Iran Agricultural Research, Vol. 31, No.1, 2012
Printed in the Islamic Republic of Iran
Shiraz University

Scaling and Fractal Concepts in Saturated Hydraulic Conductivity: Comparison of Some Models

B. GHAHRAMAN[1*], S. OMIDI[1] and A. KHOSHNOOD YAZDI[2*]**

[1]Department of Irrigation, Ferdowsi University of Mashhad, I.R. Iran
[2]Department of Soil Science, Shirvan Agriculture College, Shirvan, I.R. Iran

Received 25 December 2010, Accepted 8 November 2011, Available online November 7, 2012

ABSTRACT- Measurement of soil saturated hydraulic conductivity, K_s, is normally affected by flow patterns such as macro pore; however, most current techniques do not differentiate flow types, causing major problems in describing water and chemical flows within the soil matrix. This study compares eight models for scaling K_s and predicted matrix and macro pore K_s, using a database composed of 50 datasets. The database includes data regarding K_s, soil bulk density, particle size distribution, with textures ranging from sandy loam to clay. The results showed that among the models tested, the Saxton and Kozeny Carman models performed best for estimating soil K_s using scaling techniques. In contrast, Campbell and Cosby models were not suitable for similar K_s scaling method. Generally, Saxton, Kozeny Carman, Poulsen Saxton, Vereecken, and the Brakensiek models gave the best estimation of soil K_s. Furthermore, all models had smaller estimation deviations for loam soils than for clay loam soils. The results also showed that a sample with the average characteristics of all samples should be taken as a reference point when scaling K_s is used. Overally, the Saxton and Kozeny Carman models are recommended for scaling K_s. The performance of a simple fractal model was not suitable neither for matrix nor for macro pore hydraulic conductivity.

Keywords: Fractal, Macro-pore, Saturated hydraulic conductivity, Scaling

INTRODUCTION

Many hydrologic structural designs are based on the results of hydrologic modeling. At present, a number of estimation and numerous laboratory and field methods are available for the direct measurement of hydraulic conductivity (9, 6, 11, 17, 16). Saturated hydraulic conductivity (K_s) is a necessary key parameter for analyzing or modeling water flow and chemical transport in subsurface soil. Moreover, a measured value of K_s is usually required as a matching factor in predicting the unsaturated hydraulic conductivity function from soil water characteristic data (24,

---

*Professor, Ph. D. Student and Instructor
** Corresponding author

26, 27). This parameter is also needed for predicting unsaturated hydraulic conductivity from a large number of the existing one-parameter models (4, 18).

Currently, there are four approaches that can be used to estimate K_s:

1- estimating K_s generally by multiple nonlinear regression analysis, based on statistics from easily obtainable soil physical and chemical properties, i.e., pedotransfer functions (20);

2- developing a physico-empirical relationship between the particle size distribution and K_s. In this category, the Campbell (5) approach is based on the assumption that the particle size distribution is approximately lognormally distributed and can be represented by a geometric mean particle size (d_g) and the geometric standard deviation (σ);

3- using scaling techniques, which can be used to estimate soil hydraulic properties at different locations in a watershed by measuring these properties at one representative location and limited data at another location (1). Miyazaki (15) initiated a nonsimilar media concept (NSMC) of scaling K_s of soils with different bulk densities. The weak point of Miyazaki's model, the shape factor characterizing NSMC, however, has received an analytic treatment for estimating the shape factor τ by Zhuang et al. (28);

4- using a reserved for fractal approach. Rawls et al. (19) modified the Marshall (12) saturated hydraulic conductivity equations using the fractal properties of the Sierpinski carpet (e.g. 23) to predict matrix K_s.

Although the first two methods used for estimating K_s perform well in many cases, there seems to be no superior and generally applicable model in these categories. Therefore, attention must first be drawn to the homogeneity of the data, i.e.; **a:** all samples must use the same determination method; and **b:** there must be a model for estimating K_s that is commonly calibrated on individual data sets. The scaling and fractal techniques are thought to be significant for solving these problems because some systemic measurement deviation or some of the empirical coefficients in the regression expressions fitted using measured data can be avoided by using it.

Recent concern about groundwater pollution from agricultural activities has increased the need to develop physically based models with the ability to predict the movement of water and chemicals into and through the soil media. An added complication to such models is the existence of large pores, commonly known as macro-pores, in the soil matrix. When surface ponding of water occurs, water can flow directly through the surface-connected macro-pores to the subsoil, bypassing much of the soil matrix. The measurement of K_s is typically affected by both macro-pore and matrix flow; however, most techniques do not differentiate between these types of flow, causing major problems in describing the movement of water and chemicals in the soil. Knowledge of the matrix and macro-pore K_s is critical in describing field-scale flow process (2). Therefore, the main objectives of this study are:

1- to compare the prediction quality of different K_s models using scaling techniques; and

2- to predict matrix and macro-pore saturated hydraulic conductivity using fractal principles.

