IJST (2011) A3: 201-208
Iranian Journal of Science & Technology

http://www.shirazu.ac.ir/en

New scheme of nuclide and nuclear binding energy from quark-like model

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Abstract

Following our previous presentation of a new three-fold symmetry for derivation of the magic numbers of nuclei based upon a quark state model of nuclei, here the nuclear binding energy is obtained from such new perspective. From this point of view, since each nucleon is made of three quarks, the binding energy of nuclei contains a volume term proportional to *3A* (*A* is mass number). By considering the asymmetry in the number of up and down quarks and also coulomb correction, a new formula is presented that calculates the nuclear binding energy in terms of only *N* and *Z* numbers for most of the stable nuclides.
PACS: 21.10.Dr, 21.90.+90, 21.10.-K, 12.38.Mh, 12.90.+b

Keywords: Nuclear binding energy; Quark model; nuclear structure

1. Introduction

Nuclear shell model (NSM) and liquid drop model (LDM) are two well-known established models of nuclear physics in last century. In spite of their successes in expressing some nuclear properties, they fail to explain other peculiarities of the nuclei. For example, in the nuclear shell model, the Pauli exclusion principle is used to explain the nuclear structure based upon the quantized energy levels. This model was presented in 1949 by Eugene Paul Wigner, Maria Geoppert-Mayer and J. Hans and D. Jensen independently, and they received the Nobel Prize in 1965 [1, 2]. The nuclear shell model is similar to atomic shell model in which the configuration of the atomic electrons is explained and the stability of an atom is related to the degree of fullness of each defined shell. Perhaps the main most successful achievement of the nuclear shell model was the explanation of the magic numbers by selecting and averaging square potential well with harmonic oscillator and then adding the relativistic spin-orbit term to it. The nucleon magic numbers and some of the other nuclear properties are obtained from this model. However this model did not gain any success in the calculation of nuclear binding energy.

Liquid drop model is another successful nuclear model that was first given by George Gamov and was late developed by Niels Bohr and John Archibald Wheeler [3]. The basic assumptions of this model are that the nuclei is not made of condensed state, the nuclear force is the same for all nucleons and is charge independent, and finally, the nuclear force saturates. Based upon such assumptions, the liquid drop model succeeded in explaining some of the nuclear properties, particularly the famous nuclear binding energy formula, namely,

$$B(A, Z)=a_{\upsilon} A-a_s A^{2/3}-a_c Z(Z-1) A^{-1/3} -a_a (N-Z)^2 A^{-1} \pm\delta + \eta \quad (1)$$

But this model is not able to give any explanation about the magic numbers of the nucleus.

There are other nuclear models in the literature that have been presented to explain some other aspects of the nuclei, like Alpha-Particle Model of Nuclei [4], Nuclear Cluster Model [5], Lattice-Gas Model of Nuclei [6, 7], and the modified Debye-layer model [8] based on a first consideration of how the change of the Fermi-Dirac statistics of nucleons into the relativistic branch at little higher than nuclear density [9] may explain a quark-gluon soup instead of Wheelers assumption of a neutron star property. One of these models that has recently been presented by our group is the quark-like model (QLM) of the nuclei [10, 11] in which not only all the magic numbers are obtained, but also new

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Received: 3 November 2010 / Accepted: 27 August 2011

magic number namely 184, was predicted and later confirmed experimentally [12-15]. An indication for the quark property was based on a 3^n-relation, on which property our suggestion of a threefold quark property for a preferred shell structure of nuclei was motivated [10, 16-19]. Further development of this model is yet to be explored and could result in revealing other properties of nuclei. In this paper an attempt is made to find the nuclear binding energy in the context of the quark state of nuclei by presenting a binding energy formula in terms of atomic and mass numbers of nuclei and quark mass.

2. Determination of nuclear binding energy from quark model of nuclei

Let us consider the quark model of nuclei in which the atomic nucleons, instead of containing protons and neutrons, is made of quark-gluon soup [10, 11]. Of course the structure of nucleon as a free and bound particle has been intensively investigated, but here we are considering such nucleons as constituents of nuclide. In this state, matter is neither condensed nor free like a gas but is loosely bound.

