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A Comparative Study on Constitutive Modeling of Hot Deformation Flow Curves in AZ91 Magnesium Alloy

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Abstract: Modeling the flow curves of materials at elevated temperatures is the first step in mathematical simulation of the hot deformation processes of them. In this work, a comparative study was provided to examine the capability of three different constitutive equations in modeling the hot deformation flow curves of AZ91 magnesium alloy. As such, the Arrhenius and exponential equations with strain dependent constants, and a recently developed simple model (developed based on a power function of Zener-Hollomon parameter and a third order polynomial function of ε power of a constant number) were examined. Root mean square error (RMSE) criterion was used to assess the modeling performance of the examined constitutive equations. Accordingly, it was found that the Arrhenius equation with strain dependent constants has the best performance for modeling the hot deformation flow curves of AZ91 magnesium alloy. The results can be further used in mathematical simulation of hot deformation manufacturing processes of tested alloy.

Keywords: Constitutive equations, Hot deformation processes, Arrhenius equation, Exponential equation, AZ91 magnesium alloy

1. Introduction

High strength to weight ratio and excellent castability are the main characteristics of Mg alloys that make them a good candidate for transportation industries applications [1-4]. However, low workability of these alloys at room temperature is a restricting factor in their manufacturing processes. Thus, almost,all manufacturing processes of Mg alloys are conducted at high temperatures [1–4]. Modeling the flow curves of materials at elevated temperatures is the first step in mathematical simulation of the manufacturing processes of them. Consequently, various constitutive equations have been proposed to model the flow stress of different materials [5-9]. As explained by Lin and Chen [10], the constitutive equations can be divided into three categories including: phenomenological models, physical-based models (models which consider the mechanism of deformation such as dislocation dynamics and thermal activation) and artificial neural network (ANN) models [10]. Among these, phenomenological constitutive models are widely used in mathematical simulation of metal forming processes. In these models the flow stress of a material is expressed as a function of the forming temperature, strain-rate and strain. In the other words, in these models a mathematical function with some constants is fitted to the experimental flow curves of tested material. Arrhenius equation [11-13], exponential equation [14] and some newly developed constitutive models are some examples of this category [15-19].

The aim of the present work is to evaluate the modeling performance of a recently developed simple model for describing the hot flow curves of AZ91 magnesium alloy. This model has been recently developed based on a power function of Zener-Hollomon parameter and a third order polynomial function of strain power m (m is a constant)) and has been used to describe the flow stress of API X65 pipeline

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steel at elevated temperatures [17]. Here, the results of this model for the description of the flow stress of AZ91 magnesium alloy is compared with the results of the other two constitutive equations including the Arrhenius equation with strain dependent constants and the exponential equation with strain dependent constants. Root mean square error (RMSE) criterion is used to evaluate the modeling performance of examined models.

2. Hot compression tests

To conduct the hot compression tests, cylindrical specimens with 10 mm diameter and 15 mm length were machined from the ingot of AZ91 alloy with the chemical composition of 9.2Al, 0.8 Zn, 0.22Mn, 0.077Si (all in wt%). The tests were performed on a 250 kN Zwick tensile/compression testing machine equipped with a radiant furnace with the temperature accuracy of ±5 °C. Before the tests, the specimens were held in the furnace at 420 °C for 24 h [20-21] to dissolve β ($Mg_{17}Al_{12}$) precipitates and to homogenize the structure. After soaking for 3 minutes, the hot compression tests were conducted at temperatures of 325, 350, 375, 400 and 425 °C with different strain rates of 0.01, 0.1 and 1 s^{-1} for each of the deformation temperatures under true strain of about 0.5.

3. Results and discussion

In this section the results of the hot compression tests conducted at different deformation conditions are presented at the first. Then the results of the examined constitutive equations to describe the flow curves of AZ91 magnesium alloy at hot working conditions are presented and compared with each other.

3.1. The experimental flow curves

The experimental flow curves of AZ91 magnesium alloy obtained at different deformation conditions are shown in Fig. 1. As can be seen, the flow stress increases to a peak value and then gradually falls to a steady state stress which is an indication of the occurrence of dynamic recrystallization (DRX) and precipitate coarsening [22–24]. Moreover, as expected, the flow stress increases with an increase in strain rate and a decrease in deformation temperature.

