DIRECTION OF ARRIVAL CALIBRATION USING ITERATIVE LEARNING CONTROL^{*}

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Abstract– In this paper, an array calibration method focusing on Direction of Arrival (DOA) on narrowband signals is presented. An array calibration procedure expresses adaptive beam forming and direction finding, which includes unknown phase and gain perturbation of the active sensor. The main purpose of this study is estimation of DOA on two or more known signal sources using Iterative Learning Control (ILC) method. Due to ILC learning ability, the amount of required calculations decreases. Furthermore, ILC method has a simple structure for working on-line. As ILC is a model-free method, it requires little modeling knowledge, making it suitable for use in DOA. In this paper, a new method is proposed for finding DOA of long distance targets in sonar systems. Moreover, in the last section the simulation results approves improvement in performance of the proposed method and helps show the capabilities of the method presented.

Keywords- DOA estimation, iterative learning control, active sonar and radar

1. INTRODUCTION

Increase of the accuracy and speed of target detection is an important goal in sonar and radar systems. Currently, a high-resolution frequency method that is being used for detection, is mainly based on the structure of the distribution function of the Cohen family [1]. This article introduces a new method for finding DOA in multi-input, multi-output sonar systems and array signals in active sensor devices.

In active sonars and radars, estimation of DOA in presence of noise requires a high resolution and a robust method. Since ILC is a robust optimal method in various control fields [2], it is appropriate for solving DOA problem in active sonar and radar system.

Uncertainty in array manifold sensors can be expressed in one or any combination of parameters such as location, frequency, gain & phase and mutual coupling with sensor. These errors are inevitable in practical systems. Since the uncertainty is in the actual array system, the array manifold is only nominal and not true. Therefore, estimation of the true one is required before processing array data [3]. Furthermore, in most of the published papers, it was supposed that the array manifolds in the field of array processing systems are known [3].

In this paper, we focus on the narrow-band signals. Of course, we know that in order to find DOA on wide-band signals the output of each sensor can be converted into a narrow-band signal with the aid of Fast Fourier Transform (FFT) and filter banks [4]. Therefore, wide-band signal and its applications will be discussed.

Since the new method that is presented in this paper has not been used for sensor calibration on sonar or other applications, we will look at the history of this method, which has been employed in control systems.

^{*}Received by the editors January 26, 2013; Accepted November 25, 2014.

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Today, in modern technology smart control systems play an effective role in the development and improvement of industries. Therefore, by developing science and technology, industries are built with intelligent control algorithms which are aimed to improve the safety and operation of machines. In control engineering, studies on learning control are inspired by human capabilities and behavior. The main feature being employed in control engineering is the use of control signals to improve system performance. Iterative Learning Control (ILC) is a new field and yet stabilized in the control. In 1978 in Japan, Uchyama published the basic idea of iterative learning control in Japanese language. Later, in 1984 and 1989, Arimoto, Kawamura, et al and Goodwin, et al presented the exact definition of ILC in English [5-6]. ILC has a large number of control applications, such as robotics, control task, etc [7].

The main idea behind ILC is derived from the ability of human learning. Human beings learn by rehearsing until the end of practice. The basic idea of ILC is that when the earliest trial is completed, the algorithm uses earliest trial outputs, inputs and errors to improve performance from trial to trial.

Although all previous references for application of ILC are in control field, in this paper, control aspects are not considered and the proposed method focuses on calibration.

By storing the error signal and control input in previous iterations, ILC method calculates a new command. Thus, learning process in ILC method, is similar to human memory. There is a difference between ILC and other methods, using the idea of learning such as adaptive control and neural networks. The idea of adaptive control and neural networks is to modify the controller that is a system itself. On the other hand, in ILC the input control signal is modified at each iteration.

Note that in this article we talk about calibration process. Therefore, we try to perform a repetitive process of the calibration method, which is used with ILC. The adjustable complex vector weights in each dimension can be improved.

In this paper, ILC method is introduced and is employed along with Weighted Iterative Learning Control (WILC) to solve DOA problem. Afterwards, WILC is compared with the Weighted Least Means Square (WLMS) and Weighted Recursive Least squares (WRLS) methods. Finally, the effect of WILC method on Signal to Noise Ratio (SNR) is evaluated.

2. DOA CALIBRATION METHODS

a) DOA signal model

To define an array reference in a sonar system coordinate in Fig. 1 consider a narrow-band sonar system with arbitrarily N radiation transmitting sources and M receiving sensors for the sonar identical system. In addition, we can assume a wide-band sonar system with N transmitting orthogonal sources and decompose output of the sonar identical system by using an N point FFT in each segment into the L segment. Targets of DOA are assumed to be in the far field of the array.

