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INFLUENCE OF ROLLING VISCO-ELASTIC COUPLING ON NON-LINEAR DYNAMICS OF DOUBLE PLATES SYSTEM*

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Abstract– Multi frequency vibrations of a system of two isotropic circular plates interconnected by a rolling visco-elastic layer that has nonlinear characteristics are considered. The system with two circular plates and interconnecting discrete continually distributed rolling visco nonlinear elastic rheological elements presents the model of hybrid nonlinear system. The analytical solutions of first asymptotic approximation describing stationary behavior, in the regions around the resonances, are the principal results of the author. On the basis of those results the influence on the system dynamics of rolling coupling element was numerically analyzed. A series of the amplitude-frequency and phase-frequency curves of the two-frequency like vibration regimes were numerically obtained and presented. These curves present the evolution of the first asymptotic approximation of solutions for different nonlinear harmonics obtained by changing external excitation frequencies through discrete as well as continuous values for different values of rolling elements masses. Such an analysis proves that the presence of rolling elements in the interconnected layer of two plates causes frequency overlap of resonant regions of nonlinear modes, which at the same time causes the enlargement of the mode mutual interactions.

Keywords– Hybrid nonlinear system, circular plates, rolling visco nonlinear elastic rheological element, multi frequency, resonance, resonant jumps, mutual mode interaction, stationary resonant regimes

1. INTRODUCTION

In many engineering systems with non-linearity, high frequency excitations are the sources of multi frequency resonant regimes appearance at high as well as at low frequency modes. That is obvious from many experimental research results and also theoretical results [1, 2]. The interaction between amplitudes and phases of the different modes in the nonlinear systems with many degrees of freedom, as in the deformable body with infinite numbers frequency vibration in free and forced regimes, is observed theoretically in [3] by using averaging asymptotic methods Krilov-Bogoliyubov-Mitropolyskiy [4, 5]. This knowledge has great practical importance.

In the monograph [1] by Nayfeh a coherent and unified treatment of analytical, computational, and experimental methods and concepts of modal nonlinear interactions is presented. These methods are used to explore and unfold in a unified manner the fascinating complexities in nonlinear dynamical systems.

Identifying, evaluating, and controlling dynamical integrity measures in nonlinear mechanical oscillators are topics for researchers, [6-9]. Energy transfer between coupled oscillators can be a measure of the dynamical integrity of hybrid systems as well as subsystems [7, 10-12]. In the series of references it is possible to find a different approach to obtain solutions of the nonlinear dynamics of real systems, as well to discover nonlinear phenomena or some properties of the system dynamics. There are many

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systems which consist of a nonlinear oscillator attached to a linear system, examples of which are nonlinear vibration absorbers, or nonlinear systems under test using shakers excited harmonically with a constant force. The paper [13] presents a study of the dynamic behavior of a specific two degree-of-freedom system representing such a system. It is found that closed detached resonance curves lying outside or inside the continuous path of the main resonance curve can appear as a part of the overall amplitude-frequency response.

List of the valuable research results in a related area of the objects of the author's research is large, but in this introduction the subjective choice was mentioned.

By using averaging and asymptotic methods for obtaining system of ordinary differential equations of amplitudes and phases in first approximations and expressions for energy of the excited modes depending on amplitudes, phases and frequencies of different nonlinear modes are obtained by Hedrih [8, 9] and by Hedrih and Simonović [12]. By means of these asymptotic approximations, the energy analysis of mode interaction in the multi frequency free and forced vibration regimes of nonlinear elastic systems (beams, plates, and shells) excited by initial conditions was made, and a series of resonant jumps as well as energy transfer features were identified. Meaning that excitation was, by perturbation of equilibrium state of the double plate system at initial moment, defined by initial conditions for displacements and velocities of both plate middle surface points.

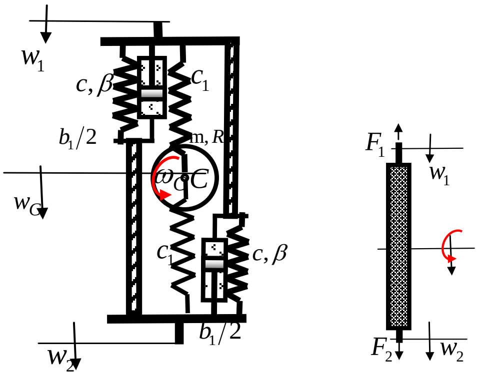
Interest in the study of coupled plates, as new qualitative system dynamics has grown exponentially over the last few years because of the theoretical challenges involved in the investigation of such systems. Recent technological innovations have caused considerable interest in the study of the hybrid dynamical processes consisting of coupled rigid and deformable bodies (plates, beams and belts) [8, 9, 14, 16-17], characterized by the interaction between subsystem dynamics and governed by coupled partial differential equations.

The study of transversal vibrations of a double, like multi plates system with elastic, visco-elastic of creep connections is important for both theoretical and pragmatic reasons. Many important structures may be modeled from composite structure and are necessary in many appliances. For example, in civil engineering for roofs, floors, walls, in thermo and acoustics isolation systems of walls, and floor constructions, orthotropic bridge decks or for building, any structural application in which the traditional method of construction is applied usage of stiffened steel. Also, it is applied in cars, planes and ship industry for sheaths of wings, for inner arrangement of plane, it is suitable for building maritime vessels or for building civil structures such as double hull oil tankers, bulk carriers, auto bodies, truck bodies or for railway vehicles.

