangular Distribution, Experienced and Inexperienced Consumers

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Abstract

Appropriate decision-making in the firm's location choice can play a crucial role in improving the competitiveness and profitability of the firms. The basic presumption of most existing location studies is the assumption of uniform distribution, which is less common in the real world. In contrast, the distribution of consumers may be in the form of a triangle in which consumers gather in the city center. On the other hand, the type of consumers in terms of experienced and inexperienced consumers can also play an effective role in the demand for firm products. Therefore, this study aims to investigate the Hotelling location model with the assumption of a triangular distribution of consumers and experienced and inexperienced consumer types. In this study, optimal location has been analyzed assuming two types of experienced and inexperienced consumers, distributed with a triangular distribution density function. The results indicate that the demand functions of two firms depend on the acquired desirability of a particular type of food and the number of experienced consumers. The unit Nash equilibrium costs are increasing compared to transportation costs. In addition, with an increase in transportation costs, firm 1 approaches the center, and firm 2 gets away from it. Furthermore, if two firms are located at the same point, they do not demand uniform equilibrium prices, and the price of each firm is more sensitive to the location of the other first than its location.

Highlights

- Unlike the results of Hotelling and other location choice models, the price and optimal location of firms also depend on the type of experienced consumers.
- The price of each firm is more sensitive to the location of other firms than its location.
- An increase in the number of experienced consumers will lead to an increase in the firm's demand producing desired goods and decreases the demand of the firm producing undesired goods.
- Even if the firms were in a symmetrical position, they had different equilibrium prices and market shares.
- By increasing transportation costs, the producer of desired products or services approaches the center, and the producer of undesired products and services gets away from the center.

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1. Introduction

Finding an optimal location for a firm in competition with other firms in the market is called competitive location. Nowadays, despite extreme competition between economic firms, there should be no mistake in decision making, and each wrong decision could have irrecoverable consequences for firms. The firm's objective of choosing a location in competitive conditions is to achieve some objectives such as getting higher profit, achieving higher market share, reducing distribution costs of goods, etc. The study of choosing the location is significant since making uninformed decisions for the establishment of industries leads to the elimination or reduction of efficiency of the economic system. Moreover, decisions related to the selection and acquisition of location specifications of a firm could have a great effect on the ability to profit and maintain competitive advantage (Choo & Mazzrol, 2003).

Although all aspects of providing services are studied when establishing these careers, failure to consider location prevents the manufacturing unit from achieving the intended profitability and lags behind its objectives (Melaniphy, 1999).

The dramatic changes of the last few decades in competitive location patterns and ever-increasing growth of these changes, and the competitiveness of the environment have forced firms to compete with others to make an effort to survive and increase market share. Here, the application of appropriate tools and methods of choosing a location is effective in fulfilling the enterprises' objectives. It's obvious that the use of a proper method of choosing the location is based on the application of real assumptions that constitute the basis of discussion.

Researchers from the 1960s have considered the location issues. In one classification, they are classified into competitive and non-competitive categories, with the latter having more share than the former. The competitive location model was first introduced by Hotelling through the game theory approach concerning the competition between two ice cream vendors in 1929. The Hotelling model is the basis of other location models, and there is always some traces of it in other studies on location. Further studies on location are mostly focused on improving one or several assumptions of the Hotelling model and proposing it in a more general area. The most important point in these generalizations is that competitive location models are inherently unstable; in other words, a slight change in an assumption or parameter will yield a completely different result.

One of the main assumptions of simplification is that one type of consumer intends to choose the best location for purchase with minimization of costs. In other words, the consumer intends to just minimize the price and his transportation costs. He doesn’t mind from which firm to purchase; however, all consumers do not have the same preference in the real world. For instance, one consumer might purchase the product of a firm which is further with the same price since he prefers that product more (based on his previous experience).

Another important simplification assumption is that in former location models, the consumers are uniformly distributed. In other words, there is one consumer at each point; while in the real world, the consumers are not equally and uniformly distributed in a street or city and the city centers are more populated than suburbs .

Therefore, this article attempts to solve the weaknesses of the Hotelling model and other location methods and propose a more appropriate solution in firms' locations in this structure. As far as in previous studies, the consumers were considered uniform, and only one type of consumer was considered. In this article, two types of consumers, experienced and inexperienced, will be investigated to present an appropriate solution for firms' locations when there are different types of consumers (experienced and inexperienced) rather than just one type of consumer. Moreover, it aims at studying the case with different types of consumers to see whether the competition between firms should still be monopolistic competition and the enterprises could have some power (even if slight) in pricing for presenting distinct services. On the other hand, meanwhile, a more realistic distribution of consumers, i.e., triangular distribution, will be used instead of uniform distribution, which will make the behavior of firms in the center (for being more population) different from margins. In the end, the results of simultaneous application of these two real assumptions will be compared with the results of Hotelling and others .

The rest of this paper is organized into five parts. In the second part, the firm's location choice studies have been reviewed. In the third part, the model has been presented with two subsections. In the fourth part, the theorems (7 theorems) in a triangular model and in the case of types of consumers have been presented. In the fifth and final part, the conclusion and recommendations are presented.

