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IMPROVED MIT BAG MODEL WITH HYPER CENTRAL INTERACTING POTENTIAL*

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Abstract – An improved MIT bag model with hyper central interactions is used to calculate the static properties of hadrons containing u, d, s and c quarks. We present a theoretical approach to the internal structure of three-body hyper central interacting quarks in a hadron, in which we take hadron as a bag. We discuss a few of the results obtained using a six-dimension harmonic oscilator (h.o) potential, having a two-body character, which turns out to be a hyper central confinement part. The other potential is six-dimensional, which is attractive for small separation, originating from the color charge of hyper color term. However the potential can easily be generalized in order to allow a systematic analysis. We calculate the relativistic wave function for quarks in a scalar-vector hyper central potential, analytically. Finally, vanishing the normal component of vector current at the surface of the baryon bag as a boundary condition equivalent to confinement, results in the static properties and the strength of hyper Coulomb like potential parameter. This depends on the mass parameters contrary to almost all previous versions. The calculated static properties for baryon are better than in the uncorrected versions of the model. PACS index 12.39 .Ba, 12.39. Ki, 12.39. P_n

Keywords – Hyper central interaction, hadron, static properties, dirac equation, charge radius, magnetic moment

1. INTRODUCTION

Because of its simplicity, the MIT bag model [1] is rather convenient for calculating various hadronic properties. The MIT bag model possesses many desirable features inspired by QCD and relativity. However, so far there is no derivation of the bag model from the first principles. Thus, it is important to test the model in a situation other than those in which the dynamics of the model were originally formulated. The success of the first attempts to calculate the static properties of hadrons lend credence to the bag model approach to hadronic phenomenology by using the normalized spin $\frac{1}{2}$ positive parity solution of the MIT bag model for studying the nuclear matter saturation mechanism based on the quark structure of the nucleon [2], and also recently, a quark- meson coupling QMC mechanism for the saturation of the nuclear was initially proposed by Guichon [3] and generalized by Fleck et al [4], Satio and Thomas [5] and Song and Su [6]. In the QMC model, the baryon is described by the static spherical MIT bag model using the normalized ground state for quark in the baryons.

Although the MIT bag model gives results which are within an acceptable range that shows the correctness of its essential ingredients, obviously it has short comings. One of the short comings is the

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neglecting of inter-quark interactions. The MIT bag model uses a free quark wave function with a no-current boundary condition in the bag's wall. Certainly quarks interact as a three-body force among themselves within the bag, which will change the standard results. In the past, certain modifications of the model were introduced, giving rise to a better agreement with experimental values [7, 8]. In this paper, a modification is proposed by extending the model to include certain residual hyper central interaction among quarks. In our model, each quark moves inside the bag in the effective field of the other quarks and gluons. The relativistic Dirac equation is considered, but with the above mentioned effective hyper central potential. In Tables 1 and 2 the static properties of the nucleon derived in the modified model are compared with the standard bag model and experimental results [7, 8]. As can be seen from the tables, the results show considerable correction over the standard results. The agreement is much better, especially for the magnetic moment. Now using $\frac{g_A}{g_V}$ as input we derive other values given in Tables 1 and 2. The improvement is rather remarkable. For example, it gives much more acceptable values for proton compared to the standard model. Further improved results can be found in Table 2.

Table 1. Comparing static properties of proton in our model and MIT bag model with experiments

Proton	MIT bag model	Our model	Experiment
m_q	0	186.241 ± 0.012 Mev	~100~350 Mev *
g_A / g_V	1.09	1.254 ± 0.006	1.254 ± 0.006
$< r_{em}^2 >^{\frac{1}{2}}$	0.73 f m	0.842 ± 0.003 fm	0.88 ± 0.03 fm
μ_p	1.9 n.m	2.693 ± 0.016 n.m	2.792 n.m
x_b	1.5 f m	1.234 ± 0.000 fm	--
α_s	0	0.753 ± 0.002	$0 \le \alpha_s \le 1$ *

*The values are not directly measured but inferred from experiment

Now the different interactions that are considered are briefly discussed. The effective interaction of quarks due to gluon exchange is assumed to be given by hyper central potential for each quark. It is assumed that the internal quark motion is described by the Jacobi coordinates ρ and λ [9, 10]. In order to describe the three quark dynamics it is convenient to introduce the hyper spherical coordinates, which are obtained by substituting the absolute values of ρ and λ in $x = \sqrt{\rho^2 + \lambda^2}$, where x is the hyper radius. If it is assumed that the confining potential is hyper central, and hence depending only on x, there are two kinds of hyper central potentials in this model, which leads to analytical solutions based on a suitable ansatz [11]. The first is six-dimensional harmonic oscillator (h.o) potential, which has a two-body character, and turns out to be exactly hyper central since

$$V_{h.o.} = \sum_{i<j} \frac{1}{2} k(r_i - r_j)^2 = \frac{3}{2} kx^2 = ax^2 \tag{1}$$