It is shown here that as a model of that structure it is possible to use two rolling visco-elastically connected plates with nonlinearity in elastic layer. This paper attempts to present the feature of interconnected layer joined with rolling elements with their inertia of rolling without sliding, and of translation of mass centers. The model of new rheological element with properties of visco- nonlinear elasticity and of rolling without sliding will be presented. Such an element has different forces on its ends in a motion. The presence of those elements in the model of interconnected layer of two plates introduces the dynamical coupling in the mathematical model of plate system dynamics. Also, this model with nonlinearity of the third order in the interconnected layer introduces the phenomenon of passing through resonant range and appearance of one or several resonant jumps in the amplitude–frequency and phase–frequency curves, as in the multi-nonlinear mode mutual interactions between amplitudes and phases of different nonlinear modes.

2. CONSTITUTIVE RELATION FOR ROLLING VISCO NONLINEAR ELASTIC RHEOLOGICAL ELEMENT AND PDE`S OF TRANSVERSAL VIBRATIONS OF A DOUBLE PLATE SYSTEM

For standard rolling visco nonlinear elastic element, Figs. 1a) and 1b), presented as a rheological model [18], we write the expressions for the velocity of translation for the centre of mass C in the form: $\dot{w}_C = (\dot{w}_2 + \dot{w}_1)/2$, and for the angular velocity around center of mass in the form: $\omega_C = (\dot{w}_2 - \dot{w}_1)/2R$.



a) b)

Fig. 1. a) The rheological model of rolling visco-elastic nonlinear discrete element; b) the rheological scheme of rolling visco-elastic nonlinear discrete element

The constitutive relations for forces on the ends of this element are in the following form:

$$F_{1(2)} = \pm\left(c + \frac{c_1}{4}\right)(w_2 - w_1) \pm b_1(\dot{w}_2 - \dot{w}_1) \pm \beta (w_2 - w_1)^3 - \frac{1}{4}\mathrm{m}\left\langle (\ddot{w}_2 + \ddot{w}_1) \mp \frac{i_C^2}{R^2}(\ddot{w}_2 - \ddot{w}_1)\right\rangle \tag{1}$$

where c and c_1 are stiffness of linear springs, b_1 is coefficient of damping force, β stiffness of nonlinear springs, m is mass of disc, $i_C^2 = \mathbf{J_C}/\mathrm{m}$ is the square of radius of inertia for the rolling element. If the rolling element is the disc then mass moment of inertia is $\mathbf{J_C} = R^2\mathrm{m}/2$ and $i_C^2 = R^2/2$.

The governing equations of the double plate system [9, 15, 16], Fig. 2, are formulated in terms of two unknowns: the transversal displacement $w_i(r,\varphi,t) = w_i$, $i = 1,2$ in direction of the axis z, of the upper plate middle surface and of the lower plate middle surface, respectively. We present the interconnecting layer as a model of distributed discrete rheological rolling visco-elastic elements with nonlinearity in the elastic part of the layer, as shown in Fig. 1a and 1b. Since elements are continually distributed on plates surfaces, the generalized resulting forces (1) are also continually distributed onto middle plate points. Our assumptions for the plates are: they are thin with the same contours and with an equal type of boundary condition and they have small transversal displacements. The system of two coupled partial differential equations is derived using d'Alembert's principle of dynamic equilibrium in the following forms:

$$\begin{gathered}
\frac{\partial^2 w_1}{\partial t^2}(1 + \tilde{a}_{11}) + \tilde{a}_{12(1)}\frac{\partial^2 w_2}{\partial t^2} + c_{(1)}^4 \Delta\Delta w_1 - 2\delta_{(1)}\left[\frac{\partial w_2}{\partial t} - \frac{\partial w_1}{\partial t}\right] - a_{(1)}^2 [w_2 - w_1] = \varepsilon\beta_{(1)}[w_2 - w_1]^3 + \tilde{q}_{(1)} \\
\frac{\partial^2 w_2}{\partial t^2}(1 + \tilde{a}_{22}) + \tilde{a}_{12(2)}\frac{\partial^2 w_1}{\partial t^2} + c_{(2)}^4 \Delta\Delta w_2 + 2\delta_{(2)}\left[\frac{\partial w_2}{\partial t} - \frac{\partial w_1}{\partial t}\right] + a_{(2)}^2 [w_2 - w_1] = -\varepsilon\beta_{(2)}[w_2 - w_1]^3 - \tilde{q}_2
\end{gathered} \tag{2}$$

where: $\tilde{a}_{ii} = \hat{a}_{ii}/\rho_i h_i$, $\tilde{a}_{12(i)} = \hat{a}_{12}/\rho_i h_i$, $\hat{a}_{12} = \mathrm{m}/4 - \mathbf{J_C}/4R^2 = \mathrm{m}/8$, $\hat{a}_{ii} = \mathrm{m}/4 + \mathbf{J_C}/4R^2 = 3\mathrm{m}/8$, $a_{(i)}^2 = (c + c_1/4)/\rho_i h_i$, $D_i = E_i h_i^3/12(1-\mu_i^2)$, $c_{(i)}^4 = D_i/\rho_i h_i$, $2\delta_i = b_1/\rho_i h_i$ and $\varepsilon\beta_{(i)} = \beta/\rho_i h_i$, for $i = 1,2$. E = Young's modulus, μ_i = Poisson's coefficient, ρ_i =density of plates material, h_i =height of plates. The form of the external loads on the plates surfaces are given as $\tilde{q}_{(i)} = \tilde{q}_{(i)}(r,\varphi,t)$.