2. A Review of the Related Literature

In what follows, a brief review of the studies focused on the location choice of firms will be presented.

The World Bank (1986) claimed that the optimal location for small industries is in the city centers and for big industries is a suburb area. Darling (2001) studied the significance of competitive location in his study entitled, "Successful competitive positioning: the key for entry into the European consumer market," focused on the significance of competitive positioning. He showed a model to marketing managers to achieve optimal location in the European market. Reggiani (2009), in his research, presents some conditions for the existence of price-location equilibrium of spokes model when product delivery is established while not all spokes are occupied. The result showed that in such an equilibrium, one firm supplies all products while others focus on their niche.

Shiode et al. (2012) considered the optimal location policy in a linear market using uniform distribution for demand and solved it through Nash pure strategy and Stackelberg equilibrium. Saidani et al. (2012), using customers' distance and quality as decision variables, performed competitive and exclusive locations with incomplete information. In their method, first, each enterprise offers its best quality to achieve the highest profit, and then in the second stage, it chooses its location with minimum distance from the customer.

Lijesen and Reggiani (2013) raised two questions about location choice models: first, where is the location of enterprises, and second, how effective are those sectors of the market that have not been covered so far by any firm the decision making of active firms in the market? They analyzed the spoke model (Chen and Riordan, 2007) to quadratic form for transportation cost. They found that all streets were connected to the city center and the customer should pass the city center for purchasing from a firm. Also, they assumed that if it is not in the consumer's street or the consumer is located in the street without firm.

Danalet et al. (2015), in their study survey location choice with longitudinal WiFi data", applied a proposed method to confront the initial problems in the choice of catering location on campus using WiFi traces. The results showed cross-validation, price elasticity, and simulation of a scenario predicting a new catering location's opening. Predicted market shares of the new catering location correspond to point-of-sale data of the first week of opening. In their location choice research, Naveen et al. (2016) proposed a new plan of classifying effective factors on location into two groups and analyzed 151 published papers in international trade and managerial journals from 1975 and achieved mentioned results. They claimed that their approach could improve further studies on location choice and enhance the literature related to this issue by providing a comprehensive model and directions for future research.

Shahbazi and Salimian (2017) expanded the firms' location theory and the product's consistency using a triangular distribution approach. They showed location choice models usually make use of uniform distribution of consumers while it is not true in reality, and mostly the consumers' accumulation is more in the city centers rather than suburb areas. They were inspired by the early model of Lijesen and Reggiani (2013) and dealt with changing consumers' distribution from uniform to triangle distribution in a two-stage game. The results showed that the increased number of streets and transportation costs leads to a price increase. If both firms are located at the same distance from the city center, they will gain the same market share and more inclined toward being closer to the city center or having a minimum distance. Moreover, when the consistency issue includes the triangular distribution, the Nash equilibrium price is less than the Rohlfs model (1974), and a greater range of consumers purchase the product.

Krenz (2019) focuses on the differences in new German manufacturing plants' location choices across the German district-free cities and districts and investigates its regional determinants. The research results showed policy implications emerge that address, in particular, the improvement of infrastructure and support to reap off benefits that arise from agglomeration externalities.

Chin (2020) in a study survey location choice of new business establishments in the United States, emphasizing local context and neighborhood conditions, and research focuses on examining the relationship between the uniqueness of certain regions, spatially bounded characteristics, and how both affect where new establishments locate. Results confirm the importance of economic, demographic, and geographic conditions at the neighborhood level, providing a better understanding of the vulnerability of the local economy.

Kim (2020) developed a model of state government competition and firm location choice combining the first-price auction among states with discrete choice by firms and estimated his model using firm-level data on accepted incentives augmented with data on state attributes. He showed that findings are consistent with the view that state government competition using incentives generates large corporate welfare and little allocative efficiency.

One of the most important assumptions in location choice models is a simplification; while, in the real world, the consumers are not equally and uniformly distributed in a street or city, and the city centers are more crowded than suburb areas. Another important simplification assumption is that consumers are evenly distributed across a street or city. There is one type of consumer who wants to choose the best location while minimizing costs. In other words, the consumer only wants to minimize the price, plus her transportation costs no matter who buys from which firm. In the real world, consumers are distributed differently all over the street or city. In other words, not all consumers have the same type of preference and have different preferences, meaning that consumers may buy the goods of a more distant firm at the same price because they prefer the product more. Hence, the contribution of this study is to enter the realistic assumptions of triangular distribution and experienced and inexperienced consumers into the Hotelling model.