3.2. Arrhenius and exponential equations with strain dependent constants

As explained elsewhere [15], equations in which the Zener–Hollomon parameter (Z) is considered as a function of stress can be used to describe the effects of temperature and strain rate on flow stress:

$$Z = \dot{\varepsilon} \exp\left(\frac{Q}{RT}\right) = f(\sigma) \quad (1)$$

where Q is the activation energy (kJ/mol), R is the universal gas constant, and T is the absolute deformation temperature. The hyperbolic sine (Arrhenius) law, the power law and the exponential law are alternatives of $f(\sigma)$ and are defined according to Eqs. (2) to (4), respectively:

$$Z = f(\sigma) = A[\sinh(\alpha\sigma)]^{\mathrm{n}} \quad (2)$$

$$Z = f(\sigma) = A'\sigma^{n'} \quad (3)$$

$$Z = f(\sigma) = A''\exp(\beta\sigma) \quad (4)$$

where $A, \alpha, n, A', n', A''$ and β are material constants. The description of flow stress by the above equations are not complete and should be rewritten for a characteristic stress (i.e. for the peak stress) or a stress corresponding to a certain strain (for example the stress corresponding to the strain of 0.3) [17, 25-26]. However, for flow stress modeling, it is suggested that the constants of constitutive equations should be expressed as polynomial functions of strain to compensate the effect of strain [26]. For this, the constitutive equations should be established for stresses corresponding to strains in a predefined interval and step size at the first and then regression analysis should be used to fit a polynomial function over the obtained constants [10, 17 and 25].