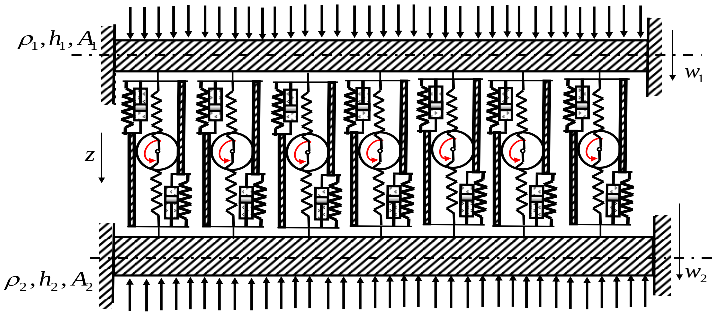


Fig. 2. Double circular plate system connected with a rolling visco-elastic nonlinear layer

3. ASYMPTOTIC APPROXIMATION OF THE SOLUTION OF PDE`S OF TRANSVERSAL VIBRATIONS OF A DOUBLE CIRCULAR PLATE SYSTEM

The systems of partial differential Eq. (2) describe the dynamics of the double plate - system with the rolling visco non-linear elastic layer. By using Bernoulli's method of particular integrals we suppose the solutions for system that are in the form of the eigen amplitude functions $\mathrm{W}_{(i)nm}(r,\varphi)=W_{i(nm)}$, $n,m=1,2,....\infty$, satisfy the same boundary conditions, expansion with time coefficients in the form of unknown time functions $T_{(i)nm}(t)=T_{i(nm)}$, and describe their time evolution [16], in the form: $w_i(r,\varphi,t)=\mathrm{W}_{(i)nm}(r,\varphi)T_{(i)nm}(t)$. After substituting this solution into the system of differential Eq. (2), keeping in mind orthogonality conditions of plate amplitude functions it makes system of DE for time function of one nm-mode of plates transversal oscillations:

$$\begin{aligned}\ddot{T}_{(1)nm}+\kappa_1\ddot{T}_{(2)nm}-2\tilde{\delta}_{(1)}\left(\dot{T}_{(2)nm}-\dot{T}_{(1)nm}\right)+\tilde{\omega}^2_{(1)nm}T_{(1)nm}-\tilde{\mathrm{a}}^2_{(1)}T_{(2)nm}=\varepsilon\tilde{\beta}_{(1)}\aleph(W_{nm})\left[T_{(2)nm}-T_{(1)nm}\right]^3+\tilde{f}_{(1)nm}\\ \ddot{T}_{(2)nm}+\kappa_2\ddot{T}_{(1)nm}+2\tilde{\delta}_{(2)}\left(\dot{T}_{(2)nm}-\dot{T}_{(1)nm}\right)+\tilde{\omega}^2_{(2)nm}T_{(2)nm}-\tilde{\mathrm{a}}^2_{(2)}T_{(1)nm}=-\varepsilon\tilde{\beta}_{(2)}\aleph(W_{nm})\left[T_{(2)nm}-T_{(1)nm}\right]^3-\tilde{f}_{(2)nm}\end{aligned} \tag{3}$$

where $\tilde{\omega}^2_{(i)nm}=\omega^2_{(i)nm}/(1+\tilde{\mathrm{a}}_{\mathrm{ii}})$ and $\omega^2_{(i)nm}=\mathrm{k}^4_{(\mathrm{i})\mathrm{nm}}\mathrm{c}^4_{(\mathrm{i})\mathrm{nm}}+\mathrm{a}^2_{(\mathrm{i})\mathrm{nm}}$, $i=1,2$ are the eigen circular frequencies of coupled plates, $\aleph(W_{nm})=\int_0^r\int_0^{2\pi}\mathrm{W}^4_{(1)nm}\,r\mathrm{d}r\mathrm{d}\varphi\Big/\int_0^r\int_0^{2\pi}\mathrm{W}^2_{(1)nm}\,r\mathrm{d}r\mathrm{d}\varphi$ is coefficient of nonlinearity influence of elastic layer, $f_{(i)nm}(t)=\int_0^r\int_0^{2\pi}\tilde{q}_i\,\mathrm{W}_{(i)nm}\,r\mathrm{d}r\mathrm{d}\varphi\Big/\int_0^r\int_0^{2\pi}\left[\mathrm{W}_{(i)nm}\right]^2 r\mathrm{d}r\mathrm{d}\varphi$ are the known function of external forces and coefficients of reduction are: $\kappa_i=\tilde{\mathrm{a}}_{12(\mathrm{i})}/(1+\tilde{\mathrm{a}}_{\mathrm{ii}})$, $\tilde{a}^2_{(i)}=\mathrm{a}^2_{(\mathrm{i})}/(1+\tilde{\mathrm{a}}_{\mathrm{ii}})$, $2\tilde{\delta}_i=2\delta_{(\mathrm{i})}/(1+\tilde{\mathrm{a}}_{\mathrm{ii}})$, $\tilde{\beta}_{(i)}=\beta_{(\mathrm{i})}/(1+\tilde{\mathrm{a}}_{\mathrm{ii}})$ and $\tilde{f}_{(i)nm}=f_{(i)nm}/(1+\tilde{\mathrm{a}}_{\mathrm{ii}})$.

Keeping in mind the form of solutions for the corresponding homogeneous system of (3) we suppose the solution of that system in the following form:

$$\begin{aligned}T_{(1)nm}=\mathrm{K}^{(1)}_{21nm}e^{-\hat{\delta}_{1nm}t}R_{1nm}(t)\cos\Phi_{1nm}(t)+\mathrm{K}^{(2)}_{21nm}e^{-\hat{\delta}_{2nm}t}R_{2nm}(t)\cos\Phi_{2nm}(t)\\ T_{(2)nm}=\mathrm{K}^{(1)}_{22nm}e^{-\hat{\delta}_{1nm}t}R_{1nm}(t)\cos\Phi_{1nm}(t)+\mathrm{K}^{(2)}_{22nm}e^{-\hat{\delta}_{2nm}t}R_{2nm}(t)\cos\Phi_{2nm}(t)\end{aligned} \tag{4}$$

