Iranian Journal of Science & Technology, Transaction A, Vol. 29, No. A3
Printed in The Islamic Republic of Iran, 2005

A NEW METHOD FOR ELECTRIC FIELD OPTIMIZATION IN HIGH POWER LASER MULTILAYERS*

M. H. MAHDIEH** AND M. ALAVI-NEJAD

Department of Physics, Iran University of Science and Technology, Narmak, Tehran, I. R. of Iran
Email: mahdm@iust.ac.ir

Abstract – Spatial distribution of electric field amplitude has an essential role in laser-induced damage multi-layer coating. A new method introduced in this paper to reduce the electric field intensity effectively within the high index layers in multi-layer reflectors that are irradiated by high power lasers. The design is based on specific non-quarter wave pairs, assuming the number of layers is held constant. Using a combination of periodic and non-periodic pairs of a non-quarter wave stack with an appropriate thickness for a specific angle of incidence, minimum electric field can be obtained within the high index layers for the S or P polarization. Different results can be obtained if the non-periodic pairs are placed adjacent to the substrate, or to the ambient. We have shown that by the former design, a lower electric field can be achieved within the layers and the mirror can be optimized. The optimization was carried out for oblique incidence and the effect of angle of incidence was examined for both S and P polarizations.

Keywords – Optical multi-layers, high power laser mirrors, optical damage, electric field reduction

1. INTRODUCTION

Multi-layer optical components such as mirrors, polarizors, and filters exposed to high power laser beams were employed in many applications [1, 2]. Optical damage in the layers due to high electric field intensity is vital in designing such components [3]. The damage of multi-layer coatings in high irradiance laser beams has been the subject of many studies [4]. In a multi-layer coating composed of absorbing or non-absorbing layers, the effect of radiation at any point is directly proportional to the electric field intensity at that point. The electric field associated with the intensity of a high power laser could induce optical damage by the Joule effect and the ionization process. It is believed that the damage of weakly absorbing multi-layer coatings by high intensity laser radiation is due to absorption and the consequent heating of the material to the point of melting, as well as thermal induced stress within the film [5-7]. At relatively long pulse lengths (≥ 10 ns), the joule heating due to absorption is dominant, but at extremely short pulses (≤ 30 ps), dielectric breakdown due to the electric field can be more effective. Therefore, the electric field within the optical layers of such components must be minimized.

Designing dielectric mirrors is based conventionally on quarter-wave stack, but for high irradiance lasers, quarter-wave layers are no longer applicable [1, 4, 8, 9]. In high power laser mirrors, one technique for increasing the laser damage threshold is the use of a quarter wave stack combined

---

*Received by the editor November 13, 2004 and in final revised form August 22, 2005
**Corresponding author

with the exterior non-quarter wave pairs. Such a technique is used to reduce the electric field within the high refractive index material [8, 10]. Although this technique increases the peak electric field strength within the low refractive index layer, the multi-layer damage threshold is increased because the low index material can resist a higher electric field. Significant work has been done on polarizer designs for high power lasers at normal incidence using this technique [2, 11-14].

We have used a different design to reduce the electric field more effectively than the conventional methods. It is shown that using combined periodic and non-periodic non-quarter wave pairs can reduce the electric field within the high index layer more effectively. In this paper, if several low and high index non-quarter wave pairs with a fixed thickness are used in the design, we call them "periodic pairs". On the other hand, if the thickness of each low and high index non-quarter wave pair is different from the next pair, we call them "non-periodic pairs". In this paper, we have presented the results of calculations for designing a multi-layer high power CO2 laser employing combined periodic and non-periodic pairs. The pairs thickness were optimized by using two different schemes. The first method is to place the non-periodic pairs adjacent to the substrate. This is called "substrate scheme" in this paper. An alternative way is to calculate the pairs by placing the non-periodic pairs on the top. This is called "top scheme". From the two approaches, different results were obtained. In designing high power laser mirrors for a particular reflection, beam polarization, and the angle of incidence, an estimation of the optimum number of non-periodic pairs is also very important from an economical viewpoint. The effects of different polarizations and the angle of incidence were also investigated, and an optimum number of non-periodic pairs was estimated.