MODELS DESCRIPTION

Scaling models

Matrix K_s

NSMC model

Miyazaki (15) presented a scaling model of K_s theoretically based on NSMC, as follows:

$$\frac{K_s}{K_{so}} = \left[(\tau\rho_s/\rho_b)^{1/3} - 1\right]^2 \left[(\tau_o\,\rho_{so}/\rho_{bo})^{1/3} - 1\right]^{-2} \tag{1}$$

where K_{so} is the measured K_s of a reference sample with bulk density ρ_{bo}, K_s is the estimated saturated hydraulic conductivity of a soil sample with a bulk density ρ_b, and ρ_s and ρ_{so} are particle densities of soils investigated and referenced, respectively. Shape factor τ was calculated by Zhung et al. (28), as follows:

$$\tau = \left[\frac{\rho_{bo}}{\rho_b}\right]^{\varepsilon} \left[1 + \left(\frac{\rho_s}{\theta_r\rho_s + \rho_b} - 1\right)\exp\left(d_g - d_g\left(1 - \theta_r - \rho_b/\rho_s\right)^{-\rho b}\right)\right]^{-1} \tag{2}$$

where

$$\theta_r = 0.015 + 0.005C + 0.014\rho_b \tag{3}$$

And

$$\varepsilon = \left([\rho_s - \rho_b]/[\rho_{so} - \rho_{bo}]\right)^{0.5} \tag{4}$$

In Eq. (3), C is the percentage content of clay particle (<2μm) of soil. In Eq. (2), d_g is the geometric mean particle diameter in mm, calculated as follows:

$$d_g = \exp\left(\sum m_i \ln d_i\right) \tag{5}$$

where d_i is the arithmetic mean diameter for particle-size class i with a mass, m_i. By using Campbell's values, d_i is 0.001 mm for clay, 0.026 mm for silt and 1.025 mm for sand.

Campbell and Kozeny-Carman Models

Campbell (5) proposed an equation based on particle size distribution data and the SMC of Miller and Miller (14) for estimating K_s. To describe the dependence of the K_s of a soil on its bulk density and particle size distribution, Campbell's model has been rewritten as follows (29):

$$\frac{K_s}{K_{so}} = \left[\left(1.3^{b-bo}\right)\left(\frac{\rho_{bo}{}^{bo}}{\rho_b{}^{b}}\right)\right]^{1.3} \exp\left[-6.9(C - C_o) - 3.7(U - U_o)\right] \tag{6}$$

where C and C_o are percentage contents of clay particle (<2µm) of soil samples investigated and referenced, respectively, U and U_o are percentage contents of silt (2-50 µm) of soil samples investigated and referenced, respectively. b and b_o are parameters related to pore-size distributions of soil samples investigated and referenced, respectively, calculated by the following formula:

$$b = d_g^{-0.5} + 0.2\sigma_g \tag{7}$$

where σ_g is the geometric standard deviation of particle diameter.

Another model for K_s estimation is the Kozeny-Carman equation, which was given a scaling form by Miyazaki (15):

$$\frac{K_s}{K_{so}} = [\rho_{bo}/\rho_b]^2 [(\rho_s - \rho_b)/(\rho_{so} - \rho_{bo})]^3 \tag{8}$$

Cosby and Vereecken Models

For the function of Cosby et al. (8), its scaling form is

$$\frac{K_s}{K_{so}} = 10^{0.126(S-S_O)-0.0064(C-C_O)} \tag{9}$$

where S and S_o indicate percentage content of sand (50-2000µm) of soil samples investigated and referenced, respectively.

For the function of Vereecken et al. (25), its scaling expression is

$$\frac{K_s}{K_{so}} = \exp\left[-0.96\ln\left(\frac{C}{C_o}\right) - 0.66\ln\left(\frac{S}{S_o}\right) - 0.46\ln\left(\frac{OM}{OM_o}\right) - 8.43(\rho_b - \rho_{bo})\right] \tag{10}$$

where OM and OM_o represent contents of organic matter of soil samples investigated and referenced, respectively.

Brakensiek and Saxton Models

The expression of Brakensiek et al. (3) is transformed into

$$\frac{K_s}{K_{so}} = \exp(a) \tag{11}$$

$$\begin{aligned} a = {} & 19.52348(\phi - \phi_o) - 0.028212(C - C_o) + 0.00018107(S^2 - S_o^2) \\ & - 0.0094125(C^2 - C_o^2) - 8.395215(\phi^2 - \phi_o^2) + 0.077718(\phi S - \phi_o S_o) \\ & - 0.00298(S^2\phi^2 - S_o^2\phi_o^2) - 0.019492(C^2\phi^2 - C_o^2\phi_o^2) + 0.0000173(S^2 C - S_o^2 C_o) \\ & + 0.02733(C^2\phi - C_o^2\phi_o) + 0.001434(S^2\phi - S_o^2\phi_o) - 0.0000035(C^2 S - C_o^2 S_o) \end{aligned} \tag{12}$$

where ∅ and $∅_o$ are the total porosity of soil samples investigated and referenced, respectively.

The scaling form for the function of Saxton et al. (22) is identical to Eq. (11), except that

$$a = -0.0755(S - S_o) + \frac{-3.895 + 0.03671S - 0.1103C + 0.00087546C^2}{0.332 - 0.0007251S + 0.1276\log_{10} C} - \frac{-3.895 + 0.03671S_o - 0.1103C_o + 0.00087546{C_o}^2}{0.332 - 0.0007251S_o + 0.1276\log_{10} C_o} \quad (13)$$