3. The Study Model

This part will be presented in two subsections. First different types of consumers and then triangular distribution will be explained

3.1 Types of Consumers (Experienced and Inexperienced)

Assume two firms are selling one type of food and shown as i, and i= 1, 2. Moreover, assume that there are L consumers who are distributed in a street of length L. Out of L consumers, there are I inexperienced and E experienced consumers (I+E= L). In addition, assume that inexperienced consumer is the one who purchases for the first time, i.e., he doesn’t mind purchasing from any firm, and the experienced consumer is the one who has previously purchased that product, i.e., the experienced consumer knows the product and it has the previous experience of purchasing that product. If the consumer is inexperienced, he doesn't care to purchase from which firm, and he just wants to minimize the price and transportation costs. However, the case is different for the experienced consumer, and if he does not purchase his favorite product, it will lose its achieved desirability up to λ. In addition, assume that the food presented by firm 1 is more desired, i.e., more consumers will prefer it in the same condition (Shahbazi and Salimian, 2018). Moreover, assume that firm 1 is located at a distance of a from point 0 and firm 2 is located at the right side of firm 1 at a distance of b from point L (Figure 1).

Figure 1. Hotelling linear city with two firms and two types of experienced and inexperienced consumers
***Source**: Shy, 1995*

The presence of different types of consumers (experienced and inexperienced) is an important issue that has not been studied in previous discussions of location. In previous studies, only location choice for one type of consumer has been considered (inexperienced consumer); however, in the real world, some consumers have previous experience of purchase (due to previous purchases, familiarity, and so on). It is clear that the entrance of this type of consumer in the model will yield more accurate and logical results. Making this assumption more realistic could be significant in using the results of optimal location in industries such as the airline industry[1] and restaurants dealing with different types of consumers (experienced and inexperienced).

3.2. The Triangular Distribution Density Function

The random variable X has triangular distribution and selects the values in S= [a, b]. Here, a is the start point of streets, and b is their endpoint, and it is assumed that a point such as c is the point connecting these streets. The probability of consumers' distribution in [a, b] subinterval linearly increases; i.e., the closer we become to the center, the more the number of consumers in that street increases. Moreover, in [c, b] subinterval, the probability of distribution of consumers linearly decreases, and by getting far away from the center, the number of consumers in that street will decrease (Evans et al., 2000). Therefore, the density function of this variable is triangular, which is shown by Tria (a, c, b), and its density function is achieved as follow:

[1] Aviation services in terms of the quality of aircraft, etc., cause different types of people to change their preferences after recognizing their services.

$$DF = \begin{cases} \frac{2(x-a)}{(b-a)(c-a)} & a \leq x < c \\ \frac{2}{b-a} & x = c \\ \frac{2(b-x)}{(b-a)(b-c)} & c < x \leq b \end{cases} \tag{1}$$

Here, the city is assumed to be linear of length L, divided into two equal parts at point L/2. Here, instead of uniform distribution of consumers on each street, two types of experienced and inexperienced consumers are considered distributed in streets with triangular distribution density function (Shahbazi and Salimian, 2017). By replacing b=L, a=0, c= L/2, the triangular distribution density function will be as equation 2:

$$(x)= \begin{cases} \frac{4x}{L^2} & 0 \leq x < \frac{L}{2} \\ \frac{4(L-x)}{L^2} & \frac{L}{2} \leq x \leq L \end{cases} \quad f \tag{2}$$

Triangular distribution density function is shown in Figure 2:

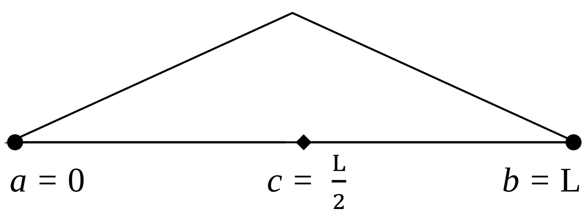

Figure 2. Triangular distribution density function
***Source**: Evans et al. (2000)*

In this state, it is assumed that consumers are distributed in 0 to L interval, and the mean point with the highest number of consumers is c which is the city center.

It should be noted that by entering the triangular distribution density function instead of uniform distribution, the assumptions of models will have more conformity with the real world since the city centers are more populated and crowded and the margins are less populated in the real world.

In general, each consumer purchases one unit of product, and in order to refer to each firm, he should pay transportation cost τ per unit of distance. Therefore, the consumer located in point x is inevitable to pay transportation cost of τ |x-a| for purchase from firm 1 and τ | x-(L-b)| for purchase from firm 2. Therefore, the desirability function of consumer located at x will be as follow:[2]

$$U^x = \begin{cases} -p_1 - \tau\,|x-a| & If\ purchase\ from\ firm\ 1 \\ -p_2 - \tau\,|\,x-(L-b)| & If\ purchase\ from\ firm\ 2 \end{cases} \tag{3}$$

[2] p_1 is fob price of firm1 product and p_2 is fob price of firm 2 product.

Now the questions are: How will be the distribution function of firms with a triangular distribution of consumers and in case of diversity of consumers? How will be the two firms' equilibrium price be, and if they are located at the same distance from the center, will they offer the same price? Will the increase of transportation costs make firm 1 (producer of desired food) closer to the center, and firm 2 (producer of undesired food) gets far away from the center? How will the increase in the number of experienced consumers affect the demand function of firms and their equilibrium prices? And finally, the optimal location of firms in this condition is the function of what factors.