where, K^s_{ijnm} are cofactors of determinant corresponding to basic homogenous coupled linear system [15], and amplitudes $R_{inm}(t)$ and phases $\Phi_{inm}(t)=q_i\Omega_{inm}t+\phi_{inm}(t)$ are unknown time functions that were obtained by use of the asymptotic Krilov-Bogolyubov-Mitropolyskiy averaging method [4, 5]. It is noted that the defined task satisfies all necessary conditions for applying asymptotic Krilov-Bogolyubov-Mitropolskiy method concerning small parameter. We suppose that the functions of external excitation at nm-mode of oscillations are the two-frequency process in the form: $\tilde{q}_{(i)nm}(t)=\mathrm{h}_{01nm}\cos[\Omega_{1nm}t+\phi_{1nm}]+\mathrm{h}_{02nm}\cos[\Omega_{2nm}t+\phi_{2nm}]$, and that external force frequencies Ω_{inm} are in the range of two corresponding eigen linear damped coupled system frequencies $\Omega_{1nm}\approx\hat{\mathrm{p}}_{1nm}$ and $\Omega_{2nm}\approx\hat{\mathrm{p}}_{2nm}$ of the corresponding linear and free system to system (3) and that initial conditions of the double plate system permit appearance of the two-frequency like vibrations regimes of the system. $\hat{\mathrm{p}}_{inm}$ are frequencies of visco-elastic coupling obtained like imaginary parts of solution $\lambda_{i,jnm}=-\hat{\delta}_{inm}\mp\mathrm{i}\hat{\mathrm{p}}_{inm}$ for characteristic equations of system (3). For details see Refs. [9, 15, 17].

The observed case is that external distributed two-frequencies force acts at upper surfaces of upper plate with frequencies near circular frequencies of coupling $\Omega_{1nm} \approx \hat{p}_{1nm}$ and $\Omega_{2nm} \approx \hat{p}_{2nm}$, and that the lower plate is free of excitation $\tilde{q}_{(2)nm}(t) = 0$. Then the first asymptotic averaged approximation of the system of differential equations for amplitudes $R_{inm}(t)$ and difference of phases $\phi_{inm}(t)$ are obtained in the following general form, [17]:

$$\begin{aligned}
\dot{a}_{1nm}(t) &= -\delta_{1nm}a_{1nm}(t) - \frac{\varepsilon P_{1nm}}{(\Omega_{1nm} + \hat{p}_{1nm})}\cos\phi_{1nm} \\
\dot{\phi}_{1nm}(t) &= (\hat{p}_{nm1} - \Omega_{1nm}) - \frac{3}{8}\frac{\alpha_{1nm}}{\hat{p}_{nm1}}a_{1nm}^2(t) - \frac{1}{4}\frac{\beta_{1nm}}{\hat{p}_{nm1}}a_{2nm}^2(t) + \frac{\varepsilon P_{1nm}}{(\Omega_{1nm} + \hat{p}_{nm1})a_{1nm}(t)}\sin\phi_{1nm} \\
\dot{a}_{2nm}(t) &= -\delta_{2nm}a_{2nm}(t) - \frac{\varepsilon P_{2nm}}{(\Omega_{2nm} + \hat{p}_{2nm})}\cos\phi_{2nm} \\
\dot{\phi}_{2nm}(t) &= (\hat{p}_{2nm} - \Omega_{2nm}) - \frac{3}{8}\frac{\alpha_{2nm}}{\hat{p}_{nm2}}a_{2nm}^2(t) - \frac{1}{4}\frac{\beta_{2nm}}{\hat{p}_{nm2}}a_{1nm}^2(t) + \frac{\varepsilon P_{2nm}}{(\Omega_{2nm} + \hat{p}_{nm2})a_{2nm}(t)}\sin\phi_{2nm}
\end{aligned} \tag{5}$$

where $a_{inm}(t) = R_{inm}(t)e^{-\hat{\delta}_{inm}t}$ is the change of variables, hence $\dot{a}_{inm}(t) = (\dot{R}_{inm}(t) - \hat{\delta}_{inm}R_{inm}(t))e^{-\hat{\delta}_{inm}t}$. The full forms of constants δ_{inm}, α_{inm}, β_{inm} and P_{inm} were presented in [17], those values for considered cases of system parameters were presented in the Table 1. Here it was underlined that those constants all rely on coefficients of coupling properties via cofactors $K_{2inm}^{(s)}$, that δ_{inm} depends on damping coefficients of visco-elastic layer $\tilde{\delta}_{(i)}$, εP_{inm} depend on excited amplitudes, and $\alpha_{inm}, \beta_{inm}$ depend on non-linearity layer properties. Coefficients β_{inm} are coefficients of mode mutual interactions.

4. NUMERICAL ANALYSIS OF THE STATIONARY REGIMES OF TRANSVERSAL VIBRATIONS OF A DOUBLE PLATE SYSTEM

For analyses of the stationary regime of oscillations, we make the right hand sides of first and third differential equations for amplitudes $R_{inm}(t)$ and second and fourth equations for difference of phases $\phi_{inm}(t)$ of system (5) equal to null. Eliminating the phases ϕ_{1nm} and ϕ_{2nm} we obtained system of two algebraic equations by unknown amplitudes a_{1nm} and a_{2nm}. Also, with elimination of amplitudes a_{1nm} and a_{2nm}, we obtained the forms for phases ϕ_{1nm} and ϕ_{2nm} in the case of two-frequencies forced oscillations in stationary regime of one nm mode of double plate system oscillations. Solving those systems of algebraic equations by numerical Newton-Kantorovic's method in computer program Mathematica, we obtained stationary amplitudes and phases curves of two-frequencies regime of one eigen nm-shape amplitude mode oscillations in double plate system depending on frequencies of external excitation force. If we fixed the value of an external excitation frequency, of two possible, we obtained amlitude-frequency curves as well as phase-frequency curves of stationary states of vibration regime in the following forms:

1* for second external excitation frequency with constant discrete value ($\Omega_{2nm} = \text{const}$) corresponding amplitude-frequency and phase-frequency curves: $a_{1nm} = f_1(\Omega_{1nm})$, $a_{2nm} = f_2(\Omega_{1nm})$, $\phi_{1nm} = f_3(\Omega_{1nm})$ and $\phi_{2nm} = f_4(\Omega_{1nm})$ and

2* for first external excitation frequency with constant discrete value $\Omega_{1nm} = \text{const}$ corresponding amplitude-frequency and phase-frequency curves: $a_{1nm} = f_5(\Omega_{2nm})$, $a_{2nm} = f_6(\Omega_{2nm})$, $\phi_{1nm} = f_7(\Omega_{2nm})$ and $\phi_{2nm} = f_8(\Omega_{2nm})$.