Poulsen-Saxton Model

Poulsen et al. (18) presented a model for estimating K_s. Its scaling form is

$$\frac{K_s}{K_{so}} = [(\theta_s - \theta_e)/(\theta_{so} - \theta_{eo})]^{3.15}[\theta_{so}/\theta_s]^{2.10} \quad (14)$$

where θ_s and θ_{so} are the saturated volumetric water content of soil samples investigated and referenced, respectively. θ_e and θ_{eo} are the volumetric water content of soil samples investigated and referenced, respectively, at -10kPa of water potential. To avoid the difficulty of measuring θ_s and θ_e, and to make Eq. (14) ready for predicting K_s by means of easily measured physical properties of soil, Zhuang et al. (29) replaced these two parameters by the values estimated by means of the function of Saxton et al. (22):

$$\theta_s = 0.332 - 0.0007251S + 0.1276\mathrm{Log}_{10}C \quad (15)$$

and

$$\theta_e = \exp\left(\frac{2.302 - \ln A}{B}\right) \quad (16)$$

with

$$A = 100\exp\left(-4.396 - 0.0715C - 0.000488S^2 - 0.00004285CS^2\right) \quad (17)$$

and

$$B = -3.14 - 0.00222C^2 - 0.00003484CS^2 \quad (18)$$

Eq. (14) is the combination of Eqs. (15) to (18) and called the Poulsen-Saxton model.

Fractal model

Rawls et al. (21), based on 11 different soil textures from literature (20), modified the Marshal K_s equation by using the fractal properties of the Sierpinski carpet to predict matrix saturated conductivity, K_s, (in cm/hr):

$$K_s = 4.41\times10^7\left(\frac{\phi^x}{n^2}\right){R_1}^2 \quad (19)$$

where R_1 is the largest equivalent pore radius for the Sierpinski carpet, x is the pore interaction exponent, and n is the total pore size classes. In order to use Eq. (19) to predict the matrix K_s, the parameters x and n need to be determined for matrix flow. Rawls et al. (21) assumed $x = \frac{4}{3}$, as proposed by Milington and Quirk (13). The largest equivalent pore radius for the Sierpinski carpet, R_1, was calculated from the capillary rise equation, following the methodology of Tyler and Wheatcraft (23):

$$R_1 = \frac{o.148}{h_b} \tag{20}$$

where h_b is the geometric mean bubbling pressure (cm). Geometric mean bubbling pressures for different soil texture classes are reported by Rawls et al. (21) (esp. Table 1). Rawls et al. (21) adopted the following equation for computing n in Eq. (19):

$$n = \frac{m\phi}{(\phi - \theta_1)} \tag{21}$$

where θ_1 is the water content at -33 kPa (1) and m is equal to 12 as proposed by Marshall, (12).

Macro-pore saturated hydraulic conductivity
Rawls model

Eq. (19) is equally valid to predict the macro-pore K_s. Rawls et al. (21) adopted $x = \frac{4}{3}$, while they proposed the following equation for computing n corresponding to macro-pore flow:

$$n = -5.7 + 77.0\,R_1 \tag{22}$$

Chu model

Chu (7) developed a capillary-tube infiltration model, based on Brooks-Corey parameters, which distinguishes between water flows in large pores from the flow in small pores. Eq. (23) describes the relationship between saturated hydraulic conductivity, K_s, and scale factor which is included in the parameter K_b:

$$K_b = K_s \frac{2 + 3f}{(\theta_s - \theta_r)f} \tag{23}$$

where K_b is the maximum pore conductivity, *f* is the pore size index (dimensionless), and θ_r is the residual volumetric water content (dimensionless).

Models Performance

The goodness of the models for scaling the K_s of various soils was evaluated by deviation times (DT), which were calculated as follows by Zhuang et al. (28):

$$\mathrm{Log}_{10}^{\mathrm{DT}} = \left\{ n^{-1} \sum_{i=1}^{n} \left[\mathrm{Log}_{10}\left(K_s^p / K_s \right) \right]^2 \right\}^{0.5} \tag{24}$$

where K_s, K_s^p are saturated hydraulic conductivity measured and predicted by the scaling models, respectively. A larger DT value means greater estimation deviation, and lower efficiency of the model.

DATABASE

Surface (0-30 cm) disturbed and un-disturbed samples were taken from different regions in the north of Iran (Amol, Babol and Karaj) (10). Samples were taken on grid points with equal distances during spring 1991. Disturbed samples were first air dried and then passed through 2-mm mesh sieves. For each sample, saturated hydraulic conductivity was measured by a constant head apparatus in 5 replications. Soil textures were determined using the standard routine method, after elimination of gypsum and organic matter (USDA, 1982). Dry bulk density was determined in triplicates after the samples were dried in an oven with a temperature of 105°C up to reaching a constant weight. A constant value of 2.65 (g.cm^{-3}) was adopted for the soils particle density. A statistical view of physical properties of the soils is presented in Table 1. Although of relatively broad soil textures, sandy loam to clay, are included in this dataset, the majority of which being loam and clay loam. The soil organic mater contents are changing from a low (0.34%) to high value (3.36%). Capillary rise equation is generally valid in non-swelling soils. Khoshnood Yazdi (10) confirmed that non-expansive clay minerals are dominant in the sample.

