IJST, Transactions of Civil Engineering, Vol. 39, No. C1, pp 81-92
Printed in The Islamic Republic of Iran, 2015

DETERMINATION OF MODAL DAMPING RATIOS FOR NON-CLASSICALLY DAMPED REHABILITATED STEEL STRUCTURES*

A. SIVANDI-POUR**1, M. GERAMI[2] AND A. KHEYRODDIN[3]

[1]Faculty of Civil Engineering, Graduate University of Advanced Technology, Kerman, I. R. of Iran
Email: abbas.sivandi@gmail.com
[2,3]Faculty of Civil Engineering, Semnan University, Semnan, I. R. of Iran

Abstract– There are many reasons for rehabilitation of existing buildings. Adding stories is one of the most common reasons. When a steel building is retrofitted by concrete jacketing for adding stories, this system contains several structural systems. These systems are composite concrete and steel systems in initial stories, welded steel system in middle stories and cold-formed steel frames in upper stories. Dynamic analysis of hybrid structures is usually a complex procedure due to various dynamic characteristics of each part, i.e. stiffness, mass and especially damping. Availability of different damping factors causes a higher degree of complication for evaluating seismic responses of hybrid systems. Due to using several structural systems, an existing building is changed to hybrid system. Damping matrix of these structures is non-classical. Also, the nonlinear software is not able to analyze these structures precisely. In this study, a method and graphs have been proposed to determine the equivalent modal damping ratios for rehabilitated existing steel buildings for adding stories.

Keywords– Hybrid buildings, damping ratio, rehabilitation, non-classical damping, nonlinear analysis

1. INTRODUCTION

Rehabilitation is necessary when the assessment of structural performance results in insufficient capacity to resist the forces of expected intensity and acceptable limits of damage. The rehabilitation of an existing building requires an appreciation for the technical, economic and social aspects and the structural condition of buildings; therefore, it is hard to develop a single typical technique for rehabilitation. Hence, many instructions have been developed in the field of seismic rehabilitation [1, 2]. One of the earliest guidelines published for the evaluation and retrofit of the buildings is ATC-40 [3]. After that, FEMA 273 [4] and FEMA 356 [5] were published respectively as guidelines and pre-standards for the seismic rehabilitation of the buildings. Afterwards, nonlinear analysis was upgraded in the ASCE 41-06 [6] as a standard for seismic rehabilitation of the buildings.

There are many important reasons for rehabilitation of existing buildings such as upgrading the design codes, changing the building usage, adding to the stories, structural damages in case of destructive events, and design or construction faults. There are various techniques for retrofitting. The retrofitting strategy of the existing buildings can be conducted by inserting lateral resistant elements (such as shear walls, braces, etc.), strengthening structural elements (such as jacketing, FRP, cover plates, etc.), decreasing demand (reducing the weight, removing the upper stories, changing the use of the buildings, using seismic isolator), etc [7].

Cold-formed steel (CFS) products are commonly used in all areas building industry. The use of cold-formed steel construction materials has become more and more popular since the initial introduction of

*Received by the editors February 2, 2013; Accepted April 13, 2014.
**Corresponding author

codified standards in 1946. These building materials encompass columns, beams, joists, studs, floor decking, built-up sections and other components. The strength of elements used for design is usually governed by buckling. Cold-formed steel members are the most useful systems for adding additional stories in existing buildings due to their light weight, economy in transportation and shipping, fast and easy erection, high strength-to-weight ratio, and being easily out sourced. If the site is severely restricted frames can be assembled on the roof. Working with cold-formed steel members is also not so weather dependent as it is with other materials [8, 9].

When a steel building is retrofitted by concrete jacketing for adding stories, this system contains several structural systems. These systems are composite concrete and steel systems in initial stories, welded steel system in middle stories and cold-formed steel frames in upper stories. By using several structural systems, an existing building is changed to hybrid system.

Dynamic response of hybrid structures has some complications. One of the reasons is the different stiffness of the parts of structure and another reason is non-uniform distribution of materials and their different features such as damping in main modes of vibration. Damping is one of the effective factors in determining dynamic response of a structure [10]. Lee et al. performed some studies by direct solution and without using time history analysis by substituting Multi Degrees of Freedom (MDOF) structure by adding dampers to Single Degree of Freedom (SDOF) structure [11]. In the field of complex eigen-vectors, Villaverde [12] presented one method for using complex modes of an irregular building by maximum response. In this method, motion equations are reviewed in the state-space and their modal specifications are evaluated in spectral analysis method. Kim et al. [13] presented a solution method to solve the eigenvalue problem raised in the dynamic analysis of non-classically damped structural systems. This method was obtained by applying the modified Newton-Raphson technique and the orthonormal condition of the eigenvectors. Huang et al. [14] reviewed a series of MDOF irregular structures in a different method in which the reinforced concrete part had lower degrees of freedom and the metal part had higher degrees of freedom. In the method presented by these researchers, in the first stage, regular damping ratio of the whole building must be obtained by trial and error method, and then the whole building is modeled by a 2-DOF system and modal damping ratio is calculated by predictive approximate method with the assumption that the normalized damping matrix is diametric.

Availability of different damping factors causes a higher degree of complication for evaluating seismic responses of hybrid systems. On the one hand, the available design regulations do not present analytic methods for determining structural systems damping and on the other hand, damping matrix of these structures is non-classical. Also, the nonlinear software is not able to analyze these structures precisely. For dynamic analysis of these structures by using the available software, an equivalent modal damping ratio must be generalized to the whole structure. One general method for determining the damping of these structures is such that two structures are modeled as three separate systems, each of them considered with its damping ratio, and the interaction between the three systems is ignored [15,16]. This method revealed many errors and is very different from the real behavior of the structure.

There are no studies concerning the determination damping ratio of non-classically damped rehabilitated buildings. This paper presents a method and graphs for determination of equivalent modal damping ratio of non-classically damped rehabilitated existing steel buildings for adding stories. In the proposed method in this study, the added storey(s) has been considered for calculation of equivalent modal damping ratios for non-classically damped hybrid rehabilitated steel buildings. In the proposed method, hybrid buildings are considered to have three structural systems of composite steel and concrete (rehabilitated storey(s)), existing steel system and added storey(s) (cold-formed steel frames). Using the proposed methods and obtained graphs for determining modal damping ratios in dynamic analysis and nonlinear analysis of rehabilitated steel buildings require more attention in comparison with the methods presented so far and the obtained response is closer to the structure's real behavior.