Asymptotic freedom is held between quarks contained in nucleons, but within the nucleus due to longer distance between the quarks, strong nuclear force acts weakly and causes the quark-gluon soup formation. Nucleons are formed and exit from the nucleus via external means such as collisions from which energy is given to the nuclei similar to the jet formations due to quark confinement. In other words, in the context of nuclear quark model, nuclei are assumed to be made of quark-gluon soup instead of nucleons [10, 11].

It should be noticed that there appears a typical duality property because we found and extend here a typical quark property for nuclei in contrast to a nucleon property on which the modified Debye model resulted in reproducing established nuclear properties [8, 9, 10, 16] such as the thickness of Hofstadter's surface structure and other facts. Within such new concept of nuclei, in order to calculate the nuclear binding energy let us make the following assumptions:

1. Nuclear binding energy is of the order of about one percent of the remining mass energy of the constituent quarks, namely, $m_q c^2$ where q stands for up and down quarks.
2. The binding energy depends upon the volume of the quark-gluon soup within the nuclei, therefore it is proportional to $3A$ where A is the mass number.
3. Due to the asymmetric distribution of up and down quarks and also the existence of the Coulomb force between them, one concludes that the binding energy depends upon terms such as $(\frac{N^2-Z^2}{Z})$.

These assumptions, plus intuitive physical curiosity led us to the following formula for the calculation of nuclear binding energy:

$$E_b = \left[3A - \left(\frac{(N^2-Z^2)+\delta(N-Z)}{Z} + 3^2\right)\right] \times \frac{m_u c^2}{100} \qquad A \geq 5 \tag{2}$$

where $m_u c^2 = 330\ (MeV)$ [20] and δ stand for nuclear beta-stability line condition and is defined as follows:

$$\delta(N-Z) = \begin{cases} 0 & for\ N \neq Z \\ 1 & for\ N = Z \end{cases} \tag{3}$$

Similar to the semi-empirical LDM, each term in Eq. (2) contains some physical insight. The presence of 3^n law with $n=1, 2$ [16-19] shows the 3-quark constituent of nucleons and perfectly fits the experimental values of binding energy. The mass of up quark appears in the above formula since up quark plays a major role in the construction of the most stable baryon, namely proton. In order to test the validity of formula (2) one has to calculate the binding energy of each nuclei for given Z and N numbers and then compare it to the experimental values. Table (1) gives samples of calculation of nuclear binding energy using formula (2), LDM and their corresponding experimental values and table (2) gives the binding energy comparison between our model (QLM) with Cluster model for some nuclides.

3. Discussion and explanation

Several questions arise that should be addressed here. First, we are confronted with the 3^2 factor in our binding energy formula which needs justification. Our explanation comes from the fact that in order to form the lightest nuclei in this model, at least three nucleon or nine quarks must participate [21]. Attention should be paid to the fact that for A less than 5, such as ${}^4_2He_2$ an ${}^3_1H_2$, the factor 3^2 in the second term of binding energy equation (2) changes to 3 and for ${}^2_1H_1$ and ${}^3_2He_1$, our given formula needs minor corrections, namely the term 3 changes to 5 and 8, which is due to the fact that for these light nuclides at least two nucleons should participate and the spherical distribution of the nucleons inside the nuclide changes. The second question concerns ignoring the surface term in our binding energy formula. The physical explanation is that such term makes sense for LDM or is essential in the modified Debye layer model [8], but here at the surface of the nuclide the number of quark bonding is negligible compared to those of

quarks in the core of the nuclide. So the surface term will be ignored.

Our binding energy per nucleon is in excellent agreement with the experimental data available over the whole range of A number, with a similar jump for $A<10$ and similar drop for $A>150$, as shown in Fig. (1) and Fig. (3).

Figures (1), (2) and (3) give the binding energy per nucleon for most of the known stable nuclei. Careful consideration of Table (1) and Figs. (2) and (3) reveals the meaningful accuracy of our quark-like model compared to liquid drop model with respect to the experimental data (Fig. (1)).

Compared to liquid drop model in which seven terms are presented for binding energy formula, our model consists of only two terms that depend only upon N and Z, indicating a more simple and comprehensive vision of the nuclide. Similar to LDM, deviations from the experimental data in this model come from two extreme sides namely, the lightest nuclides in which less than 3 quarks are needed to be formed, and from the heaviest ones in which the number of neutrons are much more than proton numbers.