Table 1. Some physical properties of soils used in this study (10)

soils tex., org., sat. hyd. conduc. and bulk density							Soil moisture at defined matric potential (kPa)					
	sand	silt	clay	OM	K_s	ρ_b	0	-5	-33	-100	-500	-1500
Factor	%				cm.h^{-1}	g.cm^{-3}	%					
Maximum	65.8	52	56	3.36	6.40	1.63	78.5	55.3	39.7	35.1	37.6	26.9
Minimum	14.8	37.2	14	0.34	0.32	1.37	36.6	33.0	20.0	16.3	11.5	9.3
Average	38.3	34.2	27.4	1.50	2.52	1.47	47.4	41.8	30.8	25.1	19.2	16.3
SD^{+}	8.6	5.5	7.6	0.70	1.25	0.05	6.7	4.2	4.1	3.7	3.1	3.4
CV^{++}	22.6	16.0	27.8	46.6	49.6	3.5	11.6	10.0	13.3	14.7	16.2	20.9

$^{+}$ **Standard deviation**
$^{++}$ **Coefficient of variation**

RESULTS AND DISCUSSION

Scaling models

Estimation Accuracy

Figure 1 shows how well the seven scaling models performed in scaling K_s of all soil textures by using average bulk density as a reference point to corresponding equations. Table 2 lists DT values of the eight models in scaling K_s (by using average bulk density as a reference point) of the 50 soil samples from Babol, Amol, and Karaj. From these results, it can be seen that different models had different accuracy in scaling K_s. Generally speaking, considering all soil textures, Saxton models had the smallest estimation deviation in terms of the mean DT, but some models were superior to the Saxton model for some specific soils (data not shown).

The second best model was the Kozeny-Carman. The Cosby model had larger DT value compared with other models (Fig. 1).

Table 2. Average values of deviation times (DT) of K_s estimated by various scaling methods and selection of different scenarios as a reference point. Bold numbers in each line are due to minimum DT among different models

Texture	Saxton	Kozeny-Carman	Poulsen-Saxton	NSMC	Vereeken	Brakensiek	Cosby	Campbell
A: average bulk density								
all	**1.792**	1.973	2.115	2.169	2.314	2.079	8.045	6.45E+22
Loam	1.688	1.497	**1.351**	1.580	1.993	1.965	3.446	3.02E+10
Clay loam	1.408	1.627	**1.372**	1.693	1.827	1.816	4.156	1.72E+12
B: minimum bulk density								
all	**1.705**	2.608	2.116	3.607	2.342	2.814	139.3	2.70E+53
Loam	1.514	1.576	**1.338**	1.712	1.914	1.976	3.901	2.77E+10
Clay loam	1.395	1.617	**1.366**	1.725	2.080	2.931	4.276	3.71E+15
C: maximum bulk density								
all	**2.003**	2.441	2.554	2.360	3.500	2.146	30.51	1.67E+34
Loam	1.928	1.821	**1.373**	1.691	3.197	2.012	3.419	3.47E+11
Clay loam	**1.529**	1.645	1.896	2.279	2.810	1.819	8.474	5.26E+17

Our analysis showed that the Campbell model failed to present a fair result. For example, for soil No. 11 (sand=42.8%, silt=32%, clay=25.2%, ρ_b=1.53 g.cm^{-3}, b=15.57, Kso=2.52 cm.hr^{-1}) we found that Ks=3.6E+10 cm.hr^{-1} (average silt and clay are 34.9% and 22.4%, respectively, $\bar{\rho}$=1.46 g.cm^{-3}, $\bar{b}$=1.84) which yields Ks/Ks$_o$ in the order of 1.43×10^{10}. Soil No. 1 was even worse (Ks/Ks$_o$= 9.35×10^{-13}). Other research (e.g. 29) does not, however, support such discrepancy. For the Campbell model, the large estimation deviation, especially when scaling K_s of clayey soils, may presumably be resulted from the unfeasibility of use of the parameter, b, because b tends to become unusually large in the case of clayey soils.

For loam and clay loam soils, when scaling K_s, the large and small estimation deviations were attributed to the Cosby model and P-S models, respectively. Unlike other models, the Cosby, Saxton, and Poulen-Saxton models do not consider bulk density or porosity as a factor in the scaling processes; thus, their sometimes superior performances, especially for the Saxton and Poullsen-Saxton models, should be attributed to the better quality of the empirical regression pedo-transfer functions (22, 18). Nevertheless, values of the DT could presumably be reduced by integrating additional independent variables, such as bulk density and organic matter content of soil, into the K_s models. Therefore, NSMC, Kozeny-Carman, Vereecken, and Brakensiek models should be recommended for investigations of K_s in fields or watersheds or as foundations for approaches to developing new models for estimating K_s in the future.