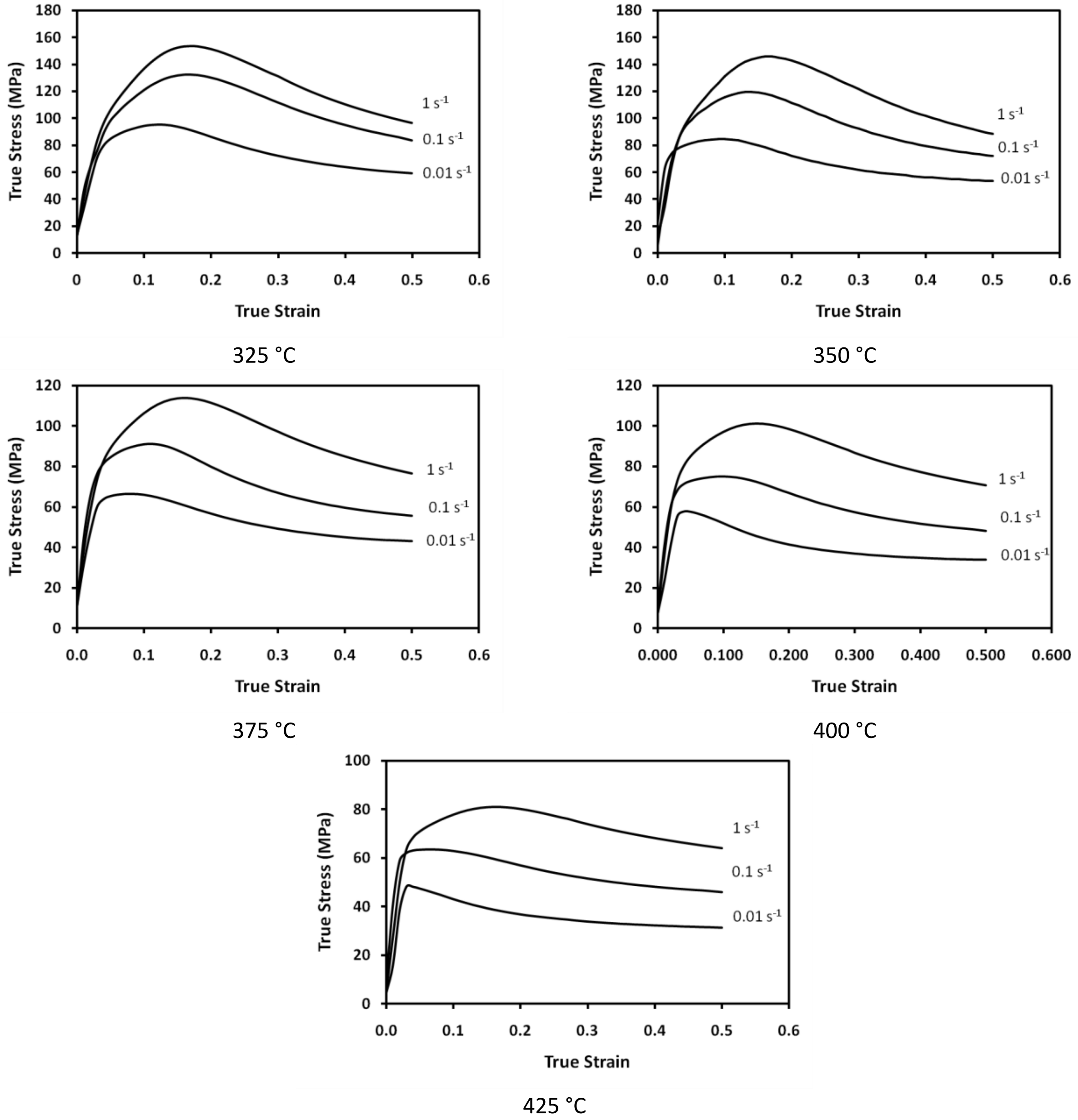


Fig. 1. Experimental flow curves of AZ91 magnesium alloy obtained at different deformation conditions [20].

Substituting $f(\sigma)$ from Eqs. (2)-(4) to Eq. (1) and taking natural logarithm, the following expressions could be derived respectively:

$$ln\dot{\varepsilon} + \frac{Q}{R}\left(\frac{1}{T}\right) = lnA + n\, ln[\sinh(\alpha\sigma)] \quad (5)$$

$$ln\dot{\varepsilon} + \frac{Q}{R}\left(\frac{1}{T}\right) = lnA' + n' ln\sigma \quad (6)$$

$$ln\dot{\varepsilon} + \frac{Q}{R}\left(\frac{1}{T}\right) = lnA'' + \beta\sigma \quad (7)$$

Taking the partial differentiations from the above equations (Eqs. (5) to (7)) gives the following relations, respectively:

$$\partial(ln\dot{\varepsilon}) + \frac{Q}{R}\partial\left(\frac{1}{T}\right) = n\, \partial ln[\sinh(\alpha\sigma)] \quad (8)$$

$$\partial ln\dot{\varepsilon} + \frac{Q}{R}\partial\left(\frac{1}{T}\right) = n'\partial ln\sigma \quad (9)$$

$$\partial ln\dot{\varepsilon} + \frac{Q}{R}\partial\left(\frac{1}{T}\right) = \beta\partial\sigma \quad (10)$$

For deriving the Arrhenius equation with strain dependent constants, the following procedure should be demonstrated to find the values of material constants. This equation for individual stresses corresponding to the strains in a predefined interval and step size (in this work, for the stresses correspond to different strains in the range of 0.05 to 0.5 with the step size of 0.05):

1) As proposed in Ref. [15], the optimum value of excess as an unknown variable of α in Eq. (8) should be determined from the following relationship:

$$\alpha = \beta/n' \quad (11)$$

where the values of n' and β can be obtained using the $ln\dot{\varepsilon}$-lnσ and $ln\dot{\varepsilon}$-σ plots, respectively (as the result of writing the Eqs. (9) and (10) for temperature constant and $\dot{\varepsilon}$ constant conditions, respectively) [25]. The average slopes obtained from these plots are considered as the values of n' and β, respectively.

2) The value of *n* should be obtained from the $ln\dot{\varepsilon}$-ln[sinh(ασ)] plot (as the result of writing the Eq. (8) for temperature constant condition). The average slope obtained from this plot is considered as the value of n.