The seven parameter fitting of the smooth LDM data with the experiment does not cover the just here reproduced grouping seen in the experimental values between 15 and 35 and that between 35 and 60 as seen in the QLM results presented here, indicating these groupings.

Attempts are made in our group to conceptualize a quark-like nuclear model capable of providing all nuclear characteristics such as binding energy per nucleon, origin of magic numbers, excited states, decay processes and magnetic moments. Our preliminary investigation of alpha decay, fission and fusion processes in this model has indicated promising results.

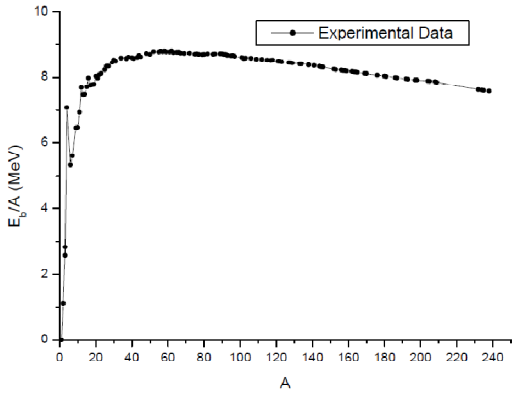

Fig. 1. Experimental data of nuclear binding energy per nucleon in terms of mass number for most of the known stable nuclei

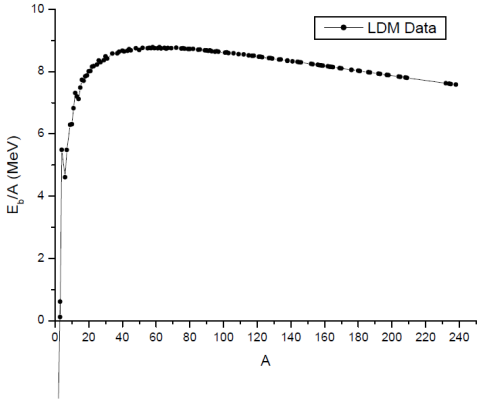

Fig. 2. LDM data of nuclear binding energy per nucleon in terms of mass number for most of the known stable nuclei

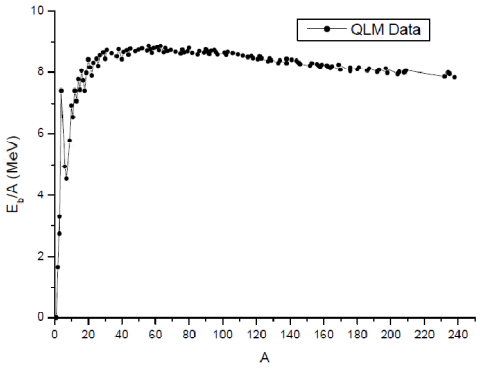

Fig. 3. Our quark-like model (QLM) data of nuclear binding energy per nucleon in terms of mass number for most of the known stable nuclei

4. Conclusion

The precise determination of the magic number and its ability of prediction of new magic number namely 184, and binding energy evaluation of quark-like model are satisfactory indications that this model may be a successful nuclear model. This model presents a new vision and a new picture of what we call nuclei. A new formula is presented that calculates the nuclear binding energy in terms of only N and Z numbers for most stable nuclides. Compared to LDM, this formula is not only simple to comprehend but also possesses the features of natural symmetry. Certainly more work should be done for this model and our group is at present investigating new features of this model.

Table 1. Nuclear binding energy per nucleon for most of the known nuclei in our model (QLM), LDM and Experimental Values