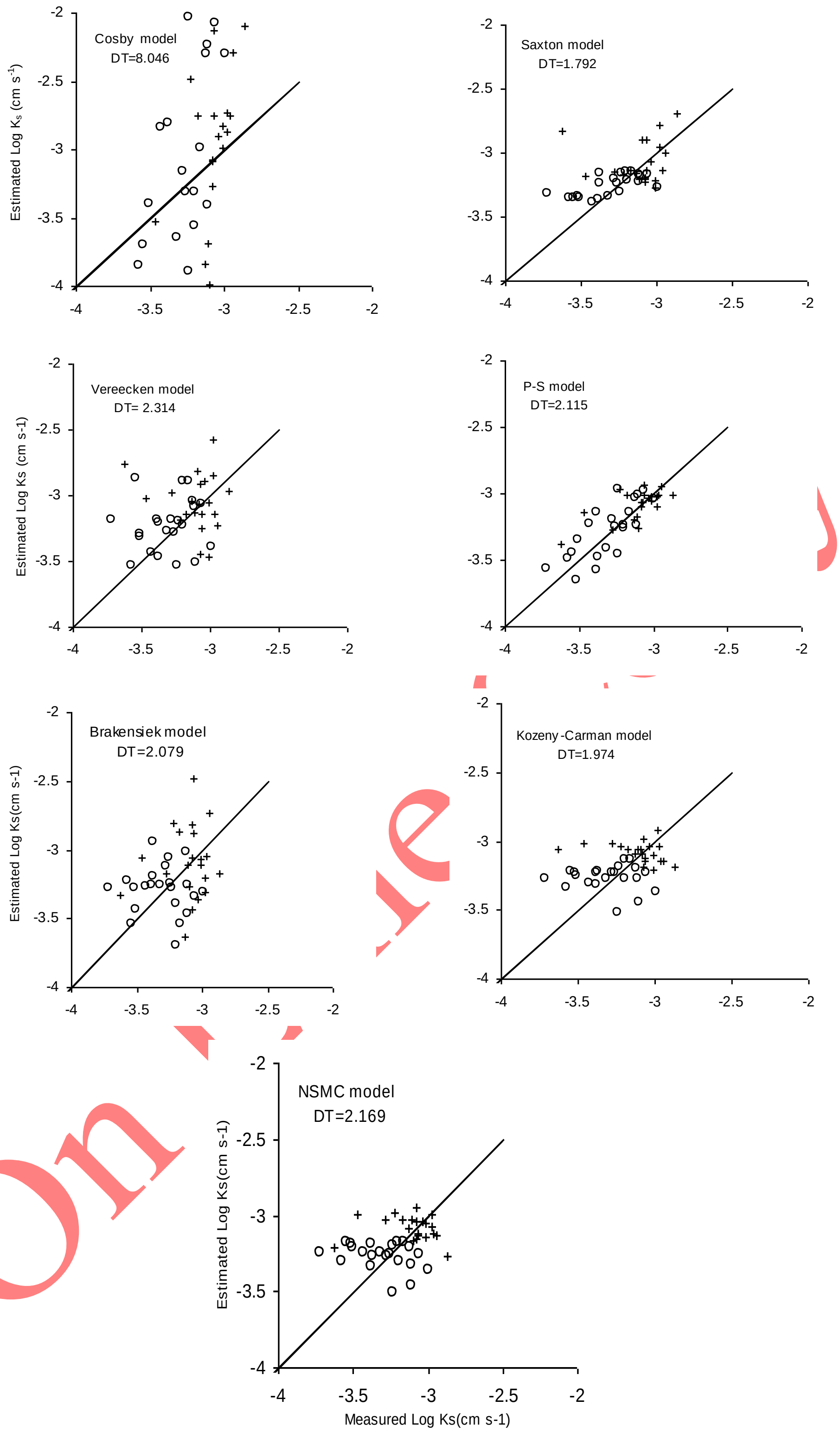

Fig. 1. Comparision of values of Log K_s measured and estimated by the seven models. (+) Loam texture, (o) Clay loam texture. Solid line represents the 1:1 line

Selection of Reference Point of Scaling

The scaling process needs a measured reference point. Defining this reference point reduces any systematic error in prediction of K_s. It is, however, difficult to define a suitable reference point and also to evaluate the consequences of adopting alternative reference points. Considering different bulk densities for identical soil textures of soil samples, one can better judge how to define reference points. Three criteria on adopting a reference point were considered. Table 2 compares differences caused by using average, minimum, and maximum bulk density values as reference points to scale K_s from various models.

On defining minimum bulk density of the soil sample, Saxton and Cosby models performed best and worst among different scaling models. In comparison, when using a reference point whose bulk density approximates to the average bulk density of the soil samples for loam texture soils, values of DT for all models decreased, while for clay loam texture soils only DT values of Saxton, Kozeny-Carman, and Poulsen-Saxton models increased. DT values for other models also decreased. It can easily be seen from Table 2 that scaling efficiency of all models decreased, as maximum bulk density was adopted as a criterion, but the Saxton model still performed better than the others. In summary, for Saxton models, it would be better to select soil samples with minimum bulk density or any samples whose bulk density is smaller than the average bulk density of all samples as a reference point. For Kozeny-Carman, Poulsen-Saxton, NSMC, and Vereecken models, it seemed better to use a sample whose bulk density approximates the averaged value of all samples as a reference point when scaling K_s. These findings confirm that there is no unique method for defining reference point for scaling, as far as different models are concerned. There is support for this result in the literature as well as reported by Zhuang et al. (29).

Adaptability of the Scaling Models to Textures

Most of the soil samples were loam and clay loam (10). Since hydraulic models are usually texture dependent, adaptability of the eight scaling models to soil textures was examined. For evaluation, the parameter $\text{Log}(K_s^p/K_s)$ was used. Results are listed in Table 3 against the geometric mean diameter (d_g) of soil particles. Evidently, most of the eight models performed better for soil textures such as silt loam or loam. In Table 3, a point worth noting is that all seven models had smaller deviation in estimating the K_s of soils with loam texture. The literature neither supports nor rejects this finding, however, the pore size, shape, and orientation of soils, especially heavy soils, depend substantially on the soil structural regime and bulk density. Soil structures can produce many coarse pores and are, as a result, prime factors influencing hydraulic conductivity. Therefore, incorporating bulk density into the specific matching factors is undoubtedly conducive to reducing the variability of the K_s predicted by the individual models. This was demonstrated further by comparing the results of the models in this study. Thus, NSMC, Kozeny-Carman, Vereecken, and Brakensiek models should be recommended for use in modeling practices of soil hydraulics, or for preferential improvements in the future. Incorporating soil structural regimes into the present models for K_s estimation is an open domain.