We will present amplitude-frequencies and phase-frequencies curves of stationary state in continuous exchange of fixed discrete values of external excitation frequencies and in that sense regard system in stationary regime, and some characteristic diagrams of that amplitude-frequency and phase-frequency curves are presented in the following Figs. 3-10.

The following analysis considers changing of rolling element masses that influence kinetic energy of interconnected layer. For further numerical calculations we present three cases of interconnecting layer rolling elements by changing their mass per unit of plates surfaces from $m = 240\,\text{kg}$ and $m = 100\,\text{kg}$ to case when we do not have rolling elements for $\mathrm{m}=0\,\mathrm{kg}$. The numerically considered plates have the same material characteristics, with radius of $1\,\mathrm{m}$, heights $\mathrm{h}_1=0{,}01\mathrm{m}$ and $\mathrm{h}_2=0{,}005\,\mathrm{m}$, maiden of still with density $\rho_i=7.849\cdot10^3\,\mathrm{kgm}^{-3}$, Poisson's ratio $\mu=0.33$ and Young's modulus $\mathrm{E}_i=21\cdot10^{10}\,\mathrm{Nm}^{-2}$. Between plates is layer of continually distributed nonlinear visco-elastic rolling elements of stiffness $\mathrm{c}=2\cdot10^5\,\mathrm{Nm}^{-1}$ and $\mathrm{c}_1=0{,}5\cdot10^5\,\mathrm{Nm}^{-1}$ and coefficient of damping $b_1=0.5\;kg\,sm^{-1}$. This is the case when the lower plate has a height two times lower than upper plate, $\mathrm{h}_2=\mathrm{h}_1/2$, and when we modify mass of rolling elements the solutions of characteristics equations of system (3) $\lambda_{1,2nm}=-\hat{\delta}_{1nm}\mp i\hat{\mathrm{p}}_{1nm}$ and $\lambda_{3,4nm}=-\hat{\delta}_{2nm}\mp i\hat{\mathrm{p}}_{2nm}$ have different values. Solved values of circular frequencies of coupling $\hat{\mathrm{p}}_{inm}$ and the coefficients α_{inm}, β_{inm}, δ_{inm} and $\varepsilon\mathrm{P}_{inm}$ are presented in the Table 1. Here we present the solutions for the case of the first eigen mode of plates oscillations for $\mathrm{n}=0$ and $\mathrm{m}=1$ for which the characteristic eigen number of clamped circular plate is $\mathrm{k}_{11}=3.196$. The value of the coefficient of nonlinearity influence is $\aleph(W_1)=0.117$, and coefficient of the nonlinearity of layer is $\beta=5\;kgm^{-2}s$, reduced values of the amplitude of excitations are $\mathrm{h}_{0i(11)}=10^7\,\mathrm{Nm}^{-3}$ for the value of the dimensionless parameters $\varepsilon=10^{-2}$.

As expected, increasing the mass of rolling elements reduces circular frequencies of couplings $\hat{\mathrm{p}}_{inm}$, and coefficients of damping influence δ_{inm}.

Table 1. The values of circular frequencies of coupling $\hat{\mathrm{p}}_{inm}$, and coefficients δ_{inm}, α_{inm} and β_{inm}, P_{inm}, for $i=1{,}2$ in first mode of plate system oscillations ($n=0, m=1$), for three different values of rolling elements masses

m(kg)	$\hat{\mathrm{p}}_{101}(s^{-1})$	$\hat{\mathrm{p}}_{201}(s^{-1})$	δ_{101}	δ_{201}	α_{101}	α_{201}	β_{101}	β_{201}	P_{101}	P_{201}
0	108.33	174.49	11	8	12210	96220	267100	17590	2945	534
100	87.33	148.42	6.273	2.151	25480	15720	91720	17470	1402	358.5
240	71.61	126.82	3.326	0.7554	18640	3538	30310	8704	1082	289

All the phenomena of the resonant transition for stationary regime need to be more evident for the same values of the amplitude of external excitations. Those are the distinctive jumps of the amplitude and phase response in the vicinity of the resonant values $\Omega_{inm}\approx\hat{\mathrm{p}}_{inm}$, appearance of the new stable and unstable branches causing more value-system responses and the emergence of two stable solutions of the system in the area of those new branches, the mutual interaction of the harmonics and the jumps of the system energies. All this phenomena are presented through the series of the amplitude-frequency and phase-frequency diagrams for both harmonics in the mentioned three cases of rolling element masses. Those characteristic shapes are the results of the modes interaction and of the particular discrete values choice of the external excitation frequencies Ω_{1nm} and Ω_{2nm}, selected from the resonant frequencies intervals, belonging to proper eigen frequencies $\hat{\mathrm{p}}_{1nm}$ and $\hat{\mathrm{p}}_{2nm}$ of the corresponding nm- th eigen amplitude shape mode of plate linear system taken in the simulations. Strong interactions between time modes in the nm-th eigen amplitude shape mode of plate, appear only in the case that both values of both external excitation frequencies Ω_{1nm} and Ω_{2nm} are chosen simultaneously in the corresponding resonant frequency interval $\Omega_{1nm}\approx\hat{\mathrm{p}}_{1nm}$ and $\Omega_{2nm}\approx\hat{\mathrm{p}}_{2nm}$. If one of the external excitation frequencies is outside of the corresponding resonant frequency interval, the interactions between modes are small. For that case a specific change of the corresponding amplitude-frequency and phase-frequency curves is not visible and is similar to the case of the single frequency external excitation in the corresponding resonant frequency interval. Hence, there is no interaction between time modes in the first asymptotic approximation. This is visible from Figs. 3-14 at the beginning or at the end of the external excitation frequency intervals.

