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**"Research Note"**

# VELOCITIES OF DUAL HOMOTHETIC EXPONENTIAL MOTIONS IN D[3*]

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**Abstract –** In this paper the concept of Homothetic Dual Exponential Motions in $D^3$ is discussed and their velocities obtained. Due to the way in which the matter is presented, the paper gives some formula and facts about dual exponential motions which are not generally known.

**Keywords –** Dual number, dual vector, homothetic motion, exponential transformation

## 1. INTRODUCTION

In modern mathematics dual numbers turn up if one poses the problem to find all algebra of rank 2 over a (commutative) field with a characteristic $p \neq 2$. It appears that there are three solutions. One of these is the algebra consisting of the dual elements $a + \varepsilon a^*$ over F, $a, a^* \in F$ and $\varepsilon^2 = 0$. If F is taken to be the field of real numbers, these elements are called dual numbers. Long before abstract algebra was included in the curriculum of all mathematical departments, dual numbers had been introduced by William Kingdon Clifford (1845-1879) as a tool for his geometrical investigations. After him, E. Study used dual numbers and dual vectors in research on line geometry and kinematics [1]; he devoted special attention to the representation of spears by dual unit vectors. In recent times several authors have been using dual quantities in their investigations concerning the analysis and synthesis of spatial mechanisms [2].

In the Euclidean space $E^n$ of n-dimensions, W. Clifford and James J. McMahon have given a treatment of a rigid body's motion generated by the most general one parameter affine transformation [3]. Another treatment was given by H.R. Müller for the same kind of motion [4].

Subsequently, the properties of the planer homothetic motions and three dimensional special homothetic motions are given by I. Olcaylar [5]. The exponential motions were given by A.P. Aydın, V. Asil [6, 7].

In this work, the concept of homothetic dual exponential motions in $D^3$ (Dual Space) is studied.

### *a) Dual numbers*

The dual numbers are a particular two-dimensional commutative unital associative algebra over the real numbers, arising from the reals by adjoining one new element ε with the property $\varepsilon^2=0$ (ε is nilpotent). Every dual number has the form

$$\hat{a} = a + \varepsilon a^* \tag{1}$$

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with a and a* uniquely determined real numbers. The algebra of dual numbers results from this definition. Two dual numbers are equal if and only if their real and dual parts are equal, respectively. The addition of two dual numbers requires the separate addition of their real and dual parts:

$$(a+\varepsilon a^*)+(b+\varepsilon b^*)=(a+b)+\varepsilon(a^*+b^*)\ . \tag{2}$$

Multiplication of two dual numbers results in,

$$(a+\varepsilon a^*)(b+\varepsilon b^*)=ab+\varepsilon(a^*b+ab^*)\,. \tag{3}$$

Hacısalihoğlu [8] presented the definition of division of two dual numbers as follows:

$$\frac{\hat{a}}{\hat{b}}=\frac{a}{b}+\varepsilon\left(\frac{a^*}{b}+\frac{ab^*}{b^2}\right), b\neq 0. \tag{4}$$

***b) Dual vectors and matrices***

An ordered triple of dual numbers $(\hat{x}_1,\hat{x}_2,\hat{x}_3)$ is called a dual vector; we write $\hat{X}=(\hat{x}_1,\hat{x}_2,\hat{x}_3)$. The numbers $\hat{x}_1,\hat{x}_2,\hat{x}_3$ are called the coordinates of $\hat{X}$.

Let $\hat{X}=(\hat{x}_1,\hat{x}_2,\hat{x}_3)$ and $\hat{Y}=(\hat{y}_1,\hat{y}_2,\hat{y}_3)$ be two dual vectors. $\hat{X}=\hat{Y}$ if and only if $\hat{x}_i=\hat{y}_i$ (i=1,2,3).

Let $\hat{\lambda}=\lambda+\varepsilon\lambda^*$ be a dual scalar. Multiplication by a dual scalar of dual vector $\hat{X}$ results in

$$\hat{\lambda}\hat{X}=(\hat{\lambda}\hat{x}_1,\hat{\lambda}\hat{x}_2,\hat{\lambda}\hat{x}_3)\,.$$

The inner product and cross-product of two dual vectors are defined as follows, respectively;

$$\langle\hat{X},\hat{Y}\rangle=\hat{x}_1\hat{y}_1+\hat{x}_2\hat{y}_2+\hat{x}_3\hat{y}_3\,, \tag{5}$$

$$\hat{X}\times\hat{Y}=(\hat{x}_2\hat{y}_3-\hat{x}_3\hat{y}_2,\hat{x}_3\hat{y}_1-\hat{x}_1\hat{y}_3,\hat{x}_1\hat{y}_2-\hat{x}_2\hat{y}_1)\ \text{[8]}.$$

Norm of $\hat{X}\neq 0$ is defined by

$$\left\|\hat{X}\right\|=\langle\hat{X},\hat{X}\rangle^{\frac{1}{2}}\ .$$

$\hat{A}$ is called dual matrix if its entries are dual numbers, and it is denoted by.

$$\hat{A}=(\hat{a}_{ij})=(a_{ij}+\varepsilon a_{ij}^{\ *}).$$

If $\hat{A}$ is a dual anti-symmetric matrix, that is $\hat{A}^T=-\hat{A}$, then we call $a_{ii}=a_{ii}^{\ *}=0$ and $a_{ij}^{\ *}=-a_{ji}^{\ *}$ for all i,j.