2. MODELING THE HYBRID REHABILITATED BUILDING

Despite the wide application of concrete jacketing in strengthening the steel columns of existing structures, research on the jacketing of such columns whilst under load is still in the early stages. From the middle of the last century, concrete jackets have been provided primarily to serve as protection against corrosion and fire, and thus were assumed not to resist structural loading [17, 18]. With present-day modern steel–concrete composite construction techniques, the stiffness and strength gain effects have been taken into account for steel columns strengthened with concrete jackets. Shear studs and the provision of an adequate amount of reinforcement have been applied during construction to ensure composite action between the existing steel column and the surrounding concrete jacket [19,20]. As was reported by Colaco et al. [21], the exterior steel columns of a 47-storey building were jacketed with reinforced concrete to meet new service requirements. One of the early methods to review the interaction between steel and concrete in composite sections is introduced by Basu and Sommerville [22]. In a research done by Liu et al., a method was presented for the second-order analysis of these sections [23].

There are two general methods for dynamic analyzing of hybrid structures with non-classical damping. The first method is the direct method. Direct method includes structural dynamic methods [24, 25] integration, [26, 27], etc.; using direct method requires the calculation of stiffness, mass and damping matrices. The calculation of stiffness and damping matrices in direct method is complicated and becomes even more complicated as the degrees of freedom of the structure increase. Hence, using direct method is very time-consuming and difficult and it is also impossible for the structures with higher degrees of freedom. The available software is not able to form damping matrices and analyze these structures because the damping matrix of these irregular structures is non-classical. The second method is to devote an equivalent damping to the whole structure and they use the available software.

In the method proposed in this study, rehabilitated structure, steel structure and additional stories are appropriately substituted with 3-DOF structure as presented in Fig. 1 in order to form a hybrid structure.

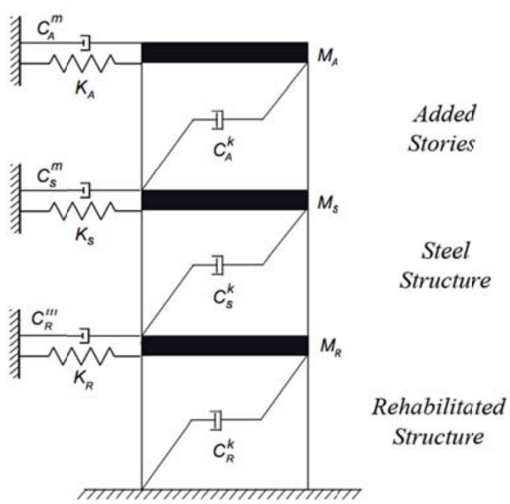


Fig. 1. Equivalent 3-DOF structure

In the next step, eigenvalues of each rehabilitated, steel structure and added stories are obtained. Eigenvalues of each part include the first mode related frequency values (ω_i^1), mass (M_i^1) and modal stiffness (K_i^1). *i* can be related either to the rehabilitated structure, steel structure and stories added. In the numerical method investigated in this study, r indicates the rehabilitated structure (composite part), s presents the stories without rehabilitation (steel part) and a presents the added stories.

3. THE PROPOSED METHOD FOR DETERMINING MODAL DAMPING

The main MDOF structure is assumed to have separate Rayleigh damping at each section. This is to say that damping ratio is proportional to stiffness and mass in each degree of freedom. So, each part of the equivalent 3 degrees of freedom has two types of damping C_i^k and C_i^m (damping proportional to stiffness and mass). Mass (M) and stiffness (K) matrices are calculated for each of the three parts of rehabilitated structure, steel structure and stories added. Each of them shows the matrix forming the related part of the overall structure. Stiffness matrix of hybrid structure is obtained from the Eqs. (1).

$$K = K^r + K^s + K^a \quad \in \mathbf{M_3}(\mathbb{R}) \tag{1-1}$$

$$K^r = [k_{i,j}^r]_{3\times 3} \quad , \quad k_{i,j}^r = \begin{cases} k_s + k_r & : i,j = 1 \\ 0 & : o.w \end{cases} \tag{1-2}$$

$$K^s = \left[k_{i,j}^s\right]_{3\times 3} \quad , \quad k_{i,j}^s = \begin{cases} -k_s & : (i = 2, j = 1)\,, (i = 1, j = 2) \\ k_s + k_a & : i,j = 2 \\ 0 & : o.w \end{cases} \tag{1-3}$$

$$K^a = \left[k_{i,j}^a\right]_{3\times 3} \quad , \quad k_{i,j}^a = \begin{cases} -k_a & : (i = 2, j = 3)\,, (i = 3, j = 2) \\ k_a & : i,j = 3 \\ 0 & : o.w \end{cases} \tag{1-4}$$

The structure's overall mass matrix is obtained from Eq. (2-1) and mass matrix of each one of the three parts forming the structure is calculated from the Eqs. (2-2) to (2-4).

$$M = M^r + M^s + M^a \quad \in \mathbf{M_3}(\mathbb{R}) \tag{2-1}$$

$$M^r = [m_{i,j}^r]_{3\times 3} \quad , \quad m_{i,j}^r = \begin{cases} m_r & : i,j = 1 \\ 0 & : o.w \end{cases} \tag{2-2}$$

$$M^s = [m_{i,j}^s]_{3\times 3} \quad , \quad m_{i,j}^s = \begin{cases} m_s & : i,j = 2 \\ 0 & : o.w \end{cases} \tag{2-3}$$

$$M^a = [m_{i,j}^a]_{3\times 3} \quad , \quad m_{i,j}^a = \begin{cases} m_a & : i,j = 3 \\ 0 & : o.w \end{cases} \tag{2-4}$$

Modal frequencies of ω_1, ω_2 and ω_3 are obtained by classic analysis method for 3-DOF structure [15, 28]. Rehabilitated story's damping matrix is calculated from Eqs. (3-1).