3) The value of Q should be obtained from the ln[sinh(ασ)]-1/T plot (as the result of writing the Eq. (8) for $\dot{\varepsilon}$ constant condition). The average slope obtained from this plot should be multiplied by R*n factor to obtain the value of Q [Eq. (8)].

4) Rewriting the Eq. (5) for the tested deformation conditions (with different temperatures and strain rates) and substituting the obtained values of α, n and Q, an optimization procedure should be used to find the proper value of lnA.

According to the obtained values of α, n, Q and lnA for stresses corresponding to different strains (obtained by repeating the stages 1 to 4), regression analysis can be used to express the obtained constants as polynomial functions of strain. Substituting these material constants as functions of strain, the following equation can be used to model the flow stress of tested material [17]:

$$\sigma = \frac{1}{\alpha} ln\left\{(Z/A)^{1/n} + \left[(Z/A)^{2/n} + 1\right]^{1/2}\right\} \quad (12)$$

This equation can be obtained from the Eq. (5) [17].

In this research, the stages of 1 to 4 was repeated to obtain the values α, n, Q and lnA at stresses corresponding to different strains in the range of 0.05 to 0.5 with the step size of 0.05. The results are presented in Fig. 2.

As depicted in this figure, the regression analysis was used to express the obtained constants as polynomial functions of strain. The results are summarized as in the follow:

$$\alpha = 0.518\varepsilon^4 - 0.746\varepsilon^3 + 0.375\varepsilon^2 - 0.062\varepsilon + 0.015 \quad (13)$$

$$n = -293.0\varepsilon^3 + 310.5\varepsilon^2 - 100.1\varepsilon + 14.49 \quad (14)$$

$$Q = 11479887\varepsilon^4 - 14{,}137{,}096\varepsilon^3 + 6358104\varepsilon^2 - 1339535\varepsilon + 286328 \quad (15)$$

$$lnA = 2281.237\varepsilon^4 - 2760.780\varepsilon^3 + 1206.529\varepsilon^2 - 243.762\varepsilon + 48.668 \quad (16)$$

Substituting the materials constants as functions of strain, Eq. (12) was used to model the flow stress of AZ91 magnesium alloy. A comparison between the experimental and calculated flow stresses (using the Arrhenius equation) at deformation conditions with two temperatures of (a) 375 and (b) 400 °C with different strain rates are presented in Figs. 3a and 3b, respectively.