The second one is the six–dimensional hyper Coulomb potential [10, 11], which is attractive for small separations originating from the color charge:

$$V_{hyC}(x) = \frac{k\alpha_s}{x} = -\frac{c}{x} \tag{2}$$

However there have been some interesting attempts to interpolate between (1) and (2), [12-17]. In Section 2 we have calculated the relativistic wave function for valence quarks. The potential parameters can be found by fitting as in Section 3.1. The results indicate that this potential is useful for quarks having masses in the range used in the phenomenological analysis of the quark model. In Section 3.2 charge radius, and in Section 3.3 the magnetic moment have been found for different quark masses. In our model it is concluded that there is a reasonable consistency between the calculated values and the experimental results.

Table 2. Comparing magnetic moment of different baryons based on the MIT bag model and the model developed in this paper with experiment

Baryon	MIT bag model	Our model	Experiment
P	1.90 n.m	2.693 ± 0.016 n.m	2.782 n.m
n	-1.273 n.m	-1.884 ± 0.008 n.m	-1.912 n.m
Λ	-0. 494 n.m	-0.618 ± 0.009 n.m	-0.614 n.m
$\sum^-$	0.684 n.m	1.149 ± 0.006 n.m	1.156 n.m
$\sum^+$	1.843 n.m	2.485 ± 0.123 n.m	2.418 n.m
$\equiv$	-1.064 n.m	-1.243 ± 0.007 n.m	-1.256 n.m
$\equiv^-$	-0.437 n.m	-0.681 ± 0.005 n.m	$-0,689$ n.m

In the following section, by solving the Dirac equation, the relativistic wave function for valence quarks has been calculated analytically. The nucleon masses and the ratio $\frac{g_A}{g_V} = 1.254 \pm 0.006$ [18] are taken as our inputs. These inputs fix the parameters in the potentials and the quark mass, and from them several other nucleon properties are derived. The numerical values and static properties of our model with three quarks potential, suitable for constituent quark's masses, which are in the range of $(\sim 100 \sim 350)$ Mev show remarkable improvement over previous results obtained by the MIT bag model quarks potential.

In Section 3.4 the strong coupling constant α_s has been found. Finally, in section (4) we give our conclusion.

2. HYPERCENTRAL RELATIVISTIC WAVE FUNCTION FOR THREE QUARKS IN A HADRON

The constituent quark model based on a hyper central approach takes into account three body force effects and the standard two-body potential contributions.

Let's represent the quark wave function satisfying the Dirac equation by $\psi(\vec{x})$, so

$$[\gamma_0 \varepsilon + i\vec{\gamma}.\vec{\nabla} - (m + U(x))]\psi(\vec{x}) = 0 \tag{3}$$

The hyper central potentials, which lead to analytical solution in our model would be

$$U(x) = \frac{1}{2}(1 + e\gamma_0)A(x) \tag{4}$$

The parameter e can take any value [19-21]. In this investigation it is taken as 1.
Hence, from Eq. (1, 2) the interaction potential can be taken as

$$A(x) = ax^2 - \frac{c}{x} \tag{5}$$

This potential has interesting properties and yields good physical results. The solution of the Dirac equation can be worked out analytically. The quark potential, $U(x)$, is assumed to depend on the hyper radius x only. The Dirac equation may transform in various ways under a Lorentz transformation. The form in common use for Scalar Hyper Central Potential $(U_0(x))$ and vector Hyper Central Potential $(V_0(x))$ is often taken as follows:

$$-i\vec{\alpha}\cdot\vec{\nabla}\psi(x) + \beta\left[m + U_0(x)\right]\psi(x) + V_0(x)\psi(x) = \varepsilon\,\psi(x) \tag{6}$$

From Eqs. (3, 4 and 5)

$$V_0(x) = U_0(x) = \frac{1}{2}A(x) \tag{7}$$

The eigenspinor of (5) denoted by $\psi_{jj_3}(x)$ is rewritten as

$$\begin{aligned}(\sigma.P)\chi + (m + U_0(x) + V_0(x))\varphi = \varepsilon\varphi \\ (\sigma.P)\varphi - (m + U_0(x) - V_0(x))\chi = \varepsilon\chi\end{aligned} \tag{8}$$