$$C^r = a_{0,r} M^r + a_{1,r} K^r \tag{3-1}$$

$$a_{0,r} = \frac{2\times\xi_r\times\omega_1\times\omega_2}{\omega_1+\omega_2} \tag{3-2}$$

$$a_{1,r} = \frac{2\times\xi_r}{\omega_1+\omega_2} \tag{3-3}$$

Steel storey's damping matrix is obtained from Eq. (4-1).

$$C^s = a_{0,s} M^s + a_{1,s} K^s + a_{2,s} K^s {m^s}^{-1} K^s \tag{4-1}$$

$$a_{0,s} = \frac{2\times\xi_s\times\omega_1\times\omega_2\times\omega_3\times(\omega_1+\omega_2+\omega_3)}{(\omega_1+\omega_2)\times(\omega_1+\omega_3)\times(\omega_2+\omega_3)} \tag{4-2}$$

$$a_{1,s} = \frac{2\times\xi_s\times\left(\omega_1^2+\omega_1\times\omega_2+\omega_1\times\omega_3+\omega_2^2+\omega_2\times\omega_3+\omega_3^2\right)}{(\omega_1+\omega_2)\times(\omega_1+\omega_3)\times(\omega_2+\omega_3)} \tag{4-3}$$

$$a_{2,s} = -\frac{2\xi_s}{(\omega_1+\omega_2)\times(\omega_1+\omega_3)\times(\omega_2+\omega_3)} \tag{4-4}$$

Added story's damping matrix is obtained from Eq. (5-1).

$$C^a = a_{0,a}M^a + a_{1,a}K^a \tag{5-1}$$

$$a_{0,a} = \frac{2\times\xi_a\times\omega_2\times\omega_3}{\omega_2+\omega_3} \tag{5-2}$$

$$a_{1,a} = \frac{2\times\xi_a}{\omega_2+\omega_3} \tag{5-3}$$

Finally, damping matrix of hybrid structure would be the summation of 3 base matrices as in Eq. (6) in which its ratios are from mass proportionality and stiffness of the overall damping matrix of 3-DOF structure.

$$C = \sum_i C^i, \quad i = r, s, a \tag{6}$$

Special frequencies' ratio R_ω and weight ratio R_m are defined as in equation 7 in order to specify the system response according to the features of three constituents.

$$R_{\omega 1} = \frac{\omega_a}{\omega_r},\ R_{m1} = \frac{M_a}{M_r}, \quad R_{\omega 2} = \frac{\omega_s}{\omega_r}, \quad R_{m2} = \frac{M_s}{M_r} \tag{7}$$

In this stage, a time history analysis is applied for equivalent 3-DOF structure according to equation 8 in order to obtain the equivalent modal damping ratios.

$$M\{\ddot{y}\} + C\{\dot{y}\} + K\{y\} = -Mr\ddot{x}_g \tag{8}$$

In the equation 8, $\{y\}$ is relative displacement vector of MDOF structure and r is equal to $\begin{bmatrix}1\\1\end{bmatrix}$ [15, 29]. The obtained results are equal to the overall acceleration and displacement in each level. Energy balance equation is defined by multiplying matrix transpose $\{\dot{y}\}$ in equation 8, as follows:

$$\frac{d}{dt}\left(\frac{1}{2}\{\dot{y}\}^{\mathrm{T}} M\{\dot{y}\} + \frac{1}{2}\{\mathrm{y}\}^{\mathrm{T}} K\{\mathrm{y}\}\right) = -\{\dot{y}\}^{\mathrm{T}}Mr\ddot{x}_g - \{\dot{y}\}^{\mathrm{T}} C\{\dot{y}\} \tag{9}$$

Equation (8) is reviewed in the state-space. The state-space method is based on transforming the N second-order coupled equations into a set of 2N first-order coupled equations [30, 31]. Equations of dynamic system motion can be recast as:

$$A\dot{u}(t) + Bu(t) = F(t) \tag{10-1}$$

where A,B $\in \mathrm{R}^{2N\times 2N}$ are the system matrices, $F(t) \in \mathrm{R}^{2N}$ the force vector and u$\in \mathrm{R}^{2N}$ is the response vector in the state-space. The parameters of the Eq. (10-1) are obtained from Eq. (10-2).

$$A = \begin{bmatrix} C & M \\ M & O_N \end{bmatrix}, B = \begin{bmatrix} K & O_N \\ O_N & -M \end{bmatrix}, F(t) = \begin{Bmatrix} -Mr\ddot{x}_g \\ O_{N\times 1} \end{Bmatrix}, u(t) = \begin{Bmatrix} y(t) \\ \dot{y}(t) \end{Bmatrix} \tag{10-2}$$

In the equation above, O_N is the N × N null matrix.

The advantage of this approach is that the system matrices in the state-space retain symmetry as in the configuration space. It should be noted that these solution procedures have exact equivalents in nature. New eigenvalues are obtained from Eq. (11).

$$B\Phi_i = -s_i A\Phi_i,\ i = 1,2,3,\ldots \tag{11}$$

In Eq. (11), s_i presents eigenvalues and Φ_i presents special vectors of complex numbers. Finally, modal damping ratio is calculated from Eq. (12).

$$\xi_i = \frac{-Re(s_i)}{|s_i|}, i = 1,2,3,\ldots \tag{12}$$

The obtained modal damping ratios are depicted in Figs. 2 to 16 for the first, second and third modes according to the previous equations in the proposed method. Colored contours represent damping ratio of hybrid structure in the three main modes. These graphs can be used in determining damping ratios of the hybrid rehabilitated steel buildings. Equation (7) is used in calculating frequency and weight ratios.

The proposed numerical method and graphs are a relatively precise method, because all of the modal quantities such as modal mass, M_i, and modal eigenvalues, Φ_i are obtained from real eigenvalues analysis that ignores the irregularity of damping matrix. Therefore, proposed method in this study is closer to the real response of the structure and has more care and credit than recent methods. Also, it is suggested that the proposed graphs in this study can be used in calculating modal damping ratios for dynamic and non-linear analysis of rehabilitated steel buildings for roof extension.