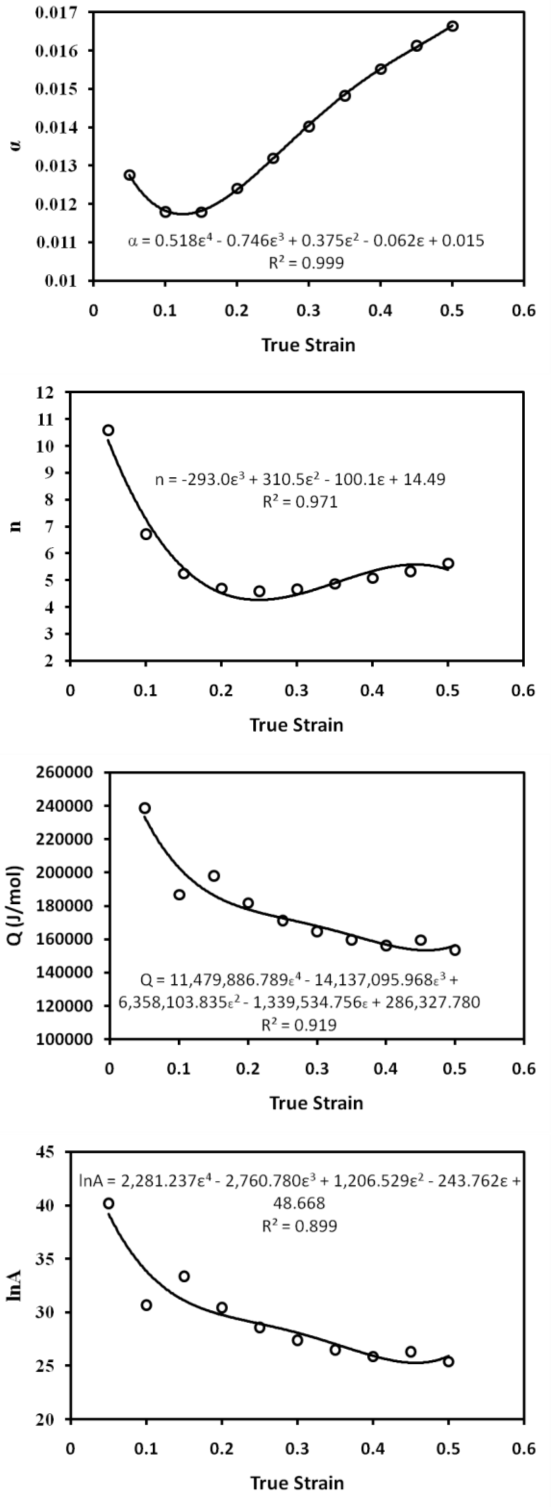
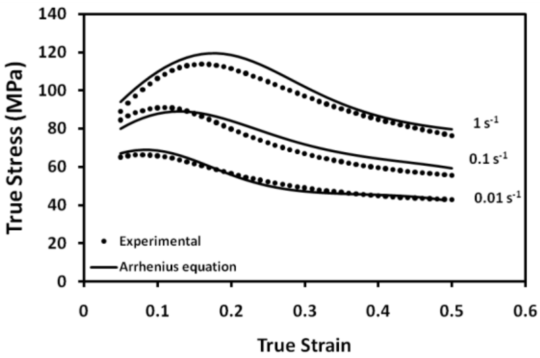
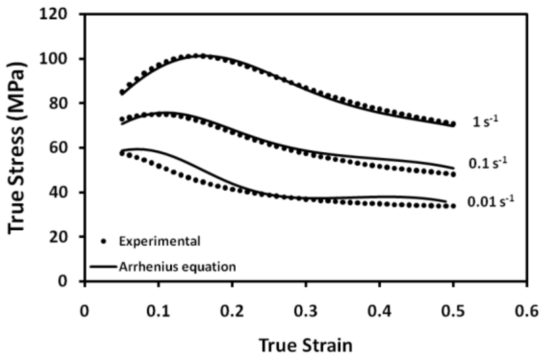


Fig. 2. Hot working constants of Arrhenius equation as functions of true strain.





(a) (b)

Fig. 3. The comparison between the experimental and modeled flow curves (using the Arrhenius equation with strain dependent constants) at deformation conditions with two temperatures of (a) 375 and (b) 400 °C with different strain rates.

According to Eq. (10), to derive the exponential equation with strain dependent constants, the following procedure should be demonstrated to find the values of material constants of β and A'' together with the values of activation energy (Q) for stresses corresponding to strains with a predefined interval and step size:

1) The value of β should be obtained from the ln$\dot{\varepsilon}$-σ plot (as the result of writing the Eq. (10) for temperature constant condition). The average slope obtained from this plot is considered as the value of β.
2) The value of Q should be obtained from the σ -1/T plot (as the result of writing the Eq. (10) for $\dot{\varepsilon}$ constant temperature condition). The average slope obtained from this plot should be multiplied by β*R factor to obtain the value of Q (Eq. (10)).
3) Rewriting the Eq. (7) for the examined deformation conditions (with different temperatures and strain rates) and substituting the obtained values of β and Q, an optimization procedure should be used to find the proper value of lnA''.

According to the obtained values of β, Q and lnA'' for stresses corresponding to different strains (conducting the above procedure), regression analysis can be used to express the obtained constants as polynomial functions of strain. Substituting these material constants as functions of strain, the following equation can used to model the flow stress of tested material [14]:

$$\sigma = (ln\dot{\varepsilon} + \frac{Q}{RT} - lnA'')/\beta \quad (17)$$

This equation can be obtained from the Eq. (7).

In this research, the stages of 1 to 3 was repeated to obtain the values β, Q and lnA'' at stresses corresponding to different strains in the range of 0.05 to 0.5 with the step size of 0.05. The results are presented in Fig. 4.