2. THEORY AND CALCULATION METHOD

There are various ways to compute the electric field distribution in multi-layers exposed to electromagnetic waves. The solution of the wave equation with appropriate boundary conditions can give the electric field distribution within the layers and interfaces. The $\mathbf{E}^+\mathbf{E}^-$ matrix method was used first by Newman [3] and Apfel [15] to compute the electric field. Lee et al. [1] designed mirrors with low electric field amplitude by analyzing admittance loci and electric field distribution. They reduced the electric field peak within the layers by calculating the appropriate thickness of a few layers that were exposed to the laser beam, and therefore maximizing the optical admittance.

We have used the matrix method [16] for calculating the electric field in a multi-layer mirror shown schematically in Fig. 1. In this method, for each stratified optical layer, a characteristic matrix can be considered. The characteristic matrix for a pair of high and low index layers with the refractive index of n_H and n_L can be given by [16]:

$$m_i(\delta z_L, \delta z_H) = \begin{pmatrix} m_{11} & im_{12} \\ im_{21} & m_{22} \end{pmatrix} = \begin{pmatrix} \cos\beta_H & i/Y_H \sin\beta_H \\ iY_H \sin\beta_H & \cos\beta_H \end{pmatrix} \begin{pmatrix} \cos\beta_L & i/Y_L \sin\beta_L \\ iY_L \sin\beta_L & \cos\beta_L \end{pmatrix} \tag{1}$$

where β_H and β_L are given by:

$$\beta_j = \frac{2\pi}{\lambda} n_j \delta z_j \cos\theta_j ,\ j = H, L \tag{2}$$

In relation (2), λ is the laser wavelength, δz_j and n_j are the thickness and the refractive index of layer j, θ_j is the angle of incidence in this layer, and Y_j is the optical admittance proportional to the refractive index of layer j. The parameter Y_j for P and S polarization is defined as follows:

$$Y_j = n_j \sec\theta_j \text{ For P} \tag{3a}$$

$$Y_j = n_j \cos\theta_j \text{ For S} \tag{3b}$$

Accordingly, the characteristic matrix of p pairs can be calculated by:

$$M(\delta z_L, \delta z_H) = \begin{pmatrix} M_{11} & iM_{12} \\ iM_{21} & M_{22} \end{pmatrix} = \prod_{i=1}^{P} m_i(\delta z_L, \delta z_H) \tag{4}$$