Some theorems are presented here that show the firms what factors do their demand function, equilibrium prices (in two states of spatial symmetry or lack of spatial symmetry), optimal locations depend, and what decision they should make on the price, optimal location, etc. This is important to note that as these points are Nash equilibrium, violation of them (rejection of theorems) will have fewer consequences (less profit) for firms; in other words, the unilateral deviation is not to the benefit of neither firm.

Now, seven theorems will be presented to respond to these questions.

4. Empirical Results

In the following, seven theorems on the triangular distribution and types of consumers will be presented. In the first step, the demand functions of the two firms will be extracted.[3]

Theorem 1: *In a triangular distribution where the consumers are distributed in 0 and L interval, demands of firm 1 and firm 2 will be as follow:*

$$\hat{X} = \frac{L^2\,(p_2 - p_1)}{4\tau\,(2L - a - b)} + \frac{L\,(L - b)}{2L - a - b} + E\,\left(\frac{\lambda\, L}{4\tau\,(2L - a - b)}\right)$$

$$L - \hat{X} = \frac{L^2\,(p_1 - p_2)}{4\tau\,(2L - a - b)} + \frac{L\,(L - a)}{2L - a - b} - E\,\left(\frac{\lambda\, L}{4\tau\,(2L - a - b)}\right)$$

Proof:

Assume that firm 1 is located at a distance of *a* from the source, and firm 2 is located at the right side of firm 1 at a distance of b from L (Figure 1). First, the indifferent consumer should be found among both experienced and inexperienced consumers. First, this will be done on inexperienced consumers. If $\hat{x}_I$ indicates an inexperienced consumer who is indifferent to purchasing from firm 1 and 2 (according to figure 1) (the indifferent consumer will be located where the price and transportation costs for purchasing from two enterprises do not differ), in this case, and as far as $(a < \hat{x}_I < L - b)$ we have:

$$-p_1 - \frac{4x}{L^2}\,\tau\,(\hat{x}_I - a) = -p_2 - \frac{4(L-x)}{L^2}\,\tau\,(L - b - \hat{x}_I) \qquad (4)$$

$$\Rightarrow \quad \hat{x}_I = \frac{L^2\,(p_2 - p_1)}{4\tau\,(2L-a-b)} + \frac{L\,(L-b)}{2L-a-b}$$

[3] All the theorems presented in this section are derived from the results of this paper.

On the other hand, the demand function of indifferent consumer among experienced consumers will be achieved as follow:

If $\hat{x}_E$ indicates an indifferent consumer from experienced ones who is indifferent in purchasing from firms 1 and 2, considering $a < \hat{x}_E < L - b$, we have:

$$-p_1 - \frac{4x}{L^2}\,\tau\,(\hat{x}_E - a) = -p_2 - \lambda - \frac{4(L-x)}{L^2}\,\tau\,(L - b - \hat{x}_E)$$

Here, as explained in the model (3-1 subsections), if the experienced consumer purchases from the firm producing an undesired product, he will be lost up to λ of his achieved desirability. Therefore:

$$\hat{x}_E = \frac{L^2\,(p_2 - p_1)}{4\tau\,(2L-a-b)} + \frac{L\,(L-b)}{2L-a-b} + \frac{\lambda\,L^2}{4\tau\,(2L-a-b)} \tag{5}$$

By summing up these two values equations (4 and 5) and since out of these L consumers, I consumers are inexperienced, and E consumers are experienced (I+E= L), the total demand function for firm 1 will be achieved:

$$\hat{x}_I + \hat{x}_E = \hat{x} = \frac{I}{L}\,\frac{L^2\,(p_2 - p_1)}{4\tau\,(2L-a-b)} + \frac{L\,(L-b)}{2L-a-b} + \frac{E}{L}\left\{\frac{L^2\,(p_2 - p_1)}{4\tau\,(2L-a-b)} + \frac{L\,(L-b)}{2L-a-b} + \frac{\lambda\,L^2}{4\tau\,(2L-a-b)}\right\}$$

Therefore, as I+E= L, we will have:

$$= \frac{L^2\,(p_2 - p_1)}{4\tau\,(2L-a-b)} + \frac{L\,(L-b)}{2L-a-b} + E\left(\frac{\lambda\,L}{4\tau\,(2L-a-b)}\right)\hat{x} \tag{6}$$

Similarly:

$$L - \hat{x} = \frac{L^2\,(p_1 - p_2)}{4\tau\,(2L-a-b)} + \frac{L\,(L-a)}{2L-a-b} - E\left(\frac{\lambda\,L}{4\tau\,(2L-a-b)}\right) \tag{7}$$

The obtained functions show that the demand of firm 1 has a direct relation with the price of firm 2 and reverse relation with the price of the firm itself. In the Hotelling model, the results are as follow:

$$= \left[\frac{p_2 - p_1}{2\tau} + \frac{L-b+a}{2}\right] \quad , \quad L - \hat{x} = \left[\frac{p_1 - p_2}{2\tau} + \frac{L+b-a}{2}\right]\hat{x} \tag{8}$$

In the triangular distribution model of Shahbazi and Salimian (in 0-1 interval), the results were as follow:

$$\hat{X} = \frac{p_2 - p_1}{4\tau\,(2-y_1-y_2)} + \frac{1-y_2}{2-y_1-y_2} \quad , \quad 1 - \hat{X} = \frac{p_1 - p_2}{4\tau\,(2-y_1-y_2)} + \frac{1-y_1}{2-y_1-y_2} \tag{9}$$

In the obtained functions (9 equation), it becomes clear that the demand of firm 1 has reverse relation with its received price (p_1), which means that the demand for the firm decreases with an increase in its price, which is also true for firm 2. These results are consistent with the results of other discussed models (Shahbazi & Salimian, 2017).