In Fig. 2, the structure of the L segment narrow-band array is shown. The array response into the plant waveform in direction of (φ, ϕ) is defined as:

$$H(f,\varphi,\phi) = W^{H^{1}} * a(f,\varphi,\phi)$$
⁽¹⁾

Where H^T is Hermitain transpose and W is adjustable complex vector weights of L dimensional:

$$W = [w_1 \, w_2 \, w_3 \dots \, w_l]^T \tag{2}$$



Fig. 2. Signal structure for narrowband beam forming

W

Based on Fig. 2 we can introduce:

$$Y = W^{H^{T}} x(t) \tag{3}$$

Where T is transposed and $a(f, \varphi, \phi)$ is a vector of L dimensional:

$$a(f,\varphi,\emptyset) = diag(g(f)) * u(f,\varphi,\emptyset)$$
(4)

where:

$$u(f,\theta,\phi) = [e^{-ik^{T}r_{1}} \dots e^{-ik^{T}r_{L}}]^{T}$$
(5)

$$g(f) = [g_1(f), \dots, g_L(f)]^T$$
(6)

$$k(f,\theta,\phi) = \left(\frac{2\pi f}{c}\right) [\cos\varphi \, \cos\phi \, \sin\varphi \, \sin\phi \, \sin\phi]^T \tag{7}$$

where c is the propagation of wave speed and $g_i(f)$ is a complex response of receiving sensor signals with an array:

$$g_i(f) = |g_i(f)|e^{j\varphi_i(f)} \tag{8}$$

And in $u(f, \varphi, \phi)$, d_m is sensor location (distance between sensor 1 and n). According to Fig. 1 we have:

$$c\delta_{nm} = d_n^T P_m \tag{9}$$

where δ_{nm} is the propagation of wave delay between nth sensors and mth sources. In Fig. 1, with simplification we can write:

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$$\delta_{nm} = \delta_n(\varphi_m) = \hat{x}_n \sin \varphi_m + \hat{y}_n \cos \varphi_m \tag{10}$$

The output of nth sensor elements can be written as:

$$x_n(t) = g_i(\sum_{m=1}^M s_m(t)e^{-jk^T r_i} + n_i(t))$$
(11)

where S_m is mth emitter signal source and n_i is ith sensor noise sample additive. $n_i(t) \begin{cases} M \\ m = 1 \end{cases}$ are mutually exclusive. By employing Fourier transforms, the output signal in the frequency domain will be:

$$X_n(f) = \frac{1}{\sqrt{T}} \int_{-\frac{T}{2}}^{\frac{T}{2}} x_m(t) e^{-j2\pi f t} dt = \sum_{m=1}^{M} e^{-j2\pi f \delta_n(\varphi_m)} S_m(f) + N_m(f)$$
(12)

Using ILC method, we find that W is a correlation of steering vector in each way. Output pattern signal can be introduced as:

$$P(\varphi) = W(\varphi)^H * d \tag{13}$$

where *d* is steering vector.

b) Iterative learning

In most repetitive processes, iterative learning algorithms are well-known methods. In control theory, they are usually used for repetitive trajectory tracking problems, but they are not limited to these cases. In this case, it is known as an iterative learning control (ILC) [2]. Inspired from human learning, the main idea of ILC is using control input from previous force of each iteration to improve the performance from iteration to iteration. Convergence of algorithms is the most important matter. Therefore, in this section we drive equation of ILC to find stability convergence conditions.



Fig. 3. ILC structure

Considering Fig. 3 and Eq. (12), we can assume the closed loop system model as:

$$\mathbf{x}_{\mathbf{k}}(\mathbf{t}) = \mathbf{G}_{\mathbf{p}}(\mathbf{\theta}) * \mathbf{u}_{\mathbf{k}}(\mathbf{t}) + \eta_{\mathbf{k}}$$
(14)

Where $G_p(\theta)$ is a $(n \times m)$ transfer function in terms of angle, which explains the relationship between input-output in each iteration and η_k denotes the noise component. Furthermore, $u_{k-1}(t)$ and $x_k(t)$ denote input and output of the system in the previous sample as illustrated in Fig. 3. $G_p(\theta)$ is the transfer function in Laplace domain, which has frequency characteristics as:

$$G_p(jw) = g_p(w)e^{i\theta_p(w)}$$
(15)

Where $g_p(w)$ is the magnitude characteristic and $\theta_p(w)$ is the phase characteristic.

As it can be seen, this configuration may be used in every feedback and closed loop form of signal processing problems as well as control problems.