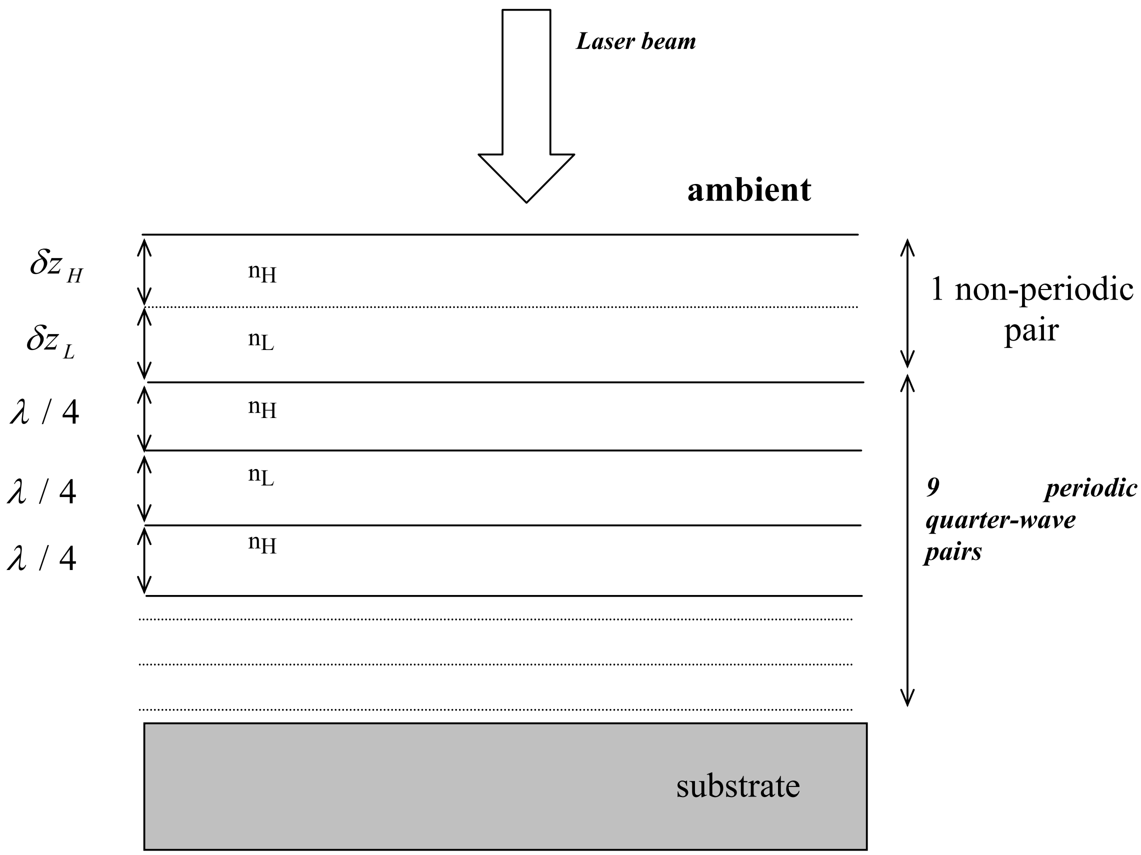

Fig. 1. Schematic diagram of a typical 10 pairs (1 non-periodic and 9 periodic pairs) multi-layer mirror

The electric and magnetic fields $\mathbf{E}_a$ and $\mathbf{H}_a$ at the interface of the top pair and the air can be related to the fields $\mathbf{E}_s$ and $\mathbf{H}_s$ at the substrate by [16]:

$$\begin{pmatrix} \mathbf{E}_a \\ \mathbf{H}_a \end{pmatrix} = M(\delta z_L, \delta z_H) \begin{pmatrix} \mathbf{E}_s \\ \mathbf{H}_s \end{pmatrix} \tag{5}$$

The optical admittance and reflection coefficient of a multi-layer with the characteristic matrix of $M(\delta z_L, \delta z_H)$ are also defined by relations (6) and (7) as:

$$Y(\delta z_L, \delta z_H) = \frac{|\mathbf{H}_a|}{|\mathbf{E}_a|} \tag{6}$$

$$r(\delta z_L, \delta z_H) = \frac{|\mathbf{E}_a^-|}{|\mathbf{E}_a^+|} = \left(\frac{Y_0 - Y(\delta z_L, \delta z_H)}{Y_0 + Y(\delta z_L, \delta z_H)} \right) \tag{7a}$$

$$R(\delta z_L, \delta z_H) = r(\delta z_L, \delta z_H) r^*(\delta z_L, \delta z_H) \tag{7b}$$

In which Y_0 is the admittance of the air.

In relation (5), $\mathbf{E}_a$ is the vector summation of the electric fields of incidence $(\mathbf{E}_a^+)$ and reflected $(\mathbf{E}_a^-)$ waves given by:

$$\mathbf{E}_a = \mathbf{E}_a^+ + \mathbf{E}_a^- \tag{8}$$

Using relations (5), (6), (7-a), and the definition of Y_s, one can write $\mathbf{E}_a$ verses $\mathbf{E}_s$ as:

$$\mathbf{E}_a^+ + \mathbf{E}_a^- = (M_{11} + iM_{12} Y_s)\mathbf{E}_s \tag{9a}$$