The other important point to be mentioned is that the demand of firm 1 has a direct relation with undesirability (λ). It means that the higher is the dissatisfaction of consumers from consuming the food of firm 2 (the firm producing undesired food), the more will be demand for firm 1 (the firm producing desired food). Moreover, it is clear that the demand functions of the two firms also depend on the number of experienced consumers (E). In other words, an increase in the number of experienced consumers will lead to an increase in demand for firm 1 (the firm producing desired food) and a decrease in demand for firm 2 (the firm producing undesired food). Therefore, these results can help the firms increase their market power in a group of target consumers (experienced consumers) (Shy, 1995). Further on, the equilibrium prices will be extracted.

Theorem 2: *In case of the presence of two types of experienced and inexperienced consumers in a triangular distribution where the consumers are distributed in the 0-L interval, Bertrand- Nash equilibrium prices of firms 1 and 2 will be:*

$$p_1 = \frac{\tau\,(12L - 8b - 4\,a)}{3L} + \frac{E\lambda}{3L} \quad , \quad p_2 = \frac{\tau\,(12L - 8a - 4\,b)}{3L} - \frac{E\lambda}{3L}$$

where these equilibrium prices are in respect to increasing transportation costs.

Proof:

It is possible to extract Bertrand- Nash equilibrium prices by extracting profit functions of two firms (according to equations 6 and 7). Firm 1 considered p_2 as given and selects p_1 in a way to maximize its profit. Therefore:

$$\max_{p_1} \pi_1 = \{\frac{L^2\,(p_2 - p_1)}{4\tau\,(2L-a-b)} + \frac{L\,(L-b)}{2L-a-b} + E\left(\frac{\lambda\,L}{4\tau\,(2L-a-b)}\right)\}(p_1) \tag{10}$$

The first-order condition will be:

$$\frac{\partial \pi_1}{\partial p_1} = -\frac{L\,(4b\tau - E\lambda + L(2p_1 - p_2 - 4\tau))}{4\tau\,(a+b-2L)} = 0$$

$$\Rightarrow \qquad p_1 = -\frac{4b\tau - E\lambda - L(p_2 + 4\tau)}{2L}$$

On the other hand, firm 2 considers p_1 as given and selects p_2 in a way to maximize its profit. Therefore, the equilibrium price of firm 2 will be as follow:

$$\frac{\partial \pi_2}{\partial p_2} = 0 \Rightarrow p_2 = -\frac{4a\tau + E\lambda - L(p_1 + 4\tau)}{2L}$$

By replacing these two prices with each other and simplifying them, we will finally have:

$$p_1 = \frac{\tau\,(12L - 8b - 4a)}{3L} + \frac{E\lambda}{3L} \tag{11}$$

$$\frac{\lambda E}{L3} - \frac{)b4 - a8 - L21(\,\tau}{L3} = {}_2p \tag{12}$$

These equilibrium prices are increasing with respect to τ. There are many costs related to searching for a lower price. The searching cost for individuals who have high time value (people who get high values per hour of extra work) is so high; therefore, they will reasonably avoid searching for lower prices and purchase the product from the first shop. On the contrary, the consumers with low searching cost (lower time value) will consider searching for purchase from the firm with lower price as profitable (Shy, 1995). These results will be as equation 13 in the Hotelling model:

$$p_1 = \frac{\tau(3L-b+a)}{3} \;,\; p_2 = \frac{\tau(3L+b-a)}{3} \tag{13}$$

Differences between the results of this study and the results of the Hotelling model are obvious in two parts. Suppose we put aside the second part of the above equations (the presence of experienced consumers). which was previously discussed. In that case, the difference in obtained equilibrium prices in the two methods is such that in the mixed method (different types of consumers and triangular distribution), the price of each firm has reverse relation with its location; however, in Hotelling uniform distribution method, this equation is direct since a is considered as the distance of firm 1 from point zero in both methods; therefore, the obtained result in the mixed method is interpreted in this way that the more a value increases, i.e., the further it becomes from point zero or the closer it becomes to the center, the more it should lower its price. Moreover, by getting away from the center and approaching point zero, the received price of the firm increases, which seems reasonable.