Nucleus	Z	A	N	B/A (QLM)	B/A(LDM)	B/A(exp)
				MeV	MeV	MeV
H	1	1	0	0	-26.461	0
H	1	2	1	1.65	-2.61276	1.1125
H	1	3	2	3.3	0.6105	2.82733
He	2	3	1	2.75	0.11753	2.57267
He	2	4	2	7.425	5.4863	7.074
Li	3	6	3	4.95	4.60667	5.33233
Li	3	7	4	4.55714	5.48336	5.60629
Be	4	9	5	5.775	6.2924	6.46278
B	5	10	5	6.93	6.30939	6.4751
B	5	11	6	6.54	6.82388	6.92773
C	6	12	6	7.425	7.31242	7.68017
C	6	13	7	7.06538	7.20223	7.46985
N	7	14	7	7.77857	7.11861	7.47564
N	7	15	8	7.44857	7.48535	7.69947
O	8	16	8	8.04375	7.73211	7.97619
O	8	17	9	7.74044	7.70438	7.75076
O	8	18	10	7.425	7.84722	7.76706
F	9	19	10	7.97018	7.87776	7.779
Ne	10	20	10	8.415	8.00775	8.03225
Ne	10	21	11	8.15571	8.01729	7.97171
Ne	10	22	12	7.89	8.15658	8.08045
Na	11	23	12	8.3087	8.17431	8.11148
Mg	12	25	13	8.437	8.22397	8.22352
Mg	12	26	14	8.20769	8.35642	8.33388
Al	13	27	14	8.54615	8.30071	8.33156
Si	14	29	15	8.64015	8.36406	8.44866
Si	14	30	16	8.43857	8.48917	8.52067
P	15	31	16	8.72194	8.4163	8.48119
S	16	34	18	8.61397	8.57741	8.5835
Cl	17	37	20	8.51494	8.58581	8.5703
Ar	18	38	20	8.75175	8.6344	8.61429
Ar	18	40	22	8.42417	8.66847	8.59528
K	19	41	22	8.65456	8.64551	8.57607
Ca	20	43	23	8.7143	8.66599	8.60067
Ca	20	44	24	8.565	8.71956	8.65818
Sc	21	45	24	8.76857	8.68134	8.61884
Ti	22	48	26	8.68125	8.74848	8.72292
V*	23	50	27	8.73209	8.69335	8.69588
Cr	24	52	28	8.77885	8.76065	8.77594
Mn	25	55	30	8.7	8.74901	8.765
Fe	26	56	30	8.86195	8.75985	8.79032
Fe	26	57	31	8.74433	8.75142	8.77026
Fe	26	58	32	8.62639	8.78243	8.79222
Co	27	59	32	8.7855	8.75084	8.76802
Ni	28	61	33	8.82383	8.7476	8.76502
Ni	28	62	34	8.71382	8.78201	8.79455
Cu	29	63	34	8.85961	8.74199	8.75214

Table 1. (continued)

Nucleus	Z	A	N	B/A (QLM)	B/A(LDM)	B/A(exp)
Cu	29	65	36	8.64653	8.76012	8.75711
Zn	30	66	36	8.79	8.7345	8.75964
Zn	30	67	37	8.68672	8.75723	8.73416
Ga	31	69	38	8.7244	8.75196	8.72458
Ge	32	72	40	8.6625	8.76636	8.73175
As	33	75	42	8.604	8.73985	8.70085
Se	34	76	42	8.73274	8.74887	8.71149
Se	34	77	43	8.64076	8.7332	8.6947
Br	35	79	44	8.67548	8.72461	8.68761
Kr	36	80	44	8.79542	8.72447	8.69293
Kr	36	82	46	8.62114	8.72887	8.71066
Rb	37	85	48	8.56951	8.70027	8.69745
Sr	38	86	48	8.68623	8.70634	8.70847
Y	39	89	50	8.63552	8.67965	8.71391
Zr	40	90	50	8.745	8.67799	8.70992
Zr	40	91	51	8.66613	8.66704	8.69326
Zr	40	92	52	8.58717	8.67739	8.69263
Nb	41	93	52	8.69528	8.65307	8.66414
Mo	42	95	53	8.72308	8.63786	8.64868
Mo	42	96	54	8.64777	8.65089	8.65395
Mo	42	97	55	8.57239	8.63619	8.63505
Ru	44	101	57	8.63094	8.60868	8.60129
Ru	44	102	58	8.55882	8.61641	8.5769
Rh	45	103	58	8.65832	8.59314	8.58411
Pd	46	106	60	8.61546	8.58669	8.57994
Ag	47	109	62	8.57433	8.55842	8.54795
Cd	48	112	64	8.53482	8.54871	8.54479
In*	49	115	66	8.49684	8.5192	8.51656
Sn	50	117	67	8.52415	8.5041	8.50963
Sn	50	118	68	8.46031	8.50705	8.51657
Sb	51	121	70	8.42513	8.4768	8.48202
Te	52	122	70	8.51425	8.47611	8.47814
Te	52	123	71	8.45277	8.46235	8.46555
I	53	127	74	8.35859	8.4321	8.44549
Xe	54	128	74	8.44575	8.43315	8.4433
Xe	54	129	75	8.38643	8.4183	8.4314
Cs	55	133	78	8.29669	8.38604	8.40998
Ba	56	134	78	8.38193	8.38827	8.40818
La*	57	138	81	8.29531	8.3425	8.37517
Ce	58	138	80	8.43306	8.35613	8.37707
Pr	59	141	82	8.40292	8.32701	8.35404
Nd*	60	144	84	8.37375	8.31093	8.32697
Nd	60	145	85	8.32017	8.29725	8.30923
Nd	60	146	86	8.26658	8.29491	8.30413
Sm	62	152	90	8.21428	8.2472	8.24411
Eu	63	153	90	8.2916	8.2354	8.22875
Gd	64	156	92	8.26587	8.21778	8.21537
Gd	64	157	93	8.21552	8.20338	8.20355
Gd	64	158	94	8.16515	8.19916	8.20187