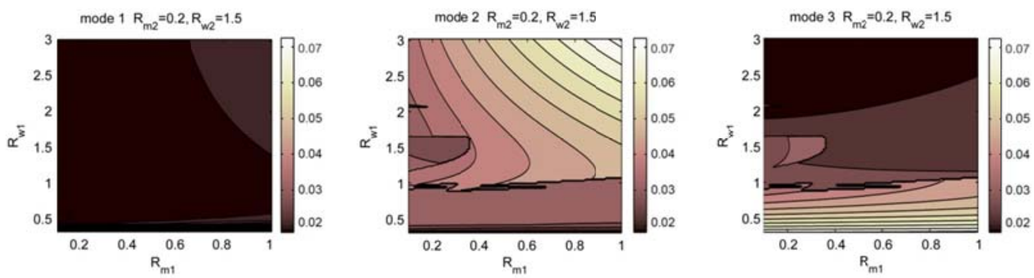


Fig. 2. The obtained modal damping ratio for R_{m2}= 0.2 and R_{w2}= 1.5

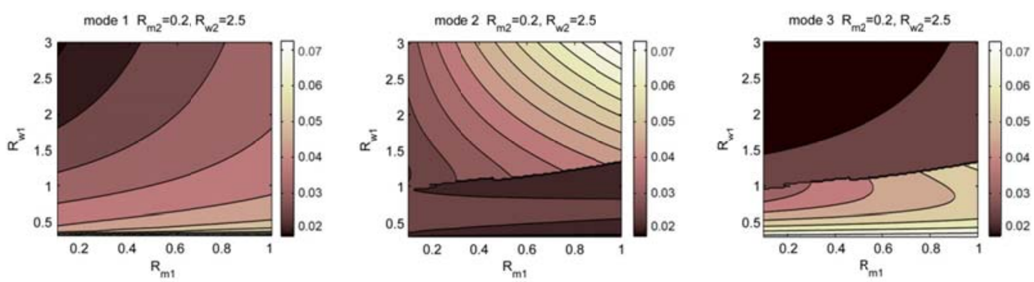


Fig. 3. The obtained modal damping ratio for R_{m2}= 0.2 and R_{w2}= 2.5

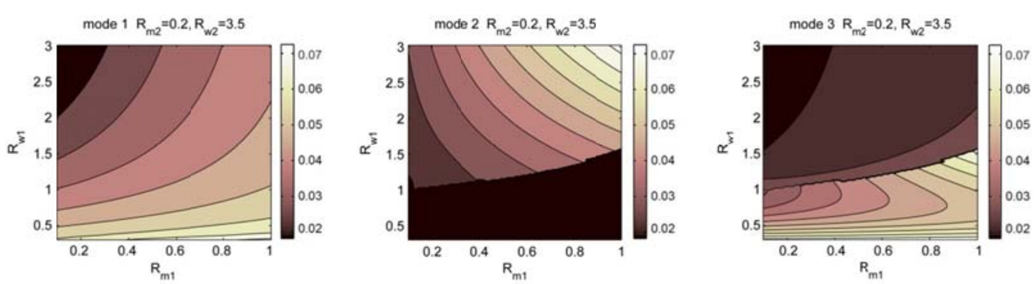


Fig. 4. The obtained modal damping ratio for R_{m2}= 0.2 and R_{w2}= 3.5

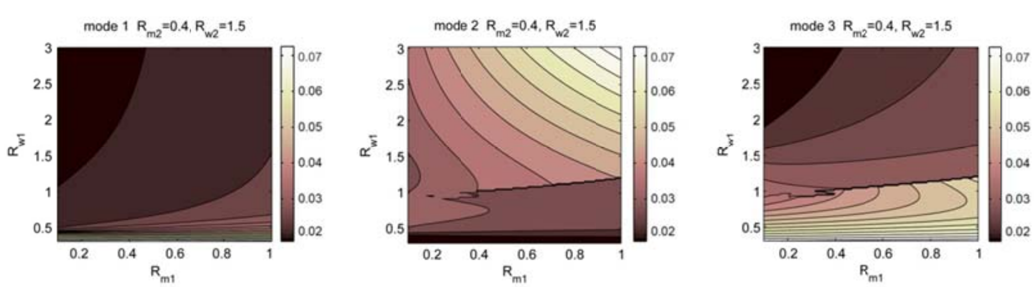


Fig. 5. The obtained modal damping ratio for R_{m2}= 0.4 and R_{w2}= 1.5

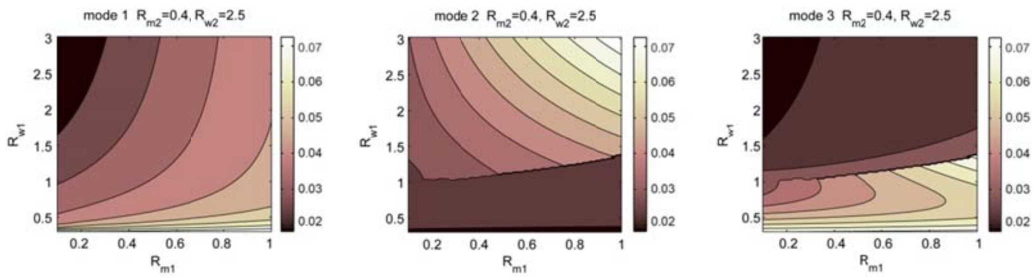


Fig. 6. The obtained modal damping ratio for R_{m2}= 0.4 and R_{w2}= 2.5

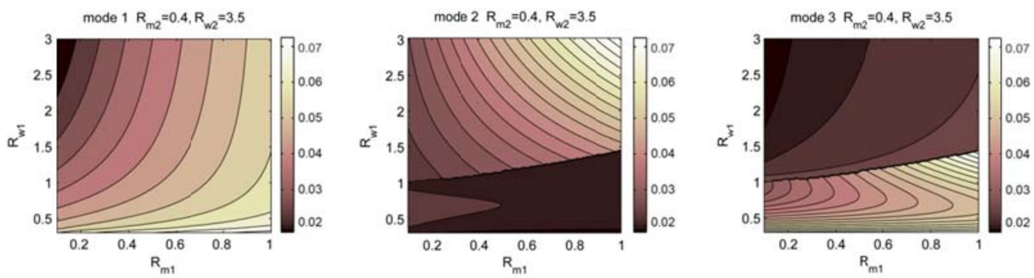


Fig. 7. The obtained modal damping ratio for R_{m2}= 0.4 and R_{w2}= 3.5

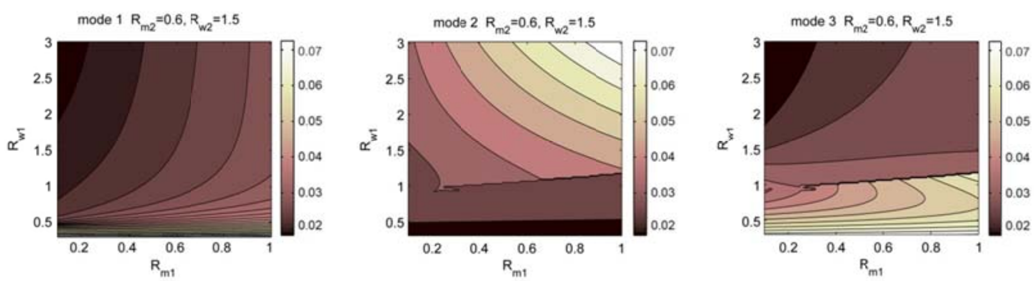


Fig. 8. The obtained modal damping ratio for R_{m2}= 0.6 and R_{w2}= 1.5

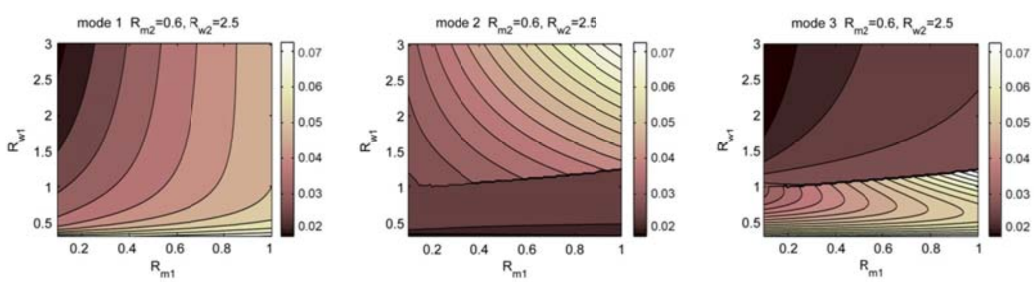


Fig. 9. The obtained modal damping ratio for R_{m2}= 0.6 and R_{w2}= 2.5