The first five figures, Figs.3-7, present amplitude and phase response for both harmonics for the case of the greatest mass of the rolling elements $\mathrm{m}=240\,\mathrm{kg}$ per unit of plate's surface. The amplitude-frequency responses for two-frequency like stationary vibration regimes contain amplitudes a_1 and a_2 presented in

Figs. 3. and 4. These figures exhibit a strong characteristic as nonlinear interactions between time modes of the two-frequency external excitation in the resonant interval of two external excitation frequencies close to the eigen linearized system frequencies. Amplitude-frequency and phase-frequency curves for the cases: $a_{1nm}(\Omega_1, \Omega_2 = 100\mathrm{s}^{-1})$, $\phi_{1nm}(\Omega_1, \Omega_2 = 100\mathrm{s}^{-1})$ presented in Figs. 3 and 5 have shapes as in the case of the corresponding single frequency amplitude-frequency and phase-frequency curves with only one pair of resonant jumps in each pair of the corresponding curves.

Comparing the first and the last diagrams in the Figs. 3, 5, 6 we may conclude that the amplitude and phase responses of the first harmonic have small changes after transient regime while the amplitude and phase responses of the second harmonics have significant changes of the values and the shapes, Fig. 4. Therefore we conclude that the influence of the first harmonics on the second, in the resonant region of the frequencies Ω_{1nm} of external excitation, is greater than in the resonant region of the frequencies Ω_{2nm} of external excitation.

In the second case for another value of rolling element masse for $\mathrm{m} = 100\,\mathrm{kg}$, Fig. 7. presents the amplitude-frequency diagrams. In this case we did not present the phase-frequency diagrams because, as we noticed on the previous series of the figures, the phase transient through resonant regime is simultaneous to those of amplitude and gives the same quantitative conclusions. In this case the difference among first $\hat{\mathrm{p}}_1 = 87.33(\mathrm{s}^{-1})$ and second $\hat{\mathrm{p}}_2 = 148.42(\mathrm{s}^{-1})$ frequencies is greater than in the previous case for $\mathrm{m} = 240\ \mathrm{kg}$. So, the overlap of the resonant region of the first $\Omega_{1nm} \in [120,210](\mathrm{s}^{-1})$ and the second $\Omega_{2nm} \in [156,175](\mathrm{s}^{-1})$ frequencies is less and mutual interactions of the modes are less obvious. The appearance of the new resonant branches has the identical mechanism as in the previous case. The new branches appear first on the right lower side of main resonant curve for the second resonant region at value $\Omega_{2nm} = 156(\mathrm{s}^{-1})$, Fig.7.

For the third and final case we practically consider the case without rolling elements at the connected layer of the two plates, $\mathrm{m} = 0\,\mathrm{kg}$. Here, the Fig. 8. also presents the amplitude-frequency diagrams of the first time harmonics. For this case we do not notice the distinctive phenomena of passing through resonant regime, there are no resonant jumps and mutual interactions of the harmonics are very small. Hence, the amplitude responses in this case is similar to the case where there is no nonlinearity, we may conclude that influence of nonlinearity in the coupling layer is insignificant for such choice of all other system parameters. The influence of the nonlinearity in the interconnected layer may be more or less present which depends on the parameters of the system.

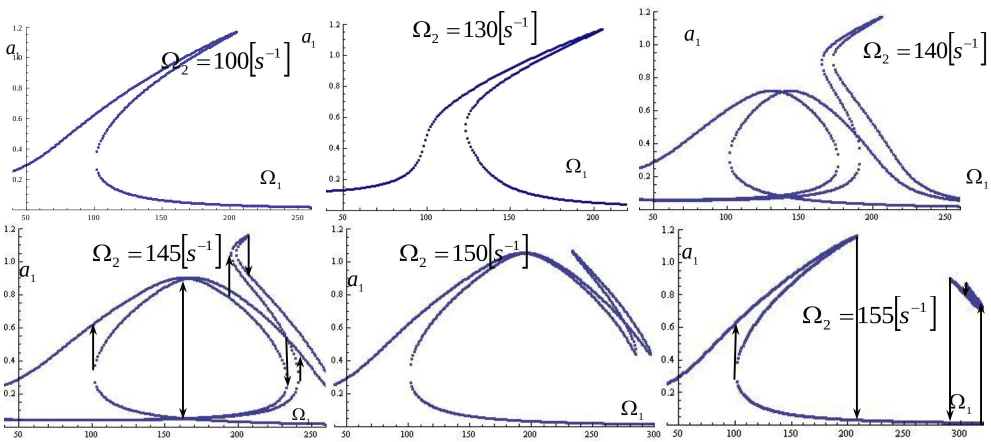


Fig. 3. Amplitude-frequency characteristic curves for the amplitudes of the first time harmonics $a_{1nm} = f_1(\Omega_{1nm})$,on the different value of excited frequency Ω_{1nm} from the interval $\Omega_{1nm} \in [50\mathrm{s}^{-1}, 250\mathrm{s}^{-1}]$for discrete value of excited frequency $\Omega_{2nm} = 100\mathrm{s}^{-1}, 130\mathrm{s}^{-1}, 132\mathrm{s}^{-1}, 135\mathrm{s}^{-1}, 140\mathrm{s}^{-1}, 145\mathrm{s}^{-1}, 150\mathrm{s}^{-1}, 155\mathrm{s}^{-1}$, with characteristic one or more resonant jumps, for $\mathrm{m} = 240\,\mathrm{kg}$. Arrows represent directions of the resonant jumps