Here
$\varphi = g_\gamma(x)Y_{jl}^{j_3}(\hat{x})$ *and* $\chi = if_\gamma(x)Y_{jl'}^{j'_3}(\hat{x})$. Now we combine with two equations (8) for the Dirac upper component and from Eqs. (7, 8) we have

$$\frac{P^2 g_\gamma(x)}{m + \varepsilon} + (m - \varepsilon + A(x))g_\gamma(x) = 0 \tag{9}$$

The internal quark motion is usually described by means of the Jacobi relative coordinates. After separating the common motion, the P^2 operator of a quark in the 3q system becomes ($\hbar = c = 1$) [10]

$$P^2 = -(\nabla_\rho^2 + \nabla_\lambda^2) = -(\frac{d^2}{dx^2} + \frac{5}{x}\frac{d}{dx} + \frac{L^2(\Omega)}{x^2}) \tag{10}$$

Hence

$$g''_{\gamma}(x)+\frac{5}{x}g'_{\gamma}(x)+\frac{L^2(\Omega)g_{\gamma}(x)}{x^2}+(\varepsilon^2-m^2-(\varepsilon+m)A(x))g_{\gamma}(x)=0 \quad (11)$$

with $A(x)$ given by (5), where $L^2(\Omega)=-\gamma(\gamma+4)$ is the grand orbital operator and γ is the grand angular quantum number given by $\gamma=2n+l_{\rho}+l_{\lambda}$. Using the method used by Znojil [21, 22] we find the upper component $g_{\gamma}(x)$ of the Dirac hyper central spinor. Now for the eigenfunction $g_{\gamma}(x)$ we make an ansatz [11, 23- 26].

$$g_{\gamma}(x)=h(x)\exp[Z(x)] \quad (12)$$

With $h(x)$ and $Z(x)$ given by

$$\begin{cases} h(x)=1+\alpha_1 x \\ Z(x)=-\dfrac{1}{2}\alpha x^2+\delta\ln x \end{cases} \quad (13)$$

This implies

$$\begin{aligned} &g''_{\gamma}(x)+\frac{5}{x}g'_{\gamma}(x)= \\ &\left[Z''(x)+Z'^2(x)+\frac{h''(x)+2Z'(x)h'(x)}{h(x)}+\frac{5(h'(x)+h(x)Z'(x)}{h(x)x}\right]g_{\gamma}(x) \end{aligned} \quad (14)$$

Then our purpose is to find the fraction of the power of x to the one on the left hand side of Eq. (11) corresponding to the potential and energy. A comparison of Eq. (14) with Eq. (11) yield α $,\alpha_1$ and γ.

$$\begin{cases} \alpha=\sqrt{a_1}=\sqrt{a(\varepsilon+m)}=\left[\dfrac{(\varepsilon+m)(\varepsilon-m)}{2(\gamma+4)}\right] \\ \alpha_1=-\left[\dfrac{2(a(\varepsilon+m)^{\frac{1}{2}}}{2\gamma+5}\right]^{\frac{1}{2}} \\ \delta=\gamma,-\gamma-4 \end{cases} \quad (15)$$

Taking $\delta=\gamma$ leads to the wave function, which is well behaved at the origin.

The potential parameters a and c in the absence of the center of mass correction are as follows:

$$a=\frac{(\varepsilon+m)(\varepsilon-m)^2}{4(\gamma+4)^2} \quad (16)$$

$$c=\left[(\frac{\varepsilon-m}{\varepsilon+m})(\frac{2\gamma+5}{\gamma+4})\right]^{\frac{1}{2}} \quad (17)$$

We try to solve this problem in the presence of the center of mass correction, where we have

$$M'=M+E_{cm}=3\varepsilon \quad (18)$$

Here M' is the corrected nucleon mass with center of mass energy E_{cm}.
For three quarks with energy ε and mass m, from Eq. (9) we have

$$\left[\sum_{i=1}^{3}\frac{d^2}{dr_i^2}+\sum_{i=1}^{3}(\varepsilon+m)A(r_i\ )-3(\varepsilon^2+m^2)\right]\prod_{i=1}^{3}\varphi_i=0\,. \tag{19}$$

From Jacobin coordinates this separates into three equations for ρ,λ and R, where one of them determines the center of mass

$$\vec{R}=\frac{1}{3}(\vec{r}_1+\vec{r}_2+\vec{r}_3) \tag{20}$$

And the other two equations, ρ and λ, have been combined as hyper central equations which were discussed previously. Let $\eta=\sqrt{3}R$ then

$$\left[-\frac{d^2}{d\eta^2}+A_1(\eta)-(\varepsilon^2-m^2)\right]\varphi(\eta)=0\,. \tag{21}$$