However, these results are not in line with the finding from the Hotelling model, such that in Hotelling uniform distribution, the more a increases, i.e., the closer this firm becomes to the middle points, the more should it increases its price. Moreover, by getting away from the middle points and approaching point zero, the received price of the firm decreases. Finally, as observed, the application of triangular distribution on customer distribution and consideration of experienced and inexperienced consumers is logical in the real world. Besides, the results are more consistent with the realities. One of these realities is the reverse relation between the location of an enterprise and its price, which is in contrast with the results of the Hotelling method. On the other hand, the higher price of firm 1 has a direct relation with the number of experienced consumers and the rate undesirability of consuming undesired food (λ). In other words, this higher price is due to the higher incline of consumers in achieving higher desirability of consuming the food of firm 1 and the number of experienced consumers. Therefore, the firm producing more desirable products could get higher prices as of $\frac{E\lambda}{3L}$. This is an important result since in the real world, it is observed that the firm producing higher quality products can receive more money. It offers higher prices because of the presence of experienced consumers and dissatisfaction (undesirability) of consumers of consuming undesired products of other firms.

If both firms are located at the same point (homogenous productions), $p_1 = p_2 = 0$ is a unique equilibrium. On the other hand, in a Bertrand game with distinct products, the profit of firms will increase by products' differentiation; i.e., the product distinction will increase the monopolistic power of firms producing trade markets by reducing the price competition between them (Shy, 1995). Therefore, the firms will get away from each other, and this confirms the results of the mixed method.

Theorem 3: *The price of each firm is more sensitive to other firms' location than its own location, and the firms reduce their equilibrium prices by approaching the center. Moreover, with the increase in the number of* *experienced consumers, firm 1 receives a higher equilibrium price, and firm 2 receives a lower equilibrium price.*

Proof:

To prove this theorem, it just suffices to differentiate equilibrium prices regarding the locations of firms 1 and 2 (a, b). Therefore, we have:

$$\frac{\partial p_1}{\partial a} = -\frac{4\tau}{3L} \quad , \frac{\partial p_1}{\partial b} = -\frac{8\tau}{3L} \tag{14}$$

On the other hand,

$$\frac{\partial p_2}{\partial a} = -\frac{8\tau}{3L} \quad , \frac{\partial p_2}{\partial b} = -\frac{4\tau}{3L} \tag{15}$$

These results indicate that the price of each firm is more sensitive to the location of other firms than its own location. On the other hand, since *a* is the distance of firm 1 from point zero and b is the distance of firm 2 from L, the more *a* increases (firm 1 approaches the center) or *b* increases (firm 2 approaches the center), the lower should they make their equilibrium prices. To prove the second part of the theorem, just it is required to differentiate the equilibrium prices in respect to the number of experienced consumers (E); therefore, we have:

$$\frac{\partial p_1}{\partial E} = \frac{\lambda}{3L} \quad , \frac{\partial p_2}{\partial E} = -\frac{\lambda}{3L} \tag{16}$$

These relations show that with the increase in the number of experienced consumers, firm 1 receives a higher equilibrium price, and firm 2 receives a lower equilibrium price. On the other hand, the differentiation of the above equations showed that with the increase in undesirability of purchasing an undesired product (λ), firm 1 gets a higher price, and firm 2 gets a lower price.

In what follow, this issue will be studied in what condition do the firms producing desired and undesired product receive the same price?

Theorem 4: *Firms 1 and 2 will get the same equilibrium prices when:*

$$\lambda = \frac{2\tau(b-a)}{E}$$

Proof:

By equating the equilibrium prices of two firms (equations 11 and 12) and solving it in respect to λ, we have:

$$\frac{\tau(12L-8b-4a)}{3L} + \frac{E\lambda}{3L} = \frac{\tau(12L-8a-4b)}{3L} - \frac{E\lambda}{3L} \tag{17}$$

$$\Rightarrow \lambda = \frac{2\tau(b-a)}{E}$$

This expression shows that only if two firms are located at the same distance from the center will they have the same equilibrium prices. In this condition, the firm cannot get a higher price from experienced consumers since it is impossible to differentiate it from an inexperienced consumer. In addition, this equation shows that the undesirability of consumers has a direct relation with transportation costs and location of firm 2 and reverse relation with the location of firm 1 and the number of experienced consumers. In this state, the higher number of experienced consumers will cause decreased undesirability. Moreover, if this equation shows that if $b > a$ (firm 2 is closer to center), in other words, the producer of undesired food is closer to the center, λ (undesirability) will increase since firm 2 was the producer of undesired food. In addition, if $a > b$ (firm 1 is closer to the center), in other words, the producer of desired food is closer to the center λ (undesirability) will decrease since firm 1 is the producer of desired food. This could be explained so that the economy of a restaurant has high commonality with the economy of consistency and standardization. This relation is due to the influence of social conditions on the demand of a consumer, which is influenced by the selection of a restaurant by another consumer (Becker, 1991). Moreover, Becker argues that social relations influence the demand for restaurants, coffee shops, sports clubs, etc. However, it does not influence on orange demand since the demand for recreational spaces is different from the demand for orange (Becker, 1974).