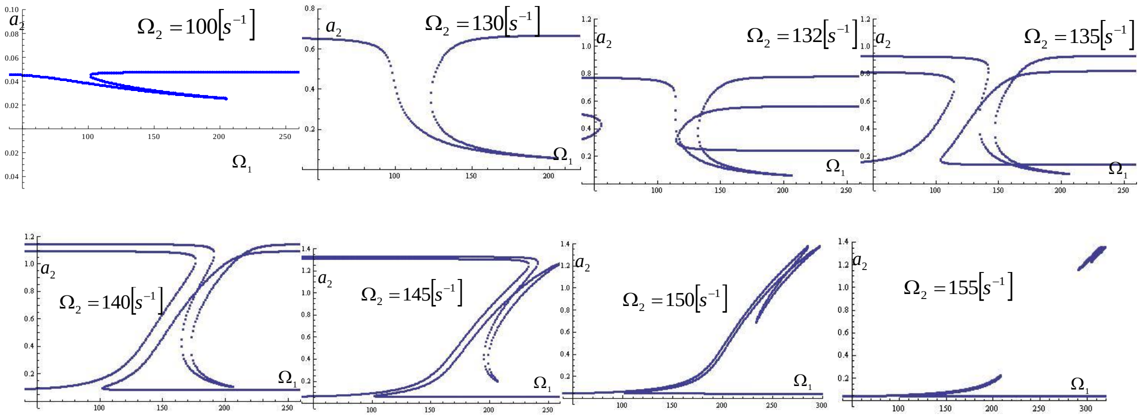


Fig. 4. Amplitude-frequency characteristic curves for the amplitude of the second time harmonics $a_{2nm} = f_2(\Omega_{1nm})$, on the different value of excited frequency Ω_{1nm} from the interval $\Omega_{1nm} \in [50\text{s}^{-1}, 250\text{s}^{-1}]$ for discrete value of excited frequency $\Omega_{2nm} = 100\text{s}^{-1}, 130\text{s}^{-1}, 132\text{s}^{-1}, 135\text{s}^{-1}, 140\text{s}^{-1}, 145\text{s}^{-1}, 150\text{s}^{-1}, 155\text{s}^{-1}$, with characteristic one or more resonant jumps, for $\text{m} = 240\,\text{kg}$

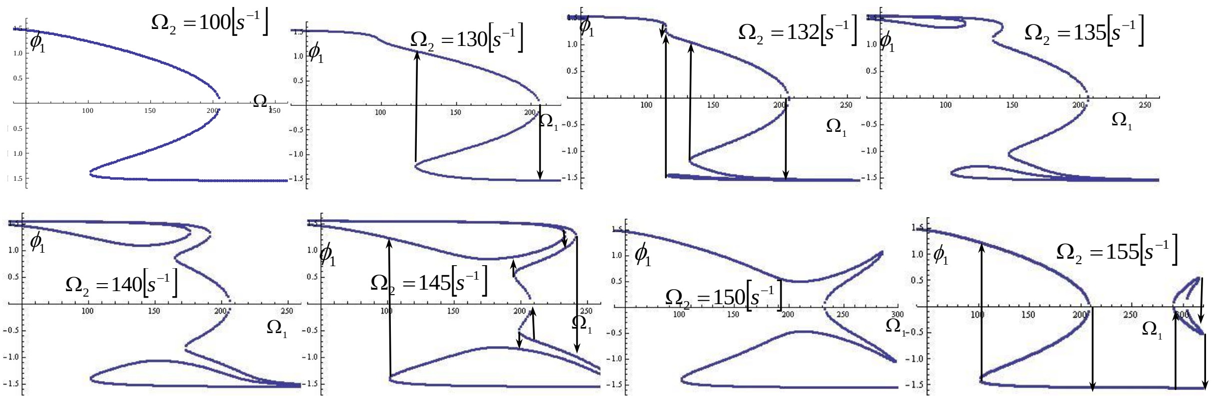


Fig. 5. Phase -frequency characteristic curves for the amplitudes of the first time harmonics $\phi_{1nm} = f_3(\Omega_{1nm})$, on the different value of excited frequency Ω_{1nm} for discrete value of excited frequency $\Omega_{2nm} = \text{const}$, with characteristic one or more resonant jumps, for $\text{m} = 240\,\text{kg}$. Arrows represent directions of the resonant jumps