As depicted in this figure, the regression analysis was used to express the obtained constants as polynomial functions of strain. The results are summarized as in the follow:

$$\beta = -5.913\varepsilon^3 + 6.204\varepsilon^2 - 1.886\varepsilon + 0.251 \quad (18)$$

$$Q = -937464.653\varepsilon^3 + 1181186.145\varepsilon^2 - 564241.074\varepsilon + 258298.730 \quad (19)$$

$$lnA'' = 181.5\varepsilon^3 - 161.9\varepsilon^2 + 21.02\varepsilon + 26.16 \quad (20)$$

Substituting the materials constants as functions of strain, Eq. 17 was used to model the flow stress of AZ91 magnesium alloy. A comparison between the experimental and calculated flow stresses (using the exponential equation) at deformation conditions with two temperatures of (a) 375 and (b) 400 °C with different strain rates are presented in Figs. 5a and 5b, respectively.

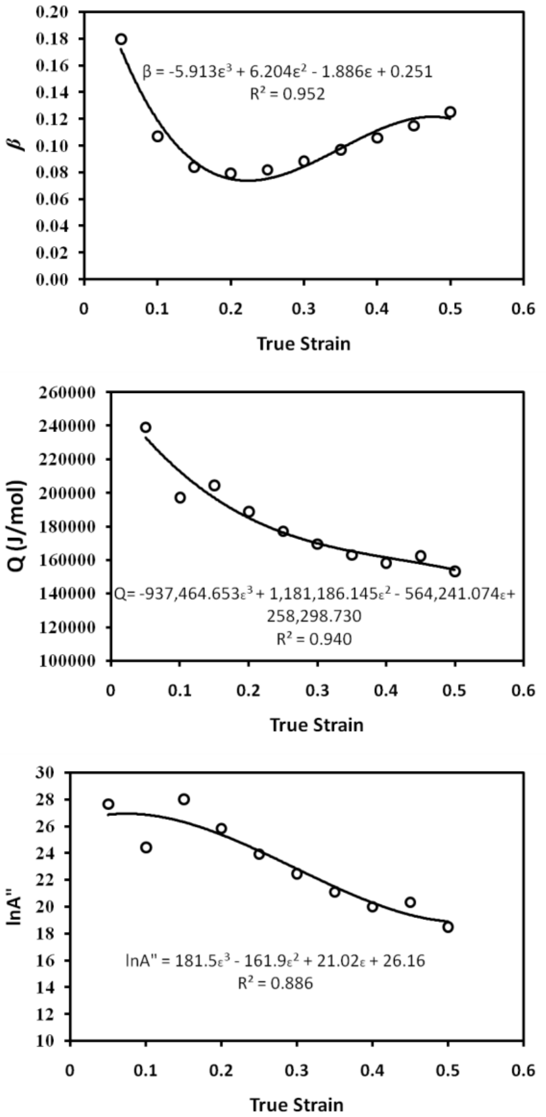
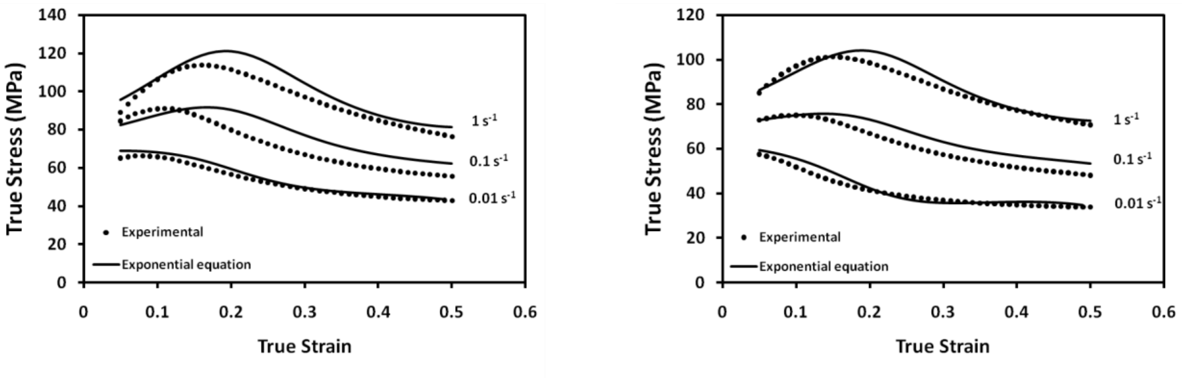


Fig. 4. Hot working constants of exponential equation as functions of true strain.