Now it is obvious that the center of mass energy is

$$E_{cm}=(\varepsilon^2-m^2)^{\frac{1}{2}}\,. \tag{22}$$

From Eq. (18), and using Bogoliuabv's assumption $M'=3\varepsilon$, and assuming $\xi=\frac{m}{\varepsilon}$, then

$$M'=M+E_{cm}=3\varepsilon=\frac{M}{3-\sqrt{1-\xi^2}}\,. \tag{23}$$

Now the potential parameters a and c and the parameter α_1 in the presence of the center of mass correction are as follows:

$$a=\frac{(1+\xi)(1-\xi)M^3}{27(3-\sqrt{1-\xi^2})[4(\gamma+4)]^2} \tag{24}$$

$$c=\left(\frac{1-\xi^{\frac{1}{2}}}{1+\xi}\right)\left(\frac{2\gamma+5}{\gamma+4}\right)^{\frac{1}{2}} \tag{25}$$

$$\alpha_1=\frac{-(1-\xi^2)^{\frac{1}{2}}M}{3(2\gamma+5)^{\frac{1}{2}}(\gamma+4)(3-\sqrt{1-\xi^2})} \tag{26}$$

Eq (24) is a relationship between quark energy and the nucleon mass M and the potential parameters a. Eqs. (12, 13, 15, 24, 25) are used to find the upper component of the Dirac hypercentral spinor $g_\gamma(x)$ of the nucleon with mass M and the parameter ξ as follows:

$$g_\gamma(x)=\left[x^\gamma-\frac{yx^{\gamma+1}}{(2\gamma+5)^{\frac{1}{2}}}\right]e^{-\frac{y^2x^2}{4}} \tag{27}$$

where

$$y = \frac{(1-\xi^2)^{\frac{1}{2}} M}{3(4+\gamma)(3-\sqrt{1-\xi^2})} \tag{28}$$

The lower component $f_\gamma(x)$ of the Dirac hypercentral spinor can be found from (8). The normalized spin $\frac{1}{2}$ positive parity solution of the quark under standard hyperspherical potential (4, 5) is introduced by the following form:

$$\psi_\gamma(x) = \begin{bmatrix} x^\gamma - \dfrac{y x^{\gamma+1}}{(2\gamma+5)^{\frac{1}{2}}} \\ \dfrac{-3i\vec{\sigma}\cdot r(3-\sqrt{1-\xi^2})}{(1+\xi)M}\left(\gamma x^{\gamma-1} - \dfrac{y}{2}x^{\gamma+1} + \dfrac{y^3x^2 - 2y(\gamma+1)x^\gamma}{2(2\gamma+5)^{\frac{1}{2}}}\right) \end{bmatrix} e^{\frac{-y^2x^2}{4}} \tag{29}$$

This wave function is different from the standard MIT bag wave function. From Eq. (29), the bag radius x_b is determined by solving the boundary equation. This shows that the normal component of vector current vanishes at the surface of the baryon bag, just like as in the MIT bag model [1].

$$\frac{\beta y^3 x_b^3}{(2\gamma+5)^{\frac{1}{2}}} - 2\left[\frac{y}{(2\gamma+5)^{\frac{1}{2}}} + \frac{\beta y^2}{2}\right] x_b^2 + 2\left(1 - \frac{\beta y(\gamma+1)}{(2\gamma+5)^{\frac{1}{2}}}\right) x_b + 2\beta\gamma = 0 \tag{30}$$

where

$$\beta = \frac{3(3-\sqrt{1-\xi^2})}{(1+\xi)M} \tag{31}$$

The bag radius is determined by solving the above equation. In order to solve this equation for different values of $\gamma(\gamma = 0,1,2,\ldots)$, first of all let's put $\gamma = 0$ for the ground state.

$$\beta y^2 x_b^2 - 2\left[y + \frac{\sqrt{5}}{2}\beta y^2\right] x_b - (\beta y - \sqrt{5}) = 0 \tag{32}$$

The bag radius (x_b) is determined by solving the above equation.

$$x_b = \frac{y + \frac{\sqrt{5}}{2}\beta y^2 - \sqrt{(y + \frac{\sqrt{5}}{2}\beta y)^2 + \beta y^3(\beta y - \sqrt{5})}}{\beta y^2} \tag{33}$$

From equations (28), (31) and (33) the bag radius (x_b) directly depends upon the quark and the nucleon masses.