Table 3. Values of Log (K_s^p/K_s) of eight models for soil with different d_g

d_g mm	Saxton	Kozeny-Carman	Poulsen-Saxton	NSMC	Vereecken	Brakensiek	Cosby
0.02-0.03	0.026	0.023	0.021	0.021	0.071	-0.017	0.024
0.03-0.04	0.014	0.010	0.004	0.009	0.022	0.002	0.010
0.04-0.05	0.064	0.055	0.022	0.053	0.077	0.052	0.056
0.05-0.06	0.007	0.004	-0.006	0.002	0.031	0.001	0.005
0.06-0.15	0.024	0.015	0.0004	0.015	0.045	0.003	0.016

***Different values of dg are equivalent to different soil textures, for example, 0.02-0.03 mm Silt to Clay loam; 0.03-0.04 mm- Silty loam; 0.04-0.05 mm- Sandy clay; 0.05-0.06 mm- Loam; 0.06-0.15 mm Sandy clay loam to Sandy loam**

Fractal approach in Matrix saturated hydraulic conductivity, K_s

Rawls et al. (21) have used geometric mean bubbling pressure, h_b, for different soil texture classes adapted from Rawls et al. (20) (e.g. h_b=11.15 and 25.89 cm for loam and clay loam, respectively). As a result, R_1 would be constant for each soil texture (e.g. 0.013 and 0.006 cm for loam and clay loam, respectively). There is no firm reason, however, for assuming a constant h_b, and therefore, a constant R_1 for every soil texture class. So we used $h_b = 1/\alpha$, where α is one of the fitting parameters of the van Genuchten SMRC model. Through this approach, a specific R_1 value can be computed for each soil sample. The results for loam soils showed that R_1 changes between 0.002 and 0.029 cm (with a mean and standard deviation of 0.011 and 0.008 cm, respectively). While for clay loam soils, the variation of R_1 was between 0.005 and 0.032 cm (with a mean and standard deviation of 0.015 and 0.007 cm, respectively). The results for loam soils showed that n changes between 23.4 and 37.4 (with a mean and standard deviation of 26.8 and 3.6, respectively). While for clay loam soils, the variation of n was between 21.8 and 31.3 (with a mean and standard deviation of 26.2 and 2.4, respectively). A comparison is made between measured (horizontal axis) and predicted matrix K_s (vertical axis) in a logarithmic scale in Figure 2. Constant R_1, as proposed by Rawls et al. (21) performed much better than variable R_1. Loam soils resemble somewhat better than clay loam ones. The other soil textures showed similar results (data not shown). DT values for the fractal model corresponding to two soil textures of loam and clay loam are listed in Table 4. As this Table is compared with Table 2, one can conclude that scaling models are performing much better than the fractal model. Further research is needed, however, after sophisticated fractal models are developed.

Table 4. Values of deviation times (DT) of K_s estimated by various scaling methods and selection of different scenarios as a reference point

Texture	Fractal model	
	Variable R1	Constant R1
Loam	7.403	1.415
Clay loam	3.330	3.568

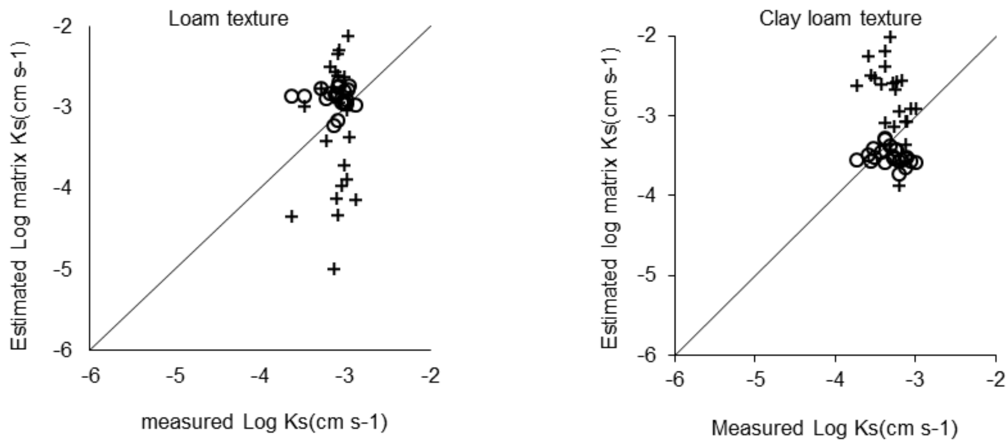

Fig. 2. Results for calculation of matrix of saturated hydraulic conductivity for loam and clay loam soils. (+) Variable R_1, (o) Constant R_1. Solid line represents the 1:1 line

Macro-pore Saturated Hydraulic Conductivity
Fractal approach conductivity, K_s

Equation [22], reported by Rawls et al. (21), yields a negative n for $R_1 \leq 0.0740$ cm which is unrealistic. Based on the data of Rawls et al. (21), another regression equation was established to compute n, imposing zero intercept, as follows:

$$n = 60.135 R_1, \qquad r=0.89 \qquad (25)$$

Combining (19) and (25), yields in:

$$K_s = 12195.1\phi^{4/3} \qquad (26)$$

which is independent of R_1. Macro-pore saturated hydraulic conductivity from a fractal concept and a capillary-tube infiltration model are plotted against the measured K_s in Fig. 3. It seems that the results of the fractal concept are not realistic. So a more sophisticated fractal approach is needed.