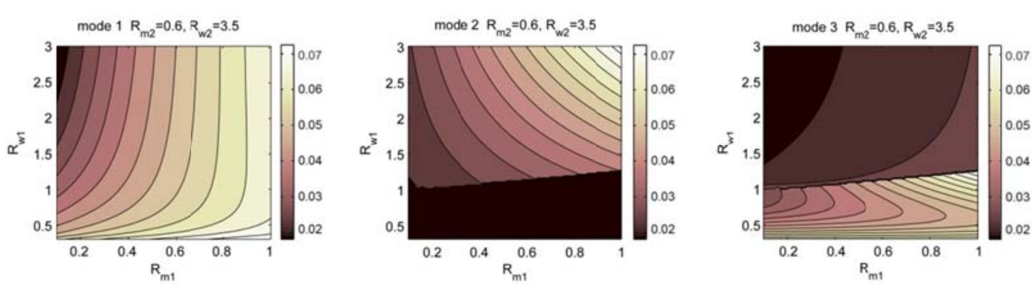


Fig. 10. The obtained modal damping ratio for R_{m2}= 0.6 and R_{w2}= 3.5

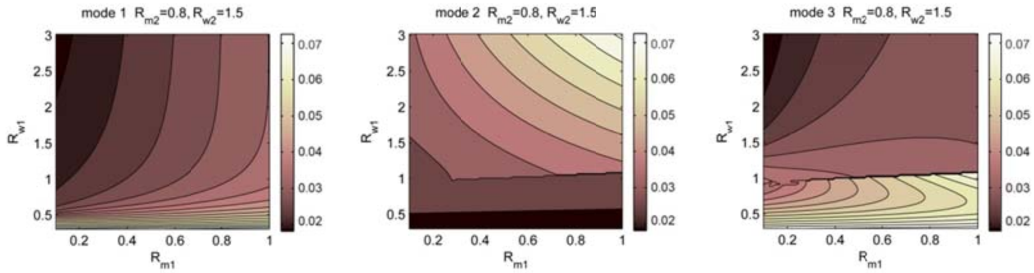


Fig. 11. The obtained modal damping ratio for R_{m2}= 0.8 and R_{w2}= 1.5

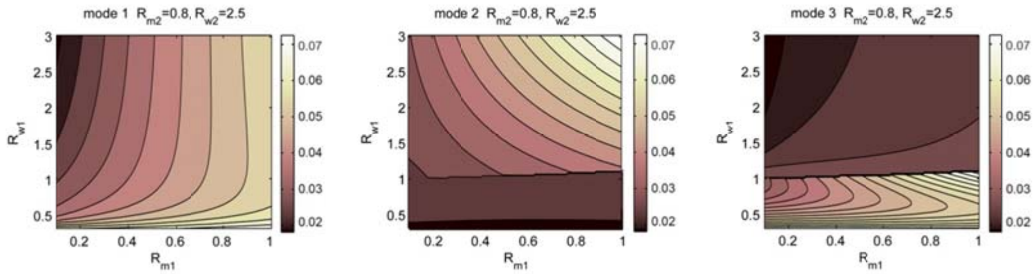


Fig. 12. The obtained modal damping ratio for R_{m2}= 0.8 and R_{w2}= 2.5

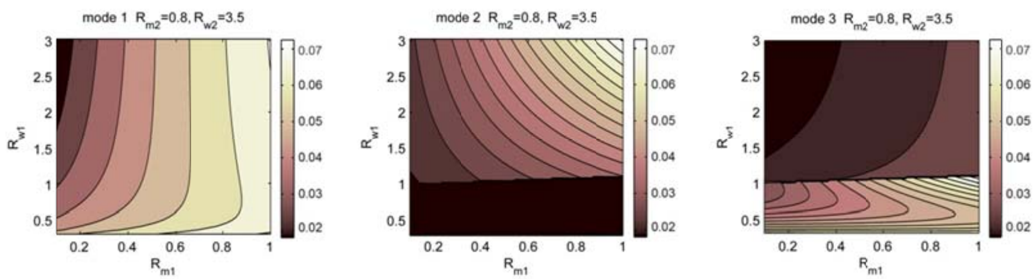


Fig. 13. The obtained modal damping ratio for R_{m2}= 0.8 and R_{w2}= 3.5

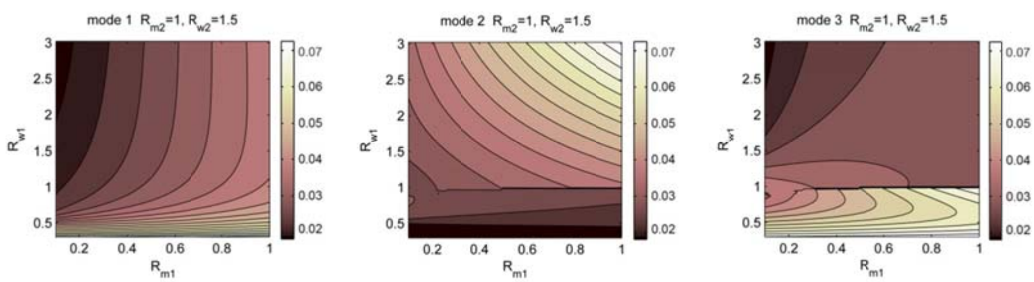


Fig. 14. The obtained modal damping ratio for R_{m2}= 1 and R_{w2}= 1.5

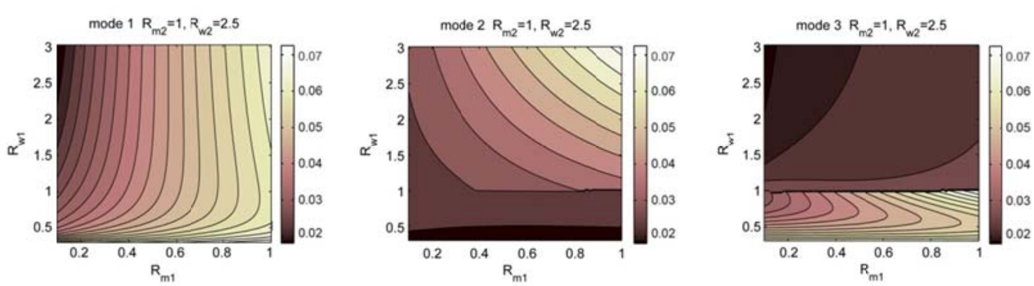


Fig. 15. The obtained modal damping ratio for R_{m2}= 1 and R_{w2}= 2.5

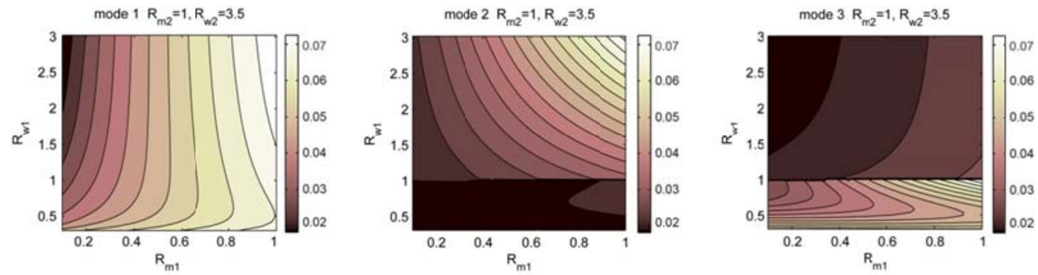


Fig. 16. The obtained modal damping ratio for R_{m2}= 1 and R_{w2}= 3.5

4. VALIDATION AND THE APPLICATION OF THE PROPOSED METHOD FOR SEISMIC ANALYSIS