$$\mathbf{E}_a^+ - \mathbf{E}_a^- = (iM_{21} / Y_0 + M_{22} Y_s / Y_0)\mathbf{E}_s \tag{9b}$$

$$Y_s = \frac{|\mathbf{H}_s|}{|\mathbf{E}_s|} \tag{9c}$$

From relations (9-a) and (9-b), the amplitude of $\mathbf{E}_a$ verses $\mathbf{E}_s$ can be computed as:

$$|\mathbf{E}_a^+|^2 = \frac{1}{4}\left[(M_{11} + M_{22}\frac{Y_s}{Y_0})^2 + (\frac{M_{21}}{Y_0} + M_{12} Y_s)^2 \right] |\mathbf{E}_s|^2 \tag{10}$$

The electric and magnetic fields $\mathbf{E}$ and $\mathbf{H}$ in an arbitrary point within the layers and at a distance *z* from the substrate can be related to the fields $\mathbf{E}_s$ and $\mathbf{H}_s$ at the substrate by [16]:

$$\begin{pmatrix} \mathbf{E} \\ \mathbf{H} \end{pmatrix} = M''(\delta z_L, \delta z_H) \begin{pmatrix} \mathbf{E}_s \\ \mathbf{H}_s \end{pmatrix} \tag{11}$$

Therefore the electric field in the arbitrary point can be calculated by:

$$|\mathbf{E}(\delta z_L, \delta z_H, z)|^2 = |(M''_{11})^2 + (Y_s M''_{12})^2| |\mathbf{E}_s|^2 \tag{12}$$

In the relation (12), M''_{11} and M''_{12} are the members of matrix M''. The matrix M'' can be found by multiplying the characteristic matrix of the entire stack by the inverse characteristic matrix of a layer with thickness Δz:

$$M'' = \begin{pmatrix} \cos\Delta\beta & -\frac{i}{Y_i}\sin\Delta\beta \\ -iY_i \sin\Delta\beta & \cos\Delta\beta \end{pmatrix} \begin{pmatrix} M_{11} & iM_{12} \\ iM_{21} & M_{22} \end{pmatrix} = \begin{pmatrix} M''_{11} & iM''_{12} \\ iM''_{21} & M''_{22} \end{pmatrix} \tag{13}$$

In relation (13), $\Delta\beta$ is the phase thickness of the incremental section and given by:

$$\Delta\beta = \frac{2\pi}{\lambda} n_j \Delta z \cos\theta_j \,, \; j = H, L \tag{14}$$

In fact, the double-primed matrix (13) represents the characteristic matrix of the stack without the thin surface layer of thickness Δz.

It is more common to use the normalized electric field strength squared $E'(\delta z_L, \delta z_H, z)$, rather than the actual field strength squared $\left|\mathbf{E}(\delta z_L, \delta z_H)\right|^2$. The normalized electric field strength squared $E'(\delta z_L, \delta z_H, z)$ is defined as:

$$E'(\delta z_L, \delta z_H, z) = \frac{\left|\mathbf{E}(\delta z_L, \delta z_H, z)\right|^2}{\left|\mathbf{E}_a^+\right|^2} \tag{15}$$

Using (10) and (12), a more convenient form of normalized field $E'(\delta z_L, \delta z_H, z)$ can be expressed as:

$$E'(\delta z_L, \delta z_H, z) = \frac{\left|\mathbf{E}(\delta z_L, \delta z_H, z)\right|^2}{\left|\mathbf{E}_a^+\right|^2} = \frac{\left|(M''_{11})^2 + (Y_s M''_{12})^2\right|}{\frac{1}{4}\left[(M_{11} + M_{22}\frac{Y_s}{Y_0})^2 + (\frac{M_{21}}{Y_0} + M_{12}Y_s)^2\right]} \tag{16}$$

If the relations (16) and (7-b) are solved simultaneously for a desired angle of incidence and polarization, and considering the following conditions:

a) Maximum reflection from relation (7-b)

b) Minimum normalized electric field from (16)

One can design a mirror with a minimum electric field within the high index layers by calculating the optimum number of pairs and the layer thickness. We have developed computer software to perform such processes, and optimize the multi-layer automatically according to the above conditions.