CONCLUSIONS

To reduce systematic errors resulting from measuring methods or the inherent uncertainty of the models for estimating K_s, the reliability of the eight models for scaling K_s was tested. Saxton and Poulsen-Saxton models generally performed best for all textural classes. On the other hand, Campbell and Cosby models were found unsuitable for a scaling practice. The results of this study showed it would be better to select a sample with average characteristics as a reference point, but for Saxton (22) and Kozeny-Carman (15) models, when the texture is clay loam, the sample with minimum bulk density should be taken as a reference point.

The measurement of saturated hydraulic conductivity is typically affected by both macro-pore and matrix flow. This study showed that the simple fractal model of

Rawls et al. (21) is not suitable for macro-pore analysis, and a more sophisticated fractal model is needed. Preferential movement of fluid takes place in maximum pore.

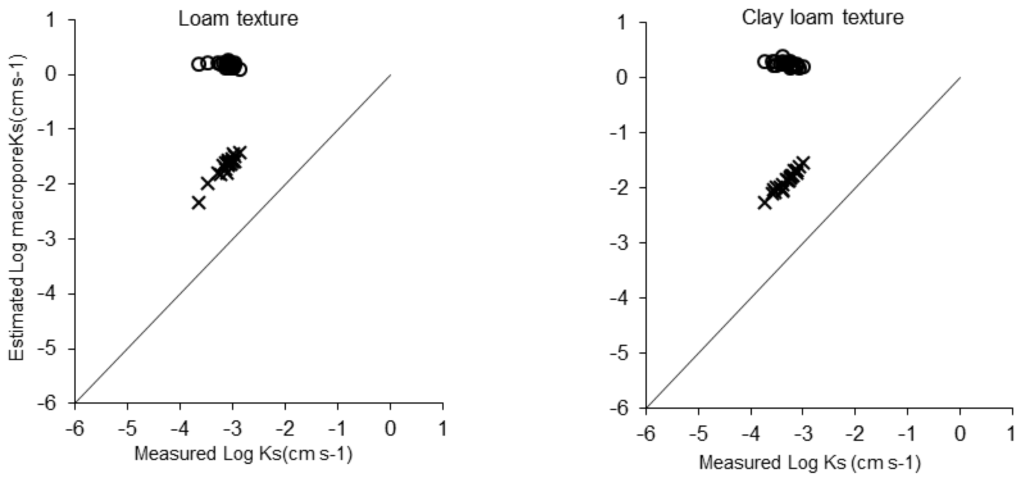

Fig. 3. Results of calculation macropore saturated hydraulic conductivity for loam and clay loam soils. (o) Rawls model, (×) Chu model. Solid line represents the 1:1 line

REFERENCES

1. Ahuja, L. R., J. W. Naney and D.R. Nielsen. 1984. Scaling soil water properties and infiltration modeling. Soil Sci. Soc. Am. J. 48: 970-973.

2. Ahuja, L. R., R. R. Bruce, D. K. Cassel and W. J. Rawls. 1988. Simpler field measurement and estimation of soil hydraulic properties and their spatial variability for modeling. ***In:*** Proc. Int. Symp. Modeling Agric., For., Rangeland Hydrol., Chicago. Dc. W. J. Rawls (***ed.***) 1988. ASAE, St. Joseph, MI.pp. 19-32.

3. Brakensiek, D. L., W. J. Rawls and G.R. Stephenson. 1984. Modifying SCS hydrologic soil groups and curve numbers for rangeland soils. ASAE paper No. PNR-84-203, st. Joseph, MI.

4. Campbell, C. S. 1974. A simple method for determining unsaturated conductivity from moisture retention data. Soil Sci. 117: 311-314.

5. Campbell, C. S. 1985. Soil Physics with basic: Transport models for Soil-Plant Systems. Elsevier, Amsterdam.

6. Carey, S. K., W. L. Quinton and N.T. Goeller. 2007. Field and laboratory estimates of pore size properties and hydraulic characteristics for subarctic organic soils. Hydrol. Process. 21: 2560–2571.

7. Chu, S. T. 1994. Capillary-Tube infiltration model with Brooks-Corey parameters. ASAE. 37(4): 1205-1208.

8. Cosby, B. J., G. M. Homberger, R. B. Clapp and T. R. Ginn. 1984. A statistical exploration of the relationship of soil moisture characteristics to the physical properties of soils. Water Resour. Res. 20: 682-690.

9. Gupta, R. K., R. P. Ruda, W. T. Dickinson, N.K. patni and G.j. Wall. 1993. Comparison of saturated hydraulic conductivity measured by various field methods. Trans. ASAE 36: 51-55.

10. Khoshnood Yazdi, A. 1991. Prediction of soil moisture retention curve from physical properties for some Iranian soils. M.Sc. Thesis, College of Agriculture, Tehran University, 140p (In Persian).

11. Lilly, A., A. Nemes, W. J. Rawls and Y. A. Pachepsky. 2008. Probabilistic approach to the identification of input variables to estimate hydraulic conductivity. Soil Sci. Soc. Am. J. 72 (1): 16-24.