Let us introduce the Laplace transform of the ILC law as follows:

$$u_{k}(s) = u_{k-1} + \gamma * \psi_{c}(s) * e_{k-1}(s)$$
(16)

Equation (16) provides an estimation of the present value of input from data provided by the previous sample. Where γ and $\psi_c(s)$ are the learning gain and the compensator in the Laplace domain, respectively. In Eq. (15), e_{k-1} is the error at k-1th iteration. The frequency characteristic of learning compensator is.

$$\psi_c(\mathbf{jw}) = k_c(w)e^{j\theta_c(w)} \tag{17}$$

where $k_c(w)$ is the magnitude characteristic and $\theta_c(w)$ is the phase characteristic. In order to find the condition of error tracking in ILC, we have:

$$e_{k(s)} = S(s) - X_k(s)$$
 (18)

Where S(s) and $X_k(s)$ are source signals (reference trajectory) and output sensor signals (plant output at iteration k), respectively. By replacing $X_k(s)$ from Eq. (14) and substituting $u_k(s)$ by Eq. 16, we have:

$$X_k(s) = G_p(s) [u_{k-1}(s) + \gamma \psi_c(s) e_{k-1(s)}] + \eta$$
(19)

Therefore, Eq. (18) can be rewritten as:

$$e_k(s) = [1 - \gamma G_p(s)\psi_c(s)]e_{k-1}(s)$$
(20)

$$\left|1 - \gamma G_p(jw)\psi_c(jw)\right| = \widetilde{G_w}^k \tag{21}$$

$$e_k(jw) = \widetilde{G_w}^k e_{k-1}(jw) \tag{22}$$

According to Goh, (1994) [8] and Hideg & Judd, (1988) [9], after taking L_2 norm of side of the above equation and using Schwarz's theorem in Inequality matrix, the tracking error at steady state and $\widetilde{G_w}^k$ converge, if the condition below is satisfied:

$$\left|\widetilde{G_{w}}\right| < 1 \quad \& \quad \lim_{w \to \infty} \left|\widetilde{G_{w}}\right| = 0 \tag{23}$$

According to condition of $|\widetilde{G_w}| < 1$ and if $\gamma > 0$ [10]

$$\gamma g_p(w) k_c(w) < 2\cos(\theta_p(w) + \theta_c(w)) \tag{24}$$

$$-90^{\circ} < \theta_{p}(w) + \theta_{c}(w) + n * 360^{\circ} < 90^{\circ}; n = 0, \pm 1, \pm 2, \pm 3, \cdots$$
(25)

According to Yongqiang Ye, et al. (2009) [10], condition of Eq. (25) is critical since when the condition is satisfied we could always find a γ (learning gain) sufficiently small to satisfy Eq. (24). So the frequency range in which condition (25) holds, ultimately learnable band is named. For minimum phase systems, in condition (25) $\theta_p(w)$ is normally negative and n = 0 and according to $\psi_c(jw)$ at Eq. (21), $\theta_c(w)$ must provide positive phase. For non-minimum phase systems that start from 360 at dc, n = -1 so that $\theta_p(w) + n * 360^\circ$ is 0 at dc and negative at other frequencies, where $\theta_c(w)$ must be positive again. In either case, development phase is needed to ensure a wider learnable band. This ensures higher tracking accuracy.

According to Hebb rule [11-12] and Eq. (3), adaptive law for weighted vector can be defined as:

$$W_{k}(jw) = W_{k-1} + \mu X_{k}(jw)e_{k}^{*}(jw)$$
(26)

where μ and e are constant gain and error, respectively. That e (error) will be introduced in Eq. (19) and * is conjugated matrix.

ILC has six types: Proportional (P), Derivative (D), Proportional-Derivative (PD), Integral (I), Proportional-Integral (PI) and Proportional-Integral-Derivative (PID). More details on the aforementioned types of ILC can be found in [2, 13-14]. Furthermore, the concept and stability of ILC are discussed in [2].

ILC method works in both time interval and discrete-time domains. In all types of ILC method, optimal gains for approximation are unknown. Therefore, by altering γ (the learning gain), convergence speed and accuracy can be changed. For example, smaller values of γ decrease learning speed and increase accuracy and vice versa. In [15], details on the trade off between smoothing noise intensity and uniform distribution of convergence in ILC by choosing learning gain can be found.