3. RESULTS AND DISCUSSION

The relations discussed in the previous section were solved simultaneously to design a 20 layer mirror with a reflectivity of ~99.99% for a 10 KW, CO2 laser ($\lambda = 10.6\mu$). The substrate is assumed to be GaSb ($n = 3.84$, $k = 0.002$), and the high threshold damage optical layers are GaP ($n = 2.9$), and KBr ($n = 1.52$).

Figure 2 shows the electric field distribution within the layers for normal incidence using three different schemes:

a) Periodic quarter wave stack

b) Periodic quarter wave stack with a non-quarter wave pair at the top ("top scheme")

c) Periodic quarter wave stack with a non-quarter wave pair at the substrate ("substrate scheme")

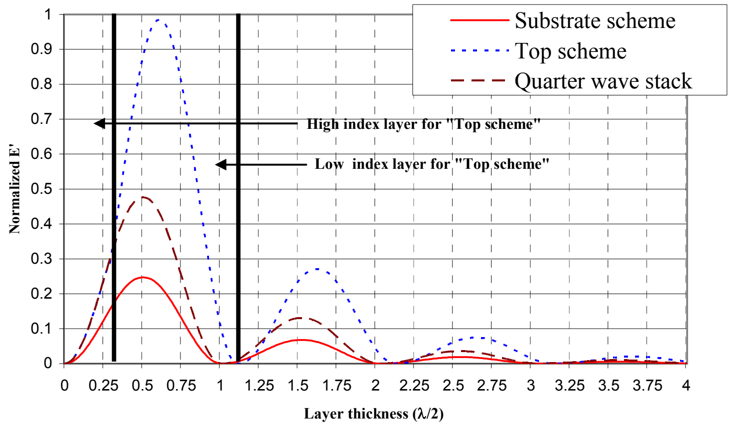

Fig. 2. The electric field distribution within the optical layers for normal incidence, using: "quarter- wave stack", "top scheme", and "substrate scheme"

The thickness of the first pair for the "top scheme" could be shown in this figure, but not for the "substrate scheme". The results are summarized in Table 1. Since the electric field is diminished severely within the layers and goes almost to zero after 4 pairs, the figure shows the electric field for only 4 pairs. From the figure, it can be seen that for the second and third design, the maximum normalized electric field E' (within the high index layers) is smaller than that of the quarter wave design. Obviously by using the third scheme, a higher optical damage threshold can be achieved for the multi-layer. The results show that the electric field, at the interface of the first high and low index layers, can be reduced by nearly 40% for the "top scheme" and 50% for the "substrate scheme". These results prove that for normal incidence, optical damage threshold of the multi-layer will be increased to its higher value if the "substrate scheme" is applied for the mirror design.

Table 1. The normalized electric field and the thickness of layers for normal and 40-degree incidence

	Quarter wave stack			"Top scheme"			"Substrate scheme"		
	0°	$40^\circ S^*$	$40^\circ P^*$	0°	$40^\circ S^*$	$40^\circ P^*$	0°	$40^\circ S^*$	$40^\circ P^*$
E'	0.47	0.43	0.51	0.28	0.24	0.23	0.25	0.21	0.21
$\delta z_{H1}\ (\lambda/2)$	0.5			0.28	0.28	0.28	0.64	0.65	0.63
$\delta z_{L1}\ (\lambda/2)$	0.5			0.82	0.93	0.90	0.24	0.26	0.31

* δz_H , and δz_L belong to the first pair at the top, or at the substrate for the "top scheme", and "substrate scheme"