A five-storey building was selected and rehabilitated for adding two stories on its roof as shown in Fig. 17. Modal damping ratios were calculated by the proposed method in this study and also fixed ratio 5% and exact method. Properties of this building are shown in Table 1.

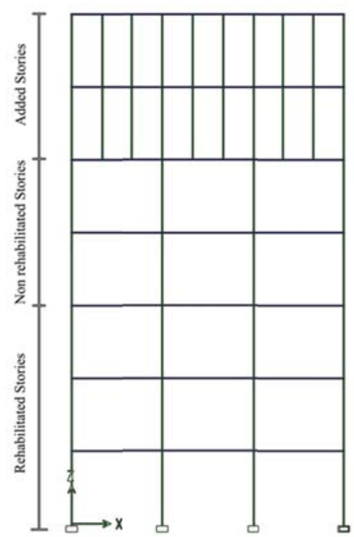


Fig. 17. Rehabilitated frame for adding stories

Table 1. Cross section for members of building

Story	Structure	System	Columns	Beams
1-3	Rehabilitated	Composite	$(50 \times 50\ , \rho_t = 0.019) + (BOX\ 30 \times 30 \times 0.1)$	IPE 30
4-5	Non rehabilitated	Steel	$BOX\ 30 \times 30 \times 0.1$	IPE 27
6-7	Added storey	LSF	$3\ UNP\ 9 \times 3.25 \times 0.01$	$UNP\ 9 \times 3.2 \times 0.01$

Transitional storey's columns also have the compound section of its upper storey's steel section and lower storey's concrete section. Modal participation ratio and period of designed structure are shown in Table 2.

