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Research Note

Gravitational radiation of a 3d harmonic oscillator in f (R) gravity

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Abstract

The energy loss of a 3-dimensional harmonic oscillator due to the quadrupole radiation in quadratic f (R) gravity is calculated.

***Keywords**:* Energy loss; f (R) gravity; gravitational radiation; 3D harmonic oscillator

1. Introduction

The f(R) gravity, has gained much interest in recent years, because of its potential ability to explain several phenomena at the cosmological scale, like dark matter, dark energy and inflation (De Felice and Tsujikawa, 2010; Sotiriou and Faraoni, 2010). For example, the model with modified Lagrangian $\mathcal{L} = f(\mathrm{R}) = \mathrm{a}_1 R + a_2 R^2$ leads to the inflating models of the universe (Starobinsky, 1980). Looking for more direct observational consequences of the f (R) models, authors in (Laurentis and Capozziello, 2011; De Laurentis and De Martino, 2011; De Laurentis, De Martino, 2013; Naf and Jetzer, 2011) have considered the energy loss of a binary system of pulsars in $f(\mathrm{R}) = \mathrm{a}_1 R + a_2 R^2$ model and compared the results with the Einstein gravity. Bearing in mind that the energy loss of binary pulsars due to the gravitational radiation is one of the successful predictions of Einstein gravity, it seems quite reasonable to study the problem of gravitational radiation in such modified theories of gravity. In this work, we consider a classical 3D harmonic oscillator and calculate its energy loss due to the gravitational radiation in $f(\mathrm{R}) = \mathrm{a}_1 R + a_2 R^2$ model. After a short review of quadrupole radiation in f (R) gravity, in the next section, we derive the energy loss of a 3D (3-dimensional) harmonic oscillator in section 3.

2. Quadrupole Radiation In f (R) Gravity

The Lagrangian density of quadratic f (R) gravity plus the matter, Lagrangian has the form (Laurentis and Capozziello, 2011)

$$\mathcal{L} = f(\mathrm{R}) = \mathrm{a}_1 R + a_2 R^2 + 16\pi G \mathcal{L}_M \tag{1}$$

The metric is assumed to be expanded around the Minkowski metric $\eta_{\mu\nu}$ as $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$. A lengthy calculation leads to the energy-momentum pseudo-tensor of the gravitational field as

$$t_{\mu\nu} = a_1 k_\mu k_\nu \dot{h}_{\alpha\beta} \dot{h}^{\alpha\beta} - a_2 \delta_{\mu\nu} (k_\alpha k_\beta \ddot{\mathrm{h}}^{\alpha\beta})^2 \tag{2}$$

Therefore, the rate of energy loss of a matter system coupled to gravity becomes (Laurentis and Capozziello, 2011)

$$-\left\langle \frac{dE}{dt} \right\rangle = \int_S d\sigma \hat{x}_i \left\langle t^{0i} \right\rangle =$$
$$= \frac{a_1}{60} G \left\langle \dddot{Q}_{ij} \dddot{Q}^{ij} \right\rangle + \frac{a_2}{30} G \left\langle \ddddot{Q}_{ij} \ddddot{Q}^{ij} \right\rangle \tag{3}$$

where the time average for a periodic motion with period T is defined via

$$\langle f \rangle = \frac{1}{T} \int_0^T f(\mathrm{t}) dt \tag{4}$$

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and the mass moment tensor of a particle with mass m has the form $Q_{ij} = m x_i x_j$.

3. Gravitational Radiation by a Harmonic Oscillator

A 3D harmonic oscillator can be modeled by means of the six identical spring lying along the axis (Fowles, 1985) (Fig. 1). For small vibrations of the particle around the origin of the system of coordinates, the equation of motion of is

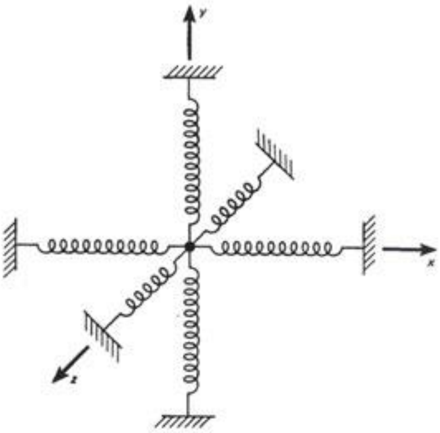

Fig. 1. Model of a 3D harmonic osillator

$$\ddot{\mathbf{x}} + \omega^2 \mathbf{x} = 0 \qquad (5)$$

which admits the solution

$$\mathbf{x} = \mathbf{A}\sin\omega t + \mathbf{B}\cos\omega t \qquad (6)$$

The particle orbit lies on the plane common to the vectors **A** and **B** (Fowles, 1985). One immediately obtains

$$\ddot{\mathbf{x}} = -\omega^2 \mathbf{x} \qquad (7a)$$

$$\ddddot{\mathbf{x}} = \omega^4 \mathbf{x} \qquad (7b)$$

Now let us look at (3). For the time derivatives of the mass moment we obtain

$$\dddot{Q}_{ij} = m(\dddot{x}_i x_j + 3\ddot{x}_i \dot{x}_j + 3\dot{x}_i \ddot{x}_j + x_i \dddot{x}_j) \qquad (8a)$$

$$\ddddot{Q}_{ij} = m(\ddddot{x}_i x_j + 4\dddot{x}_i \dot{x}_j + 6\ddot{x}_i \ddot{x}_j + +4\dot{x}_i \dddot{x}_j + x_i \ddddot{x}_j) \qquad (8b)$$