Theorem 5: *If the location of both firms is symmetric (a=b), then the equilibrium prices of the two firms will be:*

$$p_1 = \frac{\tau\,(12L - 12a)}{3L} + \frac{E\lambda}{3L} \quad , \quad p_2 = \frac{\tau\,(12L - 12a)}{3L} - \frac{E\lambda}{3L}$$

And the firms will get different market shares. In addition, the firm producing a higher-quality brand (firm 1) will demand a higher price.

Proof

Replacing a=b in equilibrium prices (equations 11 and 12) will yield the following results:

$$p_1 = \frac{\tau\,(12L-12a)}{3L} + \frac{E\lambda}{3L} \quad , \quad p_2 = \frac{\tau\,(12L-12a)}{3L} - \frac{E\lambda}{3L} \tag{18}$$

In the Hotelling model and all previous location models[4] (except types of consumers by (Shahbazi and Salimian, 2018), if two firms had symmetric location (a=b), then they would receive the same prices (τL). Here, the difference in prices is due to the difference in consumers' incline toward the foods presented by the two firms. Firm 1, which produces more desirable food, gets a higher price. In the real world, it is seen that the firms such as restaurants, coffee shops, etc., get different prices even if they have symmetric location in respect to center. This is because of market power obtained from group consumers and undesirability achieved from consuming the undesired product by consumers. If we replace the symmetric situation of two firms in-demand functions of firms 1 and 2, the result will be as follow:

$$\hat{x} = \frac{L}{2} + \frac{EL\lambda}{24\tau\,(L-a)} \quad , \quad L - \hat{x} = \frac{L}{2} - \frac{EL\lambda}{24\tau\,(L-a)} \tag{19}$$

These are different from the results of the Hotelling model, where if two firms had the same location, each would get the same market share ($\frac{L}{2}$). This difference in result is due to the differentiation in the consumers' incline toward consuming two products which are due to higher incline of consumers toward consuming food provided by firm 1 compared to firm 2; and therefore, its

[4] Salop circular city (article entitled: Monopolistic Competition with Outside Goods), Lijesen and Reggiani (article entitled: Location Choice in the Spoke Model), Shahbazi and Salimian (Triangular Distribution), Hotelling (Stability in Competition), and etc.

demand will be higher. Indeed, if the number of consumers is so high, $\frac{E\lambda}{6L\tau}$ value in both functions will be zero, and the demand of both firms will be equal. In this condition, the firms demand equal prices.

In the horizontal differentiation model, when the transportation costs are linear, the firms are willing to move toward the center (Principle of minimum differentiation); however, in the vertical differentiation model (quality), the principle of maximum differentiation will be true. The difference is due to the fact that in the vertical differentiation model, the firms gain expertise in the production of quality for a certain group of consumers, and they could increase their market price in the target consumers' group (Barbot, 2013). In what follows, by replacing the equilibrium prices through the backward induction method, the optimal location of firms in case of triangular distribution with different types of consumers will be extracted.

Theorem 6: *There is a subgame perfect Nash equilibrium in location, and this optimal location is as follow:*

$$a = -\frac{8b\tau - E\lambda - 12L\tau}{4\tau} \quad , \quad b = -\frac{8a\tau + E\lambda - 12L\tau}{4\tau}$$

The optimal location in respect to τ is increasing for firm 1 and decreasing for firm 2.

Proof

In order to achieve the optimal location of firms, it is just required to form the profit function of two firms and differentiate in respect to their location. The profit functions of firm 1 and 2 (according to equations 6 and 11 and also 7 and 12), will be respectively achieved as follow:

$$\pi_1 = (\hat{x})(p_1), \ \pi_2 = (L - \hat{x})(p_2)$$

$$\pi_1 = \left(\frac{L^2 (p_2 - p_1)}{4\tau (2L - a - b)} + \frac{L (L - b)}{2L - a - b} + E \left(\frac{\lambda L}{4\tau (2L - a - b)} \right) \right) (p_1)$$

$$\pi_2 = \left(\frac{L^2 (p_1 - p_2)}{4\tau (2L - a - b)} + \frac{L (L - a)}{2L - a - b} - E \left(\frac{\lambda L}{4\tau (2L - a - b)} \right) \right) (p_2)$$

Then by differentiating and putting equal to zero, we have:

$$a = -\frac{8b\tau - E\lambda - 12L\tau}{4\tau} \quad , \quad b = -\frac{8a\tau + E\lambda - 12L\tau}{4\tau} \qquad (20)$$

On the other hand, the following equations will be achieved by differentiating the equilibrium locations with respect to transportation costs:

$$\frac{\partial a}{\partial \tau} = -\frac{E\lambda}{4\tau^2} \quad , \quad \frac{\partial b}{\partial \tau} = \frac{E\lambda}{4\tau^2} \qquad (21)$$

It is clear that optimal location *a* is increasing concerning transportation costs, and optimal location *b* is decreasing with respect to transportation costs. These results indicate that increasing transportation costs will make firm 1 closer to the center and firm 2 far from the center. As far as firm 1 produced high-quality food, in case of an increase in transportation costs, it can attract a higher percentage of consumers by moving toward the center and maximizing its profit. Since in this condition, the inexperienced consumers will lead to increased demand of firm 1 by minimizing their transportation costs and experienced users by minimizing the total transportation costs and undesirability of consuming foods they don't prefer.