Table 2. Modal characteristics of building

Mode	Period(s)	Modal participation factor (%)	Frequency (rad/s)
1	0.76	74.79	8.203
2	0.27	12.23	23.022
3	0.23	6.59	26.805
4	0.11	4.13	54.833
5	0.10	1.35	60.508
6	0.062	0.87	100.123
7	0.057	0.032	108.946

For time history analysis of the designed structure, three earthquake records (Tabas, Elcentro and Kobe) were selected. Figure 18 shows the response spectrums of these earthquakes.

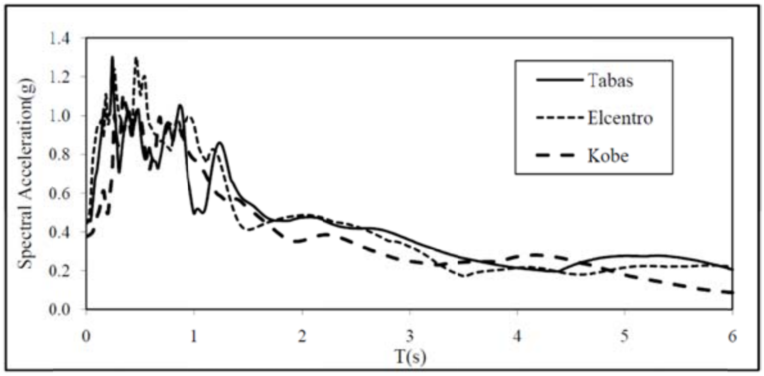
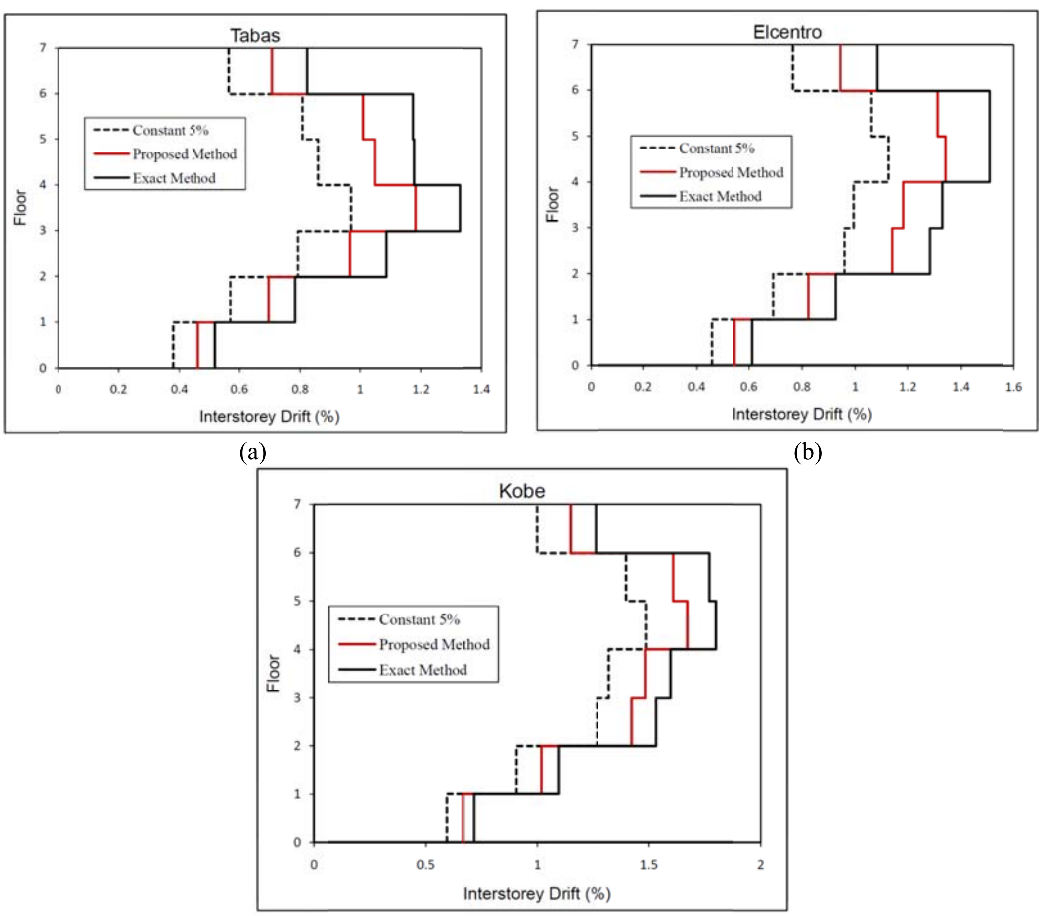


Fig. 18. Response spectrum of the selected records for time history analysis

The values of Rm_1, Rw_1, Rm_2 and Rw_2 were calculated equal to 0.23, 1.82, 0.76 and 1.96, respectively. Modal damping ratios were obtained 6.11%, 3.92% and 4.71% for the first, second and third modes, respectively.

Figure 19 shows the maximum response of the structure calculated according to the obtained damping from the proposed method and graphs, constant damping ratio 5 percent and also exact method.



(a) (b)

(c)

Fig. 19. Comparison of the obtained damping effect from the proposed method and other methods in the structure's dynamic response for (a) Tabas (b) Elcentro (c) Kobe earthquake

The average error of dynamic analysis by considering constant damping ratio, 5%, in Tabas, Elcentro and Kobe earthquakes are respectively equal to 20.2%, 17.8% and 16.3%. The average error of dynamic analysis by using proposed modal damping ratios in Tabas, Elcentro and Kobe earthquakes are respectively equal to 11.8%, 10.3% and 6.4%.

6. CONCLUSION

Developing various instructions all around the world, scholars and researchers have devoted much attention to rehabilitation of buildings. There are many reasons for rehabilitation of existing buildings. Adding story(s) is one of the most common reasons. When a steel building is retrofitted by concrete jacketing for adding story(s), this system contains several structural systems. These systems are composite sections in initial stories, welded steel sections in middle stories and cold-formed steel sections in stories added. Due to using several structural systems, an existing rehabilitated building is changed to the hybrid structure. In this paper a method was proposed for determining equivalent modal damping ratio of rehabilitated steel buildings for adding story(s) by considering the effect of the retrofitting initial stories and some graphs were also extracted. Validation of the proposed method with exact method and also the former methods showed the high accuracy of the proposed method. Therefore, the proposed method in this study is closer to the real response of the structure. Also, it is suggested that the proposed graphs in this study can be used in determination of modal damping ratios for dynamic and non-linear analysis of rehabilitated steel buildings for roof extension.