The calculations proceeded for oblique incidence with different polarization S and P. This is very useful for designing a mirror as reflector, and a splitter for high power lasers. The conditions of Fig. 2 are applied again for oblique incidence. Figures 3a, and 3b show the electric field within the optical layers for a laser beam with an incident angle of 40°, and with polarization S and P respectively. It must be noted that according to the relation (2), for the case of oblique incidence, the periodic pairs are quarter-wave no longer and must be modified. Therefore, for both the "top", and "substrate" scheme, 9 periodic non-quarter wave pairs are used with one different non-quarter wave pair. These results are also summarized in Table 1. It can be seen that for the S polarization at the interface of the first two layers, a reduction of roughly 45% and 52% for the electric field were achieved for the "top scheme" and "substrate scheme" respectively. Similar results were attained for the P polarization.

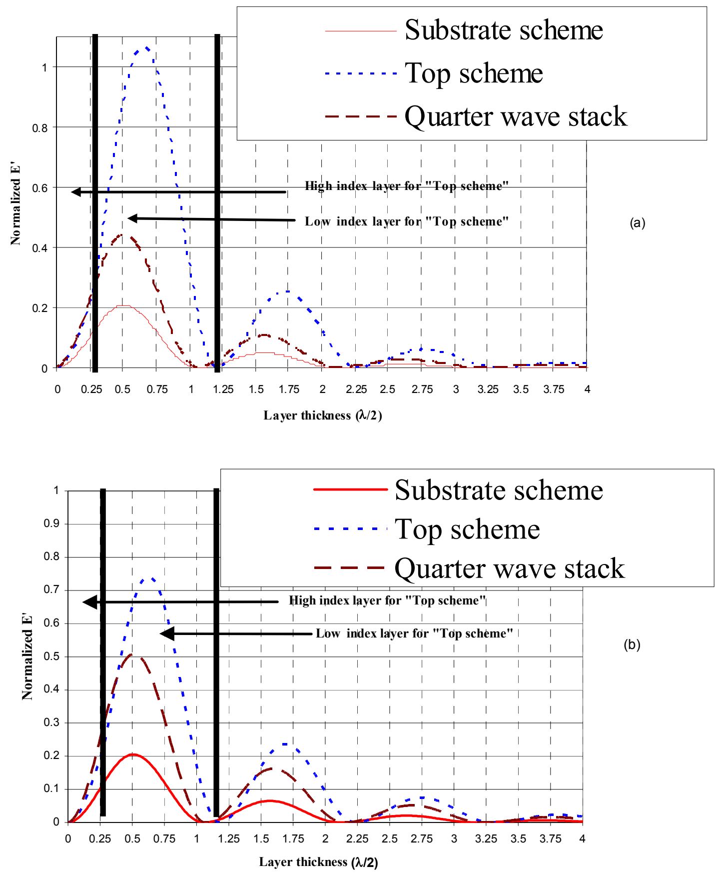

Fig. 3. The electric field within the optical layers for a laser beam with an incident angle of 40° with: (a) polarization S, and (b) polarization P

The optical damage threshold of the multi-layer can be reduced more effectively by increasing the number of non-periodic pairs for both the "top scheme" and "substrate scheme". It must be noted again that the total number of pairs is kept fixed in the design and if the number of non-periodic pairs is increased, the number of periodic pairs must be decreased. Applying "substrate scheme" for the normal incidence, we carried out the calculation for a 10 pair multi-layer, but using 1 to 9 non-periodic pairs. Figure 4 shows some results of these calculations. It can be seen that the normalized electric field E' decreased by increasing the number of non-periodic pairs. Using 2 non-periodic pairs, the electric field E' diminished by a factor of 40%, while 3 and 4 pairs can provide further reduction to 50%, and 60%. It can be concluded that increasing the number of non-periodic pairs will dampen the electric field within the high index layers more effectively. Nevertheless, an optimum number of such pairs give an optimum electric field within the layers. Figure 5a and 5b give graphs of the peak of electric field E' (for normal incidence as well as oblique incidence) verses the number of non-periodic pairs using the "substrate scheme" design for both polarizations. The total number of periodic and non-periodic pairs is again held constant (10 pairs). For oblique incidence and S polarization, similar results to that of the normal incidence were obtained. The trend of reduction for the electric field is almost the same for all angles, but greater angles give smaller values for E'. The P polarization behaves somehow differently. Although the trend of E' verses the number of non-periodic pairs is the same as the S polarization, the performance is not necessarily similar to P polarization for different incident angles. From Figs. 5a and 5b it can be inferred that considerable reduction in the electric field is achievable by increasing the number of non-periodic pairs from 1 to 4. From these figures, it can be also seen that further non-periodic pairs have a minor effect in decreasing the electric field.