(a) (b)

Fig. 5. The comparison between the experimental and modeled flow curves (using the exponential equation with strain dependent constants) at deformation conditions with two temperatures of (a) 375 and (b) 400 °C with different strain rates.

3.3. Recently developed constitutive equation

As explained in Ref. [16], the flow stress of different materials at hot working conditions can be described by the constitutive equations with the general form of:

$$\sigma = f(\varepsilon, \dot{\varepsilon}, T) = f(\dot{\varepsilon}, T) \times f(\varepsilon) \quad (21)$$

where the first term is selected to compensate the effects of temperature and strain rate and the second term is selected to compensate the effect of strain. Accordingly, a new phenomenological constitutive equation has been developed by Rakhshkhorshid [17] that in it a power function of Zener-Hollomon parameter has been used to compensate the effects of temperature and strain rate on the flow stress. Moreover, a third order polynomial function of ε power m (a constant number) has been proposed to compensate the effect of strain. Hence, the following equation has been proposed as a constitutive equation to model the flow stress of different materials at hot working conditions [17]:

$$\sigma = Z^q \times (a + b\varepsilon^m + c\varepsilon^{2m} + d\varepsilon^{3m}) \quad (22)$$

In addition, it is suggested that the constants of Eq. (22) can be obtained through a Newtonian optimization procedure so as to minimize the sum squared error between the modeled and experimental flow curves. This model has been previously applied to describe the flow stress of API X65 pipeline steel [17]. In this study, minimizing the sum squared error between the modeled and experimental flow curves for the fifteen flow curves of AZ91 magnesium alloy (see Fig. 1). The following equation was derived to describe the flow stress of this alloy at hot working conditions:

$$\sigma = \dot{\varepsilon}^{\,0.122} exp(\frac{0.122\times 144998}{RT}) \times (-3.333 + 41.335\varepsilon^{0.4} - 74.551\varepsilon^{0.8} + 41.322\varepsilon^{1.2}) \quad (23)$$

A comparison between the experimental and modeled flow curves at deformation conditions with the temperatures of (a) 375 and (b) 400 °C with different strain rates are presented in Figs. 6(a) and 6(b), respectively. As can be seen in Figs. 6(a) and 6(b), the predicted peak strains for all the strain rates are almost the same. This is one of the restrictions of this model over the other considered constitutive equations.

3.4. Modeling performance of the examined constitutive equations

Root mean squared error (RMSE) criterion was used to evaluate the modeling performance of examined constitutive equations:

$$\mathrm{RMSE} = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(t_i - y_i)^2} \quad (24)$$

where t_i is the target output and y_i is the model output. The RMSE values obtained for the fifteen flow curves of AZ91 magnesium alloy (Fig. 1) by the constitutive equations examined in this work is presented in Table 1.