3. 1. RATIO OF g_A TO g_V FOR NUCLEON

In this section, it is explained how the ratio $\frac{g_A}{g_V}$ is used as an input .The ratio of axial vector coupling constant g_A to vector coupling constant g_V in the relativistic case satisfies [1]

$$\frac{g_A}{g_V} = \frac{5}{3}(1 - 2\langle l_z \rangle) = \frac{5}{3}(1 - 2\langle \psi_\gamma | l_z | \psi_\gamma \rangle) \tag{34}$$

Eq (34) contains unknown parameters ξ and the hadrons mass M. In order to find the parameter ξ for proton (M=938 Mev), as an example, $\frac{g_A}{g_V}$ can be taken as 1.254 ± 0.006 (or $\langle l_z \rangle = 0.123 \pm 0.002$). This value is measured experimentally [18], so we get $\xi = 0.597 \pm 0.012$ and from Eqs. (16) and (17) the parameters of the potential a and c can be found for $\gamma = 0$. Now from these we can get $m_q = (186.241 \pm 0.012)$ Mev. This result is well within the expected range.

3. 2. NUCLEON CHARGE- RADIUS

The mean-square charge radius for a hadron $< r_{em}^2 >$ is defined as:

$$\langle r_{em}^2 \rangle = \sum e_q \langle r^2 \rangle_q \tag{35}$$

where

$$\langle r^2 \rangle_q = \int_{bag} r^2 \psi_\gamma^*(\vec{r}) \psi_\gamma(\vec{r}) d^3 r \tag{36}$$

By using the upper and lower components of the spinor (29) and $m_q = 186.241 \pm 0.012$ MeV from the above, the charge-radius of proton Eq. (36) we obtained $< r_{em}^2 >^{\frac{1}{2}} = 0.842 \pm 0.003$ fm. This result is closer to the observed value $0.88 \pm 0.03 fm$ than the previous one of the MIT bag model by 13%.

From Eq. (33) and $\xi = 0.597 \pm 0.012$ the bag radius for proton is $x_b = 1.112 \pm 0.008$ fm

3. 3. NUCLEON MAGNETIC MOMENT

By using the standard definitions of the magnetic moment, it can be shown that the general expression for the magnetic moment of a quark in its ground state is as follows:

$$\mu_q = -\frac{2}{3} e_q N^2 \int_{bag} r^3 f_\gamma(r) g_\gamma(r) dr \tag{37}$$

From the upper and lower components of the spinor (29), the magnetic moment of a proton for $\gamma = 0$ would become $\mu_p = 2.693 \pm 0.016$ nm, close to the experimental value 2.782 nm [8]. There is a considerable improvement over the result of the MIT bag model as shown in Table 1.

The magnetic moment results of our model and the MIT bag model for several baryons are summarized in Table 2. In all cases the values of our model are much closer to observed ones.

3. 4. THE STRONG COUPLING CONSTANT α_s

The strength of hyper Coulomb potential was calculated with regard to the parameter ξ and the grand angular quantum number γ. The short hyper Coulomb-like term in our potential (1) is $\frac{c}{r}$ where c is the color factor, which is $\frac{2}{3}\alpha_s$ for the nucleon, assuming 3 flavors for quarks. Using Eqs. (2) and (17) the effective strong coupling constant α_s is obtained:

$$\alpha_s = \frac{3}{2}\left[\left(\frac{1-\xi}{1+\xi}\right)\left(\frac{2\gamma+5}{\gamma+4}\right) \right]^{\frac{1}{2}}. \tag{38}$$

In the model for the proton where $m_q = 186.241 \pm 0.012$ Mev, it is found that $\alpha_s = 0.753 \pm 0.002$ for a ground state in which $\gamma = 0$. This is in the acceptable range, $0 \le \alpha_S \le 1$. It should be noted that α_s is an effective coupling constant in our model and is not a fundamental quantity.

4. CONCLUSIONS

We present a theorical approach to the internal structure for three valance quarks in a bag with residual hypercentral interaction, among them are asymptotic freedom and confinement. Hence, the results have been improved from the MIT bag model to get better results than many of the improved bag models to a large extent. From Table 1 the static properties of proton, as an example, is seen to be very close to the experimental results. This method, just like MIT bag model, can be applied to a wide variety of baryons and mesons. In general we have considered the proton only, however, in all cases our results are much closer to the experimental values than the other improved bag models. For a few other hadrons we have checked the magnetic moments which show a remarkable improvement.

Comparing Tables 1 and 2 with the MIT bag model and experimental results shows that our model certainly improves the MIT bag model, and hence, can also improve the models in references [2, 3].

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