$\hat{A}$ is a dual orthogonal matrix for which $\hat{A}\hat{A}^T=\hat{A}^T\hat{A}=I$.

## 2. DUAL HOMOTHETIC EXPONENTIAL MOTIONS

**Definition 2.1.** Let $g(t)=e^{t\hat{A}}$ and $H(t)=h(t)g(t)$ be dual orthogonal matrices where $h(t)\neq$ constant, $\hat{A}$ dual anti-symmetrical matrix, $t\in R$.

$$\hat{Y}=H\hat{X}+\hat{C} \tag{6}$$

which is called dual homothetic exponential motion in $D^3$.

$\hat{X}, \hat{Y}$ and $\hat{C}$ are 3x1 dual matrices. The homothetic scale $h$ and the elements of $g$ are continuously differentiable functions of a real parameter $t$. $\hat{X}$ and $\hat{Y}$ correspond to the dual position vectors of the same point with respect to the rectangular coordinate systems of the moving dual space $R_0$ and the fixed dual space R, respectively. At the initial time $t = t_0$ we consider the dual coordinate systems of $R_0$ and R coincident. We assume that $h(t) \neq$ constant, and to avoid the cases of pure translation and pure rotation we also assume for $g' = \hat{A} g$ and $\hat{C} \neq 0$

$$H' = \frac{dH}{dt} = h'g + hg' = (h' + h\hat{A})g \tag{7}$$

where (‘) indicates $\frac{d}{dt}$. On the other hand, since $h = h(t)$ is scalar matrix, its inverse and transpose are

$$h^{-1} = \frac{1}{h} I, h^T = h \tag{8}$$

respectively. Since g is a dual orthogonal matrix, the inverse of H is

$$H^{-1} = h^{-1} g^T, g^{-1} = g^T. \tag{9}$$

**Theorem 2.1.** The dual homothetic exponential motions are regular for all n.

**Theorem 2.2.** If g is a dual orthogonal nxn matrix, the $n^{th}$-order derivatives of H are given by

$$H^{(n)} = \left[ \sum_{k=0}^{n} \binom{n}{k} h^{(n-k)} \hat{A}^k \right] g. \tag{10}$$

**Theorem 2.3.** In the dual space of n-dimensions the high order velocities of dual homothetic exponential motions are given by

$$Y^{(n)} = \sum_{k=0}^{n} \left[ \sum_{i=0}^{n-k} \binom{n-k}{i} h^{(n-k-i)} \hat{A}^i \right] g \hat{X}^{(k)} + \hat{C}^{(n)}. \tag{11}$$

**Theorem 2.4.** If $\hat{A}$ and $g = e^{t\hat{A}}$ are a dual anti-symmetric and dual orthogonal matrices, respectively,

i) $\hat{A} g = \hat{A}$

ii) $\hat{A}^2 = \hat{A}^3 = ... = \hat{A}^n = 0$

Taking space in Theorem 2.3 $D^3$, if we apply the first and second derivatives of $\hat{Y}$ to Theorem 2.4, we obtain the following results:

$$\hat{Y}' = (h'g + h\hat{A})\hat{X} + \hat{C}' + hg\hat{X}' \tag{12}$$

where $\hat{Y}'$ is absolute velocity, $(h'g + h\hat{A})\hat{X} + \hat{C}'$ is the sliding velocity and $hg\hat{X}'$ is the relative velocity of the point $\hat{y}$ whose position vector is $\hat{Y}$.

$$\hat{Y}'' = (h''g + 2h'\hat{A})\hat{X} + \hat{C}'' + 2(h'g + h\hat{A})\hat{X}' + hg\hat{X}'' \tag{13}$$

where $\hat{Y}''$ is the absolute acceleration, $(h''g + 2h'\hat{A})\hat{X} + \hat{C}''$ is sliding acceleration, $2(h'g + h\hat{A})\hat{X}'$ is the relative acceleration and ${}_{hg}\hat{X}''$ is the Coriolis acceleration of the point $\hat{y}$ whose position vector is $\hat{Y}$.

## REFERENCES


1. Study, E. (1903). Geometrie der Dynamen. *Leipzig*.
2. Dimentberg, F. M. (1950). *Determination of the motion of spatial mechanism* (Russian). Moscow, Akad. Nauk.
3. Clifford, W. & McMahon, J. J. (1961). The rolling of one curve or surface up on another. *Am. Math*. *68*(23A 2134), 338-341.
4. Müller, H. R. (1966). Zur Bewenguns geometrie in Röümen Höhere dimensions. Mh. Math, *70* band, 1 heft, 47-57.
5. Olcaylar*, I. (1967). Homothetic motions in three dimensional space, Asc. thesis. Faculty of science, Ankara, Turkey, Middle East Technical Univ.*
6. Aydın, A. P. (1987). Homothetic exponential motions and their velocities. *The journal of Fırat University*, *2*(2), 33-39.
7. Aydın, A. P. & Asil, V. (1999). Infinitesimal complex homothetic exponential motion. *Jour. of Inst. of Math and Comp. Sci. (Math. Ser.)*, *12*(2), 167-169.
8. Hacısalihoğllu, H. H. (1983). *Motions and quaternions theory*. Ankara, Gazi Univ.