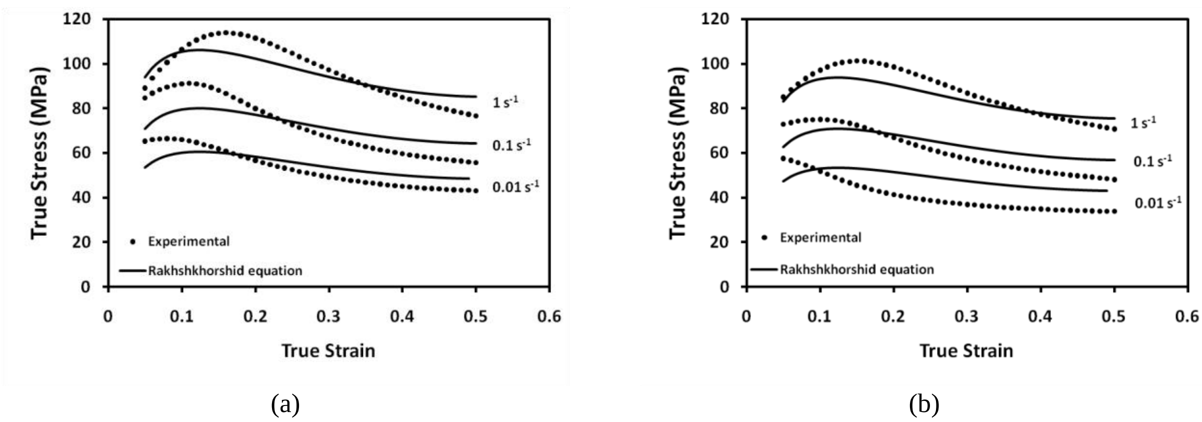


Fig. 6. The comparison between the experimental and modeled flow curves (using the recently developed simple constitutive equation) at deformation conditions with two temperatures of (a) 375 and (b) 400 °C with different strain rates.

Table 1. Root Mean Square Error (MPa) between the experimental and modeled flow curves of tested alloy.

Constitutive equation	Root Mean Square Error (MPa)
Arrhenius equation with strain dependent constants	4.96
Exponential equation with strain dependent constants	5.88
The recently developed constitutive equation	10.51

As presented in Table 1, the Arrhenius equation with strain dependent constants has the best performance for modeling the hot deformation flow curves of AZ91 magnesium alloy; while, the recently developed model has the worst. However, the simplicity of the recently developed simple model (developed based on a power function of Zener-Hollomon parameter and a third order polynomial function of strain power m) is its advantage over the other considered equations. There are seven constants in this equation that can be easily determined using an optimization procedure.

4. Conclusion

In this research, the flow stress of AZ91 magnesium alloy at hot deformation conditions was described through the use of three different constitutive models including the Arrhenius equation with strain dependent constants, the exponential equation with strain dependent constants and a recently developed simple model. The values of RMSE criterion for these three constitutive equations were obtained as 4.96, 5.88 and 10.51 MPa, respectively. This shows that the Arrhenius equation with strain dependent constants has the best performance for modeling the hot deformation flow curves of AZ91 magnesium alloy. The results can be used in finite element simulation of the manufacturing processes of the tested alloy.

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مطالعه مقایسه‌ای مدل‌سازی منحنی‌های سیلان تغییر شکل گرم آلیاژ منیزیمی AZ91 توسط مدل‌های جامع

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چکیده: مدل‌سازی منحنی‌های سیلان تغییر شکل گرم مواد مختلف، اولین مرحله در شبیه سازی فرآیندهای تولید آن‌ها است. در این تحقیق، عملکرد سه معادله جامع مختلف جهت مدل سازی منحنی های سیلان تغییر شکل گرم آلیاژ منیزیمی AZ91 با یکدیگر مقایسه شده است. بدین‌منظور، منحنی های سیلان تغییر شکل گرم این آلیاژ توسط معادله آرنیوسی با ثابت‌های وابسته به کرنش، معادله نمایی با ثابت های وابسته به کرنش و یک مدل ساده جدید (که بر مبنای یک تابع توانی از پارامتر زنر-هولومون و یک تابع درجه سه از کرنش به توان یک عدد ثابت توسعه یافته است) مدل شده اند. از معیار ریشه میانگین مربعات خطا جهت ارزیابی عملکرد مدل‌های مورد مطالعه استفاده شد و معین گردید که معادله آرنیوسی با ثابت‌های وابسته به کرنش بهترین عملکرد را برای مدل‌سازی منحنی‌های کارگرم آلیاژ منیزیمی AZ91 دارد. از نتایج تحقیق حاضر می توان جهت شبیه سازی فرآیندهای تولید آلیاژ مورد مطالعه استفاده کرد.

کلمات کلیدی: منحنی‌های سیلان، تغییر شکل گرم، مدل‌های جامع، آلیاژ منیزیمی AZ91.