In this paper, we focus on traditional P, D and PD-types of ILC for solving DOA problem.

c) D-type ILC

Since the derivative of error is used in D-type, fewer numbers of iterations are required for the tracking process, Also, less error is made compared to P-type. In 1984, Arimoto et al. [5] proposed the learning law of D-type. In 2001 and 2005 by Chen et al and Wang et al. [16-17], the convergence condition of D-type in Laplace domain was derived and finally in 2009 it was revised by Yongqiang Ye et al. [10]. In a similar fashion for D-type according to Fig. 4 we have:



Fig. 4. D-type structure

$$u_k(t) = u_{k-1}(t) + \mathbf{L}\dot{e}_{k-1}(t)$$
(27)

$$\left|1 - \mathcal{L}G_p(jw)jw\right| < 1 \tag{28}$$

Finally, we have:

$$-180^{\circ} < \theta_{p}(w) + n * 360^{\circ} < 0^{\circ}; n = 0, \pm 1, \pm 2, \pm 3, \cdots$$
⁽²⁹⁾

d) P-type ILC

The scheme of (28) is called P-type, since a derivative is obviously not used for tracking error. According to D-type and with slight manipulation of D-type equations, for P-type (Fig. 5) we have:



Fig. 5. P-type structure

Direction of arrival calibration using...

$$u_k(t) = u_{k-1}(t) + Le_{k-1}(t)$$
(30)

$$\left|1 - \mathcal{L}G_p(jw)\right| < 1 \tag{31}$$

and

$$-90^{\circ} < \theta_{p}(w) + n * 360^{\circ} < 90^{\circ}; n = 0, \pm 1, \pm 2, \pm 3, \cdots$$
(32)

e) PD-type ILC

In the improved control scheme, PD-type of ILC is the full form of the P-type and PD feedback control using current tracking error since PD-type ILC works with previous tracking error for updating laws. In this scheme, the control law for next iteration is obtained from the control input information on current iteration and the first and higher-order derivatives of the error. The PD-type of ILC is offered for nonlinear systems with perturbation [18-19]. With combination of P&D-type, for PD-type (Fig. 6) we have:



$$u_k(t) = u_{k-1}(t) + Le_{k-1}(t) + L_1 \dot{e}_{k-1}(t)$$
(33)

$$\left|1 - LG_p(jw) - L_1 jwG_p(jw)\right| < 1$$
(34)

By employing frequency characteristics $G_p(jw)$ have:

$$(L + L_1 w)N_p(w) < 2(\cos(\theta_p(w) + 90^\circ) + \cos(\theta_p(w)))$$

$$(35)$$

$$(L + L_1 w) N_p(w) < \frac{2}{\sqrt{2}} \left(\cos(\theta_p(w) + 45^\circ) \right)$$
(36)

and

$$-45^{\circ} < \theta_p(w) + n * 360^{\circ} < 45^{\circ}; n = 0, \pm 1, \pm 2, \pm 3, \cdots$$
(37)

3. SIMULATION RESULT

Now we try to extend the aforementioned control algorithms to a DOA problem. Consider a linear array with n=8 sensors for N=500 (N denotes the number of samples), $d=\lambda/2$ (λ is wavelength) and m=2 represents far-field source that works in f=50 Hz, with SNR 10dB for each source. Two targets are located in $\varphi_1 = -20$ and $\varphi_1 = 20$. We use ILC to find DOA, we use i=200 in which i shows number of iterations in ILC method. In Fig. 7 - learning curve is shown for P, D and PD-type.

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Fig. 7. Normalized learning curve. P-Type, D-Type, PD-Type WILC

In Fig. 8 normalized DOA is shown for three types of ILC (P, D and PD), WMLS and RLS with the same iteration and the same parameters.



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In Fig. 9 square error of WILC is shown in the last iteration (i=300), Fig. 10 exhibits the mean value of the array square error for WILC, WLMS and WRLS, respectively in each iteration.



Fig. 9. Square error in the last iteration (i=300). (a) P-Type WILC. (b) D-Type WILC. (c) PD-Type WILC



Fig. 10. Mean value of array square error versus iteration

In Fig. 11, root mean square error (RMSE) is presented in terms of ranging from -60dB to 60dB. Simulation results of all five methods are averaged form 100 Monte Carlo runs [20]. RMSE is defined as

$$\left(E\left\{\sum_{m=1}^{2}\left|\hat{\theta}_{m}(k)-\hat{\theta}_{m}\right|^{2}\right\}\right)^{\overline{2}}$$
(38)

Where θ and m are estimations of θ in the kth run of Monte Carlo and number of sources, respectively.

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4. CONCLUSION

Iterative Learning Control is a powerful method in control systems. In this paper this method was used as a new innovative method for DOA estimation. It was employed for Narrow-band and wide-band sonar and radar systems. In control application, ILC method was used for tracking the reference trajectory by repeating in a finite time interval. ILC has a simple structure and high computational speed, which makes it suitable for online systems. ILC is a free-model method; thus, little model knowledge is required. In this paper, two well-known basic methods, WLMS and WRLS, were applied to DOA problem and compared with WILC. Finally, the effect of providing WILC was investigated using an example of DOA problem. Simulation results proved the WILC abilities in DOA problem.

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