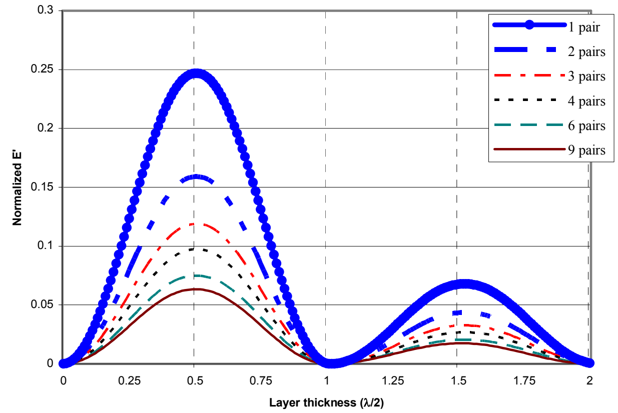

Fig. 4. The electric field distribution within the optical layers for normal incidence using: "substrate scheme" with 1 to 9 non-periodic non-quarter wave pairs

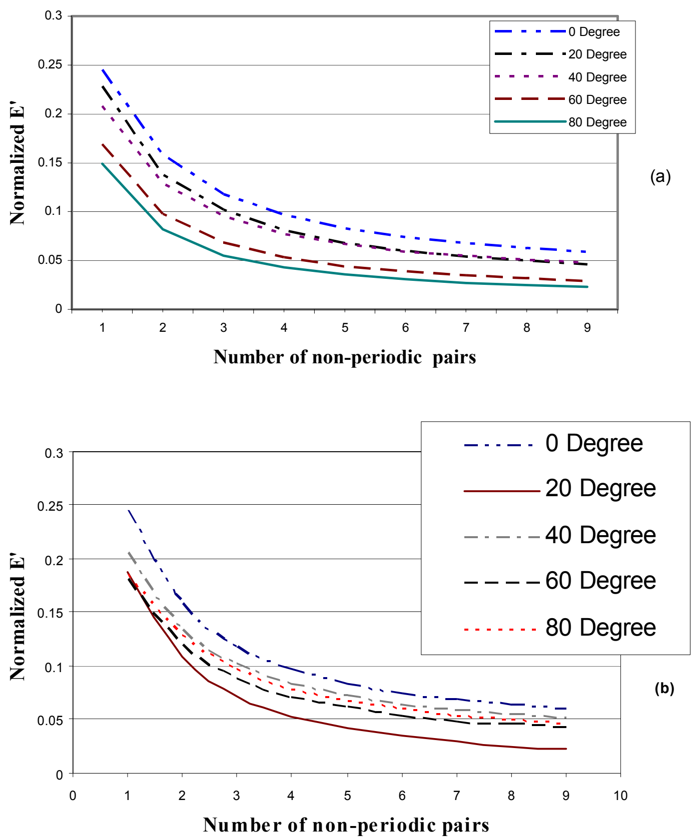

Fig. 5. The peak of the electric field E' for normal and oblique incidence verses the number of non-periodic pairs, using the "substrate scheme" design for: (a) polarization S, and (b) polarization P

4. CONCLUSIONS

In this paper, we have presented the results of calculations for designing a multi-layer high power CO2 laser using a new technique. The matrix method is used for calculating the electric field within the layers in multi-layer high power reflectors. For oblique incidence, employing combined periodic and non-periodic non-quarter wave pairs of high and low index materials, multi-layer mirrors for high power lasers can be designed with a minimum electric field within the high index material. Obviously for the normal incidence, periodic pairs would be quarter-wave stack. The optical damage threshold of the multi-layer can be increased if the thickness of the non-periodic pair is optimized. Two different schemes, namely "top scheme", and "substrate scheme" were used to optimize the pair thickness.