REFERENCES

1. Shakib, H., Dardaei Jogha, S., Pirizade, M. & Moghaddasi Musavi, A. (2011). Seismic rehabilitation of semi-rigid steel framed buildings—A case study. *Journal of Constructional Steel Research*, Vol. 67, pp. 1042-1049.
2. Shibin, L., Lili, X., Maosheng, G. & Ming, L. (2010). Performance-based methodology for assessing seismic vulnerability and capacity of buildings. *Journal of Earthquake Engineering and Engineering Vibration*, Vol. 9, pp. 157-165.
3. ATC-40. (1996). Seismic evaluation and retrofit of concrete buildings, Applied Technology Council.
4. FEMA 273. (1997). *Guidelines for the seismic rehabilitation of buildings*. Federal Emergency Management Agency.
5. FEMA 356. (2000). *Pretended and commentary for the seismic rehabilitation of buildings*. Federal Emergency Management Agency.
6. ASCE/SEI 41-06. (2006). *Seismic rehabilitation of existing buildings*. American Society of Civil Engineers.
7. FEMA 547. (2006). *Techniques for the seismic rehabilitation of existing buildings*. Federal Emergency Management Agency.
8. Sears J., Seek, M. & Murray, T. M. (2008). *Design guide for cold-formed steel roof framing systems*. American Iron and Steel Institute, Washington, D.C.
9. American Iron and Steel Institute (1996). *Specification for the design of cold-formed steel structural members*. Washington, D.C. Published.
10. Hajjar, J. F. (2002). Composite steel and concrete structural systems for seismic engineering. *Journal of Constructional Steel Research*, Vol. 58, pp. 703-723.
11. Lee, S. H., Min, K. W., Hwang, J. S. & Kim, J. (2004). Evaluation of equivalent damping ratio of a structure with added dampers. *Journal of Engineering Structures*, Vol. 26, pp. 335-46.
12. Villaverde, R. (2008). A complex modal superposition method for the seismic analysis of structures with supplemental dampers. *Proceedings of the 14th World conference on earthquake engineering*, 14WCEE, Beijing, China.

13. Kim, M. C., Jung, H. J., Lee, I. W. (1999). Solution of eigenvalue problem for non-classically damped system with multiple frequencies. *Journal of Sound and Vibration*, Vol. 219, pp. 196-111.
14. Huang, B. C., Leung, A. Y. T., Lam, K. L. & Cheung, V. K. (1996). Analytical determination of equivalent modal damping ratios of a composite tower in wind-induced vibrations. *Journal of Computers and Structures*. Vol. 59, pp. 311-316.
15. Papageorgiou, A. & Gantes, C. (2005). Decoupling criteria for dynamic response of rehabilitated/steel structural systems. *Proceedings of 4th European workshop on the seismic behavior of irregular and complex structures*.
16. Zona, A., Barbato, M. & Conte, J. P. (2008). Nonlinear seismic response analysis of steel-concrete composite frames. *Journal of Structural Engineering*, Vol. 14, pp. 986-997.
17. Johnson, S. M. (1965). *Deterioration, maintenance and repair of structures*. New York: McGraw-Hill.
18. Ong, K. C. G. & Kang, J. H. (2004). Jacketing of preloaded steel columns. *Journal of Constructional Steel Research*, Vol. 60, pp. 109–124.
19. Griffis, L. G. (1992). Composite frame construction. *Constructional Steel Design, an international guide*. Amsterdam, pp. 523–53.
20. Husem, M., Pul, S., Yoz Gat, E., Gorkem, S. E. (2011). Fracture of connection between steel and reinforced concrete shear walls under the cyclic loading. *Iranian Journal of Science and Technology Transactions of Civil Engineering*, Vol. 36, pp 97-102.
21. Colaco, J. P., Ford, W. & Robertson, G. (1997). Composite supercolumns and steel diagonals retrofit a 47-story steel framed structure. *Proceedings of the Engineering Foundation Conference*, pp. 526–38.
22. Basu, A. K. & Sommerville, W. (1996). Derivation of formulae for the design of rectangular composite columns. *Proc. Instn Civ. Engrs, Supp*., pp. 233-280.
23. Liu, S. W., Liu, Y. P. & Cha, S. L. (2012). Advanced analysis of hybrid steel and concrete frames Part 1: Cross-section analysis technique and second-order analysis. *Journal of Constructional Steel Research*. Vol. 70, pp. 326-336.
24. Chopra, A. K. (1995). *Dynamics of structures: theory and application to earthquake engineering*. Upper-Saddle River, NJ: Prentice-Hall.
25. Clough, R. W. & Penzien, J. (1993). *Structural dynamics*. New York: McGraw Hill.
26. Adhikari, S. & Wagner, N. (2004). Direct time–domain integration method for exponentially damped linear systems. *Journal of Computers and Structures*, Vol. 82, pp. 2453-2461.
27. Cortes, F., Mateos, M. & Elejabarrieta, M. J. (2009). A direct integration formulation for exponentially damped structural systems. *Journal of Computers and Structures*, Vol. 87, pp. 391-394.
28. Kamgar, R. & Rahgozar, R. (2013). A simple approximate method for free vibration analysis of framed tube structures. *The Structural Design of Tall and Special Buildings*, Vol. 22, pp. 217-234.
29. Daneshjoo, F. & Gerami, M. (2003). Higher mode effects on seismic behavior of MDOF steel moment resisting frames. *Journal of Seismology and Earthquake Engineering (JSEE)*, Vol. 5, pp. 22-34.
30. Foss, K. A. (1958). Co-ordinates which uncouple the equations of motion of damped linear dynamic systems. *Transaction of ASME, Journal of Applied Mechanics*, Vol. 25, pp. 361-364.
31. Newland, D. E. (1989). *Mechanical vibration analysis and computation*. Longman, Harlow and John Wiley, New York.