12. Marshall, T. J. 1958. A relationship between permeability and size distribution of pores. Soil Sci. 9: 1-8.

13. Milington, R. J. and J. P. Quirk 1961. Permeability of porous solids. Trans. Faraday Soc. 57: 1200-1206.

14. Miller, E. E. and R. D. Miller 1956. physical theory for capillary flow phenomena. Appl. Phys. 27: 324-332.

15. Miyazaki, T. 1996. Bulk density dependence of air entry suctions and saturated hydraulic conductivities of soils. Soil Sci. 161: 484-490.

16. Motaghian, H. R. and J. Mohamadi. 2011. Spatial estimation of saturated hydraulic conductivity from terrain attributes using regression, kriging, and artificial neural networks. Pedosphere 21(2):170–177.

17. Petersen C. T., A. Trautner and S. Hansen. 2008. Spatio-temporal variation of anisotropy of saturated hydraulic conductivity in a tilled sandy loam soil. Soil & Tillage Research. 100:108–113

18. Poulsen , T. G., P. Moldrup and O. H. Jacobsen. 1998. One-parameter models for unsaturated hydraulic conductivity. Soil Sci. 163: 425-435.

19. Rawls, W. J. 1983. Estimating soil bulk density from particle size analyses and organic matter content. Soil Sci. 135:123-125.

20. Rawls, W. J., D. L. Brakensiek and K. E. Saxton. 1982. Estimation of soil water properties. Trans. ASAE 25:1316-1320 and 1328.

21. Rawls, W. J., D. L. Brakensiek, and S. D. Logsdon, 1993. Predicting saturated hydraulic conductivity utilizing fractal principles. Soil Sci. Soc. Am. J. 57: 1193-1197.

22. Saxton, K. E., W. J. Rawls, J.S. Romberger and R. I. Papendick. 1986. Estimating generalizing soil water characteristics from texture. Soil Sci. Soc. Am. J. 50(4): 1031-1036

23. Tyler, S. W. and S. W. Wheatcraft. 1990. Fractal processes in soil water retention. Water Resour. Res. 26: 1047-1054.

24. van Genuchten, M. Th. 1980. A closed-form equation for predicting the hydraulic conductivity of unsaturated soils. Soil Sci. Soc. Am. J. 44: 892-898.

25. Vereecken, H., J. Maes and J. Feyen. 1990. Estimating unsaturated hydraulic conductivity from easily measured soil properties. Soil Sci. 149: 1-12.

26. Zand Parsa, Sh. and A. R. Sepaskhah. 2004. Soil hydraulic conductivity function based on specific liquid-vapour interfacial area around the soil particles. Geoderma. 119: 143-157.

27. Zand-Parsa, Sh. 2006. Improved soil hydraulic conductivity function based on specific liquid–vapour interfacial area around the soil particles. Geoderma. 132: 20-30.

28. Zhuang, J., G. R. Yu, T. Miyazaki, and K. Nakayama. 2000b. Modeling effects of compaction on soil hydraulic properties—a NSMC scaling method for saturated hydraulic conductivity. Adv. Geoecol. 32: 144-153.

29. Zhuang, J., K. Nakayama, G. R. Yu and T. Miyazaki. 2000a. Scaling of saturated hydraulic conductivity: A comparison of models. Soil Sci. 165(9): 718-727.

مفاهیم مقیاس‌بندی و فرکتال در هدایت هیدرولیکی اشباع خاک: مقایسه چند مدل

بیژن قهرمان[1*]، سمیرا امیدی[1] و اصغر خوشنود یزدی[2*]**

[1] بخش مهندسی آب، دانشکده کشاورزی، دانشگاه فردوسی مشهد، جمهوری اسلامی ایران

[2] بخش خاکشناسی، دانشکده کشاورزی شیروان، جمهوری اسلامی ایران

چکیده - در این مقاله هشت مدل برای مقیاس کردن هدایت هیدرولیکی اشباع خاک، K_s و K_s ماتریکس و منافذ درشت خاک با استفاده از اطلاعات در ۵۰ نقطه انجام شده است. این اطلاعات شامل K_s، چگالی ظاهری خاک، توزیع اندازه ذرات، و با محدوده بافتی لوم تا لوم رس بود. نتایج نشان داد که با استفاده از اصول مقیاس بندی، مدل‌های ساکستون و کوزنی کارمن بهترین عملکرد را در برآورد K_s داشتند. از طرف دیگر، مدل‌های کمبل و کازبای در مقیاس سازی K_s موفق نبودند. به طور کلی، مدل-های ساکستون، کوزنی کارمن، پائولسن ساکستون، وریکن، و براکنزیک منجر به بهترین برآورد برای K_s خاک گشتند. در مقایسه با خاک‌های لوم رسی، خاک‌های لومی در تمامی مدل‌ها انحراف کمتری داشتند. همچنین نتایج نشان داد که در مقیاس‌سازی K_s نمونه ای که از میانگین تمامی خاک ها استفاده شود، می‌تواند به عنوان نقطه مرجع انتخاب گردد. در نتیجه برای مقیاس کردن K_s، مدل‌های ساکستون و کوزنی کارمن پیشنهاد می‌شود . کارایی مدل ساده فرکتال برای هیچکدام از هدایت هیدرولیکی ماتریکس و منافذ درشت مناسب نبود.

واژه های کلیدی: فرکتال، مقیاس سازی، منافذ درشت، هدایت هیدرولیکی اشباع

* به ترتیب استاد، دانشجوی دکتری و مربی خاکشناسی

** مکاتبه کننده