From the two approaches, different results were obtained. The results show that the number of non-periodic pairs in the design (holding the total pairs constant) is crucial. The effects of different polarizations and the angle of incidence were also evaluated, and the optimum number of non-periodic pairs was estimated. From the results it can be concluded that for either normal or oblique incidence, the "substrate scheme" more efficiently diminishes the electric field within the high index layer and interfaces, therefore increasing the optical damage threshold of the multi-layer. Such results have not been reported yet as we are aware. Therefore applying the "substrate scheme" one can enhance high power multi-layer lasers more effectively.

REFERENCES

1. Lee, C. & Cheu, C. W. (1987). High power CO2 Laser Mirror: A design. *Applied Optics 26* (13), 2544.
2. Stolz, C. J. (1999). Brewster's angle thin film plate polarizer design study from an electric field perspective. *SPIE Vol. 3738*, 347.
3. Newmam, B., Gill, D. H. & Faulkner, G. (1976). Influence of standing wave fields on the laser damage resistance of dielectric films. *NBS Special Publication 435*.
4. Arnon, O. & Baumericter, P. (1980). Electric field Distribution and the reduction of laser damage in multi-layers. *Applied Optics 19* (11), 1853-1855.
5. Fan, Z., Zhao, Q. & Wu, Z. L. (1997). Temperature field design of optical thin film coatings. *SPIE Vol. 2966*, 362.
6. Bauer, S. M., Kolvalev, A. M., Tovstic, P. E., Petror, M. B. & Tikhomirov, V. V. (1985). Numerical investigation of the temperature deformation of an optical mirror. *Sov. J. Opt. Technol., 52*(9), 529-531.
7. Hue, J., Mertin, J. & Kovac, B. (1994). Critical temperature of mirrors under high power continuous wave CO2 laser irradiation. *SPIE Vol. 2114*, 201.
8. Apfel, J. H. (1977). Optical coating design with reduced electric field intensity. *Appl. Opt. 16*, 1880.
9. Demichelis, F., Demichelis, F., Mezzehi, E., Tallone, L. & Tresso, E. (1984). Optimization of optical parameters and electric field distribution in multi-layers. *Appl. Opt. 23*, 165.
10. Gill, D. H. & Mcleod, J. (1978). Use of non-quarter wave design to increase the damage resistance of reflectors at 532 and 1064 nm. In laser induced damage in optical materials: Glass, A. J. & Guenther, A. H. (1977). eds., *NBS Spec. Publ. 509*, 260-270.
11. Motokoshi, S., Yoshida, H., Yoshida, K. & Nakai, S. (1993). Development of newly designed polarizer for high power laser. *Jpn. J. Appl. Phys. 32* L1660-L1662.
12. Gu, P. F. & Tang, J. F. (1994). Laser induced damage resistance of thin film polarizes prepared by ion assisted deposition. *Opt. Letters 19*, 81-83.

13. Buchman, W. W. (1975). Index matching reduces peak electric fields in thin film polarizer. *App. Opt. 14*, 1220-1224.
14. Monga, J. C. (1989). Multi-layer thin film polarizers with reduced electric field intensity. *J. of Moderm Optics, 36*, 769-784.
15. Apfel, J. H. (1976). Electric fields in multi-layers at oblique incidence. *Appl. Opt. 15* (10), 2339.
16. Born, M. & Wolf, E. (1985). Principles of Optics. Sec.1-6 Sixth edition, *Pergamon Press*.