By the help of (7) we get

$$\dddot{Q}_{ij} = -4m\omega^2(\dot{x}_i x_j + x_i \dot{x}_j) \qquad (9a)$$

$$\ddddot{Q}_{ij} = 8m\omega^2(\omega^2 x_i x_j - \dot{x}_i \dot{x}_j) \qquad (9b)$$

which, after taking the time average yields

$$\left\langle \dddot{Q}_{ij} \dddot{Q}^{ij} \right\rangle = 32m^2\omega^4 \left[\langle \dot{\mathbf{x}}\cdot\dot{\mathbf{x}}\mathbf{x}\cdot\mathbf{x} \rangle + \langle \mathbf{x}\cdot\dot{\mathbf{x}}\mathbf{x}\cdot\dot{\mathbf{x}} \rangle \right] \qquad (10a)$$

$$\left\langle \ddddot{Q}_{ij} \ddddot{Q}^{ij} \right\rangle = 64m^2\omega^4\omega^4 [\langle \mathbf{x}\cdot\mathbf{x}\mathbf{x}\cdot\mathbf{x} \rangle - 2\omega^2 \langle \mathbf{x}\cdot\dot{\mathbf{x}}\mathbf{x}\cdot\dot{\mathbf{x}} \rangle + +\langle \dot{\mathbf{x}}\cdot\dot{\mathbf{x}}\dot{\mathbf{x}}\cdot\dot{\mathbf{x}} \rangle] \qquad (10b)$$

From the definition of time average, Eq (4), one easily obtains

$$\left\langle \sin^4 \omega t \right\rangle = \left\langle \cos^4 \omega t \right\rangle = \frac{3}{8} \qquad (11a)$$

$$\left\langle \sin^2 \omega t \cos^2 \omega t \right\rangle = \frac{1}{8} \qquad (11b)$$

$$\left\langle \sin^3 \omega t \cos \omega t \right\rangle = \left\langle \sin \omega t \cos^3 \omega t \right\rangle = 0 \qquad (11c)$$

Then, by means of the above relations, we find

$$\langle \mathbf{x}\cdot\mathbf{x}\mathbf{x}\cdot\mathbf{x} \rangle = \frac{1}{8}\left[3\mathbf{A}^4 + 3\mathbf{B}^4 + 4(\mathbf{A}\cdot\mathbf{B})^2 + 2\mathbf{A}^2\mathbf{B}^2\right] \qquad (12a)$$

$$\langle \dot{\mathbf{x}}\cdot\dot{\mathbf{x}}\dot{\mathbf{x}}\cdot\dot{\mathbf{x}} \rangle = \frac{\omega^4}{8}\left[3\mathbf{A}^4 + 3\mathbf{B}^4 + 4(\mathbf{A}\cdot\mathbf{B})^2 + 2\mathbf{A}^2\mathbf{B}^2\right] \qquad (12b)$$

$$\langle \mathbf{x}\cdot\dot{\mathbf{x}}\mathbf{x}\cdot\dot{\mathbf{x}} \rangle = \frac{\omega^2}{8}\left[\mathbf{A}^4 + \mathbf{B}^4 + 4(\mathbf{A}\cdot\mathbf{B})^2 - 2\mathbf{A}^2\mathbf{B}^2\right] \qquad (12c) \qquad (0.1)$$

$$\langle \mathbf{x}\cdot\mathbf{x}\dot{\mathbf{x}}\cdot\dot{\mathbf{x}} \rangle = \frac{\omega^2}{8}\left[\mathbf{A}^4 + \mathbf{B}^4 - 4(\mathbf{A}\cdot\mathbf{B})^2 + 6\mathbf{A}^2\mathbf{B}^2\right] \qquad (12d)$$

$$\langle \dot{\mathbf{x}}\cdot\mathbf{x}\mathbf{x}\cdot\mathbf{x} \rangle = 0 \qquad (12e)$$

Thus, equations (10a) and (10b) take the form

$$\left\langle \dddot{Q}_{ij} \dddot{Q}^{ij} \right\rangle = 8m^2\omega^6 \left(\mathbf{A}^2 + \mathbf{B}^2\right)^2 \qquad (13a)$$

$$\left\langle \ddddot{Q}_{ij} \ddddot{Q}^{ij} \right\rangle = 32m^2\omega^8 \left(\mathbf{A}^2 + \mathbf{B}^2\right)^2 \qquad (13b)$$

Therefore, from the equations (3), (13a) and (13b), one obtains the energy loss of a 3D oscillator as

$$-\left\langle \frac{dE}{dt} \right\rangle = \frac{2}{15} m^2\omega^6 G\left(a_1 + 8\omega^2 a_2\right)\left(\mathbf{A}^2 + \mathbf{B}^2\right)^2 \qquad (14)$$

For a motion along the straight line (1D oscillator), setting either **A** = 0 or **B** = 0, the loss rate reduces to

$$-\left\langle \frac{dE}{dt} \right\rangle = \frac{2}{15} m^2\omega^6 A^4 G\left(a_1 + 8\omega^2 a_2\right) \qquad (15)$$

We observe that for $a_2 = 0$, the energy loss has the well-known form based on the Einstein gravity, i.e. (De Sabbata and Gasperini, 1985)

$$-\left\langle \frac{dE}{dt} \right\rangle \square\, m^2 \omega^6 A^4 G \qquad (16)$$

Another interesting case is a circular motion, which implies $\mathbf{A} = \mathbf{B}$ and $\mathbf{A}.\mathbf{B} = 0$. Thus the rate of energy loss becomes

$$-\left\langle \frac{dE}{dt} \right\rangle = \frac{8}{15} m^2 \omega^6 A^4 G \left(a_1 + 8\omega^2 a_2 \right) \qquad (17)$$

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