In this condition, it is better for firm 2 to get far away from firm 2 and go to margin to maximize its profit. The cost minimization condition by users will proceed firm 2 to make such a decision.

Theorem 7: *In a triangular distribution with a diversity of consumers where they are distributed in the 0-L interval, the profit of firm 1 increases by getting closer to the center if the following condition is established:*

$$\frac{L\,(E\lambda + 4L\tau - 4b\tau)}{\tau} \geq 0$$

And the profit of firm 2 increases by moving toward the center when:

$$\frac{L\,(12L\tau - E\lambda - 12a\tau)}{\tau} \geq 0$$

Proof

As far as the profit function of firm 1 is in the form of equation 10, by differentiation in respect to *a*, it becomes clear that the following conditions should be established to make the profit of this firm positive, i.e.:

$$\frac{\partial \pi_1}{\partial a} \geq 0 \iff \frac{L\,(E\lambda+4L\tau-4b\tau)}{\tau} \geq 0 \tag{22}$$

And for firm 2, the following condition should be established:

$$0 \geq \frac{(\tau a21-\lambda E-\tau L21)\,L}{\tau} \iff 0 \geq \frac{{}_2\pi\partial}{b\partial} \tag{23}$$

This means that in each location *a* and *b*, firm 1 could increase its profit by moving toward firm 2 to achieve greater market share (the principle of minimum differentiation) since the firms produce products of minimum differentiation by moving toward the center. The above conditions declare that if firm 1 gets extremely close to firm 2, there would be no equilibrium, and if it is exactly at the point of firm 2, its profit will decrease (for the presence of experienced consumers, the profit will not be zero) which indicates that it should move to the left side (Shy, 1995).

Here, moving toward other firms is for the attraction of more inexperienced consumers. The profit of the firm will decrease for this reason because the competition of firms to attract inexperienced consumers will make the firms enter the price combat. In the end, this low price should be taken from experienced consumers, too (profit will be zero) and it is better for them to get far away from each other. Therefore, in order to have equilibrium, the firms cannot be so close to each other (Shy, 1995 & LaFountain, 2005).

Reggiani (2009) presented the conditions for price-location equilibrium from the spoke model at the time of product delivery where all spokes are not occupied by the firms. The results indicated that in equilibrium conditions, one of the firms supplies all products while others focus on their niche. Anderson & Neven (1986) showed that if the transportation costs are sufficiently convex, the equilibrium will include some distinctions between firms.

5. Concluding Remarks

One of the most effective factors in the decision-making of individuals and firms is location. The location is defined as the selection of location for one or several firms with consideration of other firms and the existing constraints so that a special target will be optimized. This target could be transportation cost, getting more profit, presenting fair services to customers, getting the highest market share and etc. As far as making the policies for the establishment of industries without any knowledge will lead to the elimination or reduction of the efficiency of the economic system, the significance of location studies becomes highlighted. In this paper, two deficiencies or weaknesses of models in simplifying the location models in respect of consumers' distribution and types of consumers have been studied and discussed.

As far as similar price conditions, some consumers prefer a product to another, two types of consumers, experienced and inexperienced, have been considered. Moreover, since in the real world, the distribution of consumers is not uniform in the street or city. Usually, the city centers are more populated than margins, the assumption of uniform distribution of consumers was studied through triangular distribution method, and more comprehensive results were achieved.

The results of the mixed method (types of consumers and triangular distribution) showed that in case of the presence of experienced and inexperienced consumers in a triangular density function, the demand functions of two firms depend on the achieved desirability of a certain type of food and the number of experienced consumers (the increase in the number of experienced consumers will increase demand for firm 1 and decrease it for firm 2). Moreover, there is a unit Nash equilibrium in prices, and these equilibrium prices are increasing with respect to transportation costs. Moreover, if two firms are located at the same place, due to differences in inclines of consumers, they will not demand the same equilibrium prices. With the increase in transportation costs, firm 1 approaches the center, and firm 2 gets far away from the center, and in the end, the price of each firm is more sensitive to the location of other firms than its location.

Finally, because the demand function of firms, profit, price, and equilibrium value of firms depend on the types of consumers (experienced and inexperienced) and the manner of consumers' distribution, the firms (especially restaurants, airlines, etc.) with high quality and desirable products, are recommended to raise their price through increasing the number of experienced consumers, approaching the center or when the producer of low-quality product gets closer to the center. The opposite is true for producers of undesired products.

Another important point for firms in the above-mentioned mixed method is that the symmetrical situation of firms is not the reason for presenting uniform prices. Therefore, the producers of desired goods or services are recommended to get higher prices in case of presenting better and high-quality products (for the presence of experienced consumers). Moreover, it is recommended that with an increase in transportation costs, the producer of desired products or services approaches the center, and the producer of undesired products and services gets away from the center. It is also suggested that in future research, more realistic assumptions, such as streets with unequal length and other statistics distribution of consumers, etc., should be considered to determine the optimal location choice of firms.

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