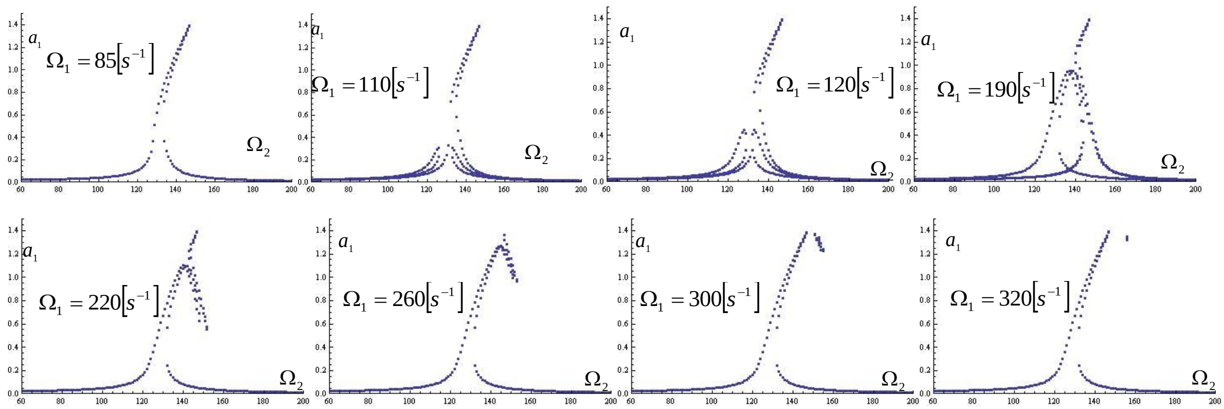


Fig. 6. Amplitude-frequency characteristic curves for the amplitudes of the first time harmonics $a_{1nm} = f_5(\Omega_{2nm})$, on the different value of excited frequency Ω_{2nm} continuously in the interval $\Omega_{2nm} \in [60\text{s}^{-1}, 200\text{s}^{-1}]$ for discrete value of excited frequency $\Omega_{1nm} = 85\text{s}^{-1}, 110\text{s}^{-1}, 120\text{s}^{-1}, 190\text{s}^{-1}, 220\text{s}^{-1}, 260\text{s}^{-1}, 300\text{s}^{-1}, 320\text{s}^{-1}$, with characteristic one or more resonant jumps, for $m = 240\,kg$

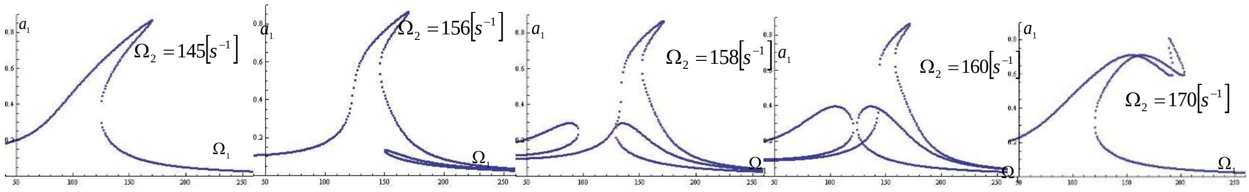


Fig. 7. Amplitude-frequency characteristic curves for the phases of the first time harmonics $a_{1nm} = f_1(\Omega_{1nm})$, on the different value of excited frequency Ω_{1nm} continuously in the interval $\Omega_{1nm} \in [50\mathrm{s}^{-1}, 250\mathrm{s}^{-1}]$ for discrete value of excited frequency $\Omega_{2nm} = 145\mathrm{s}^{-1}, 156\mathrm{s}^{-1}, 158\mathrm{s}^{-1}, 160\mathrm{s}^{-1}, 170\mathrm{s}^{-1}$, with characteristic resonant jumps, for $\mathrm{m} = 100\,\mathrm{kg}$

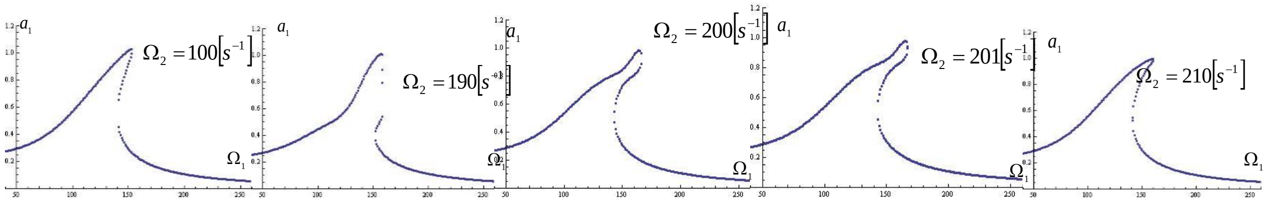


Fig. 8. Amplitude-frequency characteristic curves for the phases of the first time harmonics $a_{1nm} = f_1(\Omega_{1nm})$, on the different value of excited frequency Ω_{1nm} continuously in the interval $\Omega_{1nm} \in [50\mathrm{s}^{-1}, 250\mathrm{s}^{-1}]$ for discrete value of excited frequency $\Omega_{2nm} = 100\mathrm{s}^{-1}, 190\mathrm{s}^{-1}, 200\mathrm{s}^{-1}, 201\mathrm{s}^{-1}, 210\mathrm{s}^{-1}$, with characteristic resonant jumps, for $\mathrm{m} = 0\,\mathrm{kg}$

5. CONCLUSION

To analyze stationary regimes of nonlinear oscillations for presented model, we solved system of PDE`s (3) semi analytically in averaged asymptotic first approximation. Then part of the solution was obtained numerically and amplitudes-frequency and phase-frequency characteristics were presented with obvious interaction of the nonlinear component modes. For the case of the external excitation by two frequency forces and resonant range of the frequencies, we conclude complexity in the system nonlinear response, depending on initial conditions and also on other system kinetic parameters and on the corresponding relation between these sets of the kinetic parameters.

For the system of two circular plates connected with nonlinear rolling visco-elastic layer on the basis of obtained results in this paper we can conclude that nonlinearity in the interconnecting distributed layer introduced in the system resonant jumps, as well as resonant oscillatory jumps, trigger of coupled singularities, as well as coupled triggers of coupled singularities, which are characteristic phenomena of passing through resonant regime. Passing through resonant frequency ranges of the external excitation, unique values of the amplitudes and phases lose stability and splits into trigger of the coupled three singularities, two stable values and one unstable, saddle type of the amplitudes (or phases) for simple case without nonlinear interactions between time modes. But, in the case when there are resonant interactions between modes more than one pair of the resonant jumps appear, and there are possibilities for appearance of the coupled triggers of the coupled singularities containing an odd number of the alternating coupled stable and unstable singularities.

The presented model of new features in interconnected layer introduced with rolling elements with its inertia of rolling without sliding and of translation of mass center is the novelty in modeling of the rheological elements. The presence of rolling elements in the interconnected layer introduces the part of the dynamic coupling into system of obtained PDE's. On the basis of the presented numerical comparison we consequently conclude that dynamic coupling intensifies the phenomena of the resonant transition caused by the mutual interaction of the harmonics.

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