Table 1. (continued)

Nucleus	Z	A	N	B/A (QLM)	B/A(LDM)	B/A(exp)
Tb	65	159	94	8.2409	8.18856	8.18885
Dy	66	162	96	8.21667	8.17041	8.17349
Dy	66	163	97	8.16779	8.15587	8.16182
Dy	66	164	98	8.1189	8.15099	8.15875
Ho	67	165	98	8.19313	8.14135	8.14701
Er	68	170	102	8.07529	8.1028	8.112
Tm	69	169	100	8.24165	8.11017	8.11451
Yb	70	176	106	8.03411	8.05472	8.06411
Lu*	71	176	105	8.15097	8.05062	8.05906
Hf	72	180	108	8.085	8.02743	8.03498
Ta	73	181	108	8.15372	8.01643	8.02343
W	74	186	112	8.04573	7.97987	7.98861
Re*	75	187	112	8.11318	7.96947	7.97795
Os	76	192	116	8.00847	7.93248	7.94852
Ir	77	193	116	8.07469	7.92257	7.93812
Pt	78	198	120	7.97308	7.88531	7.91418
Au	79	197	118	8.12012	7.89091	7.91566
Hg	80	204	124	7.93941	7.83841	7.88555
Tl	81	205	124	8.00327	7.82921	7.8784
Pb	82	208	126	7.98648	7.80973	7.86746
Bi*	83	209	126	8.04826	7.79827	7.84799
Th*	90	232	142	7.86532	7.62579	7.61503
U*	92	234	142	7.9796	7.60908	7.60071
U*	92	235	143	7.94427	7.59957	7.59091
U*	92	238	146	7.83825	7.58055	7.57013

Table 2. Binding energy comparison between our model (QLM) with Cluster model for some nuclides

Element	Neutrons	Protons	B (MeV) (Experimental)	B (MeV) (Cluster Model)	B (MeV) (Quark-like Model)
He	2	2	28.295674	28.295674	29.7
Be	4	4	56.49951	56.591348	49.5
C	6	6	92.161728	84.887022	89.1
O	8	8	127.619336	113.182696	128.7
Ne	10	10	160.644859	141.47837	168.3
Mg	12	12	198.25689	169.774044	207.9
Si	14	14	236.53689	198.069718	247.5
S	16	16	271.78066	226.365392	287.1
Ar	18	18	306.7157	254.661066	326.7
Ca	20	20	342.052	282.95674	366.3
Ti	22	22	375.4747	311.2524	405.9
Cr	24	24	411.462	339.548088	445.5
Fe	26	26	447.697	367.843762	485.1
Ni	28	28	483.988	396.139436	524.7
Zn	30	30	514.992	424.43511	564.3
Ge	32	32	545.95	452.730784	603.9
Se	34	34	576.4	481.026458	643.5
Kr	36	36	607.1	509.322132	683.1
Sr	38	38	638.1	537.617806	722.7
Zr	40	40	669.8	565.91348	762.3
Mo	42	42	700.9	594.209154	801.9
Ru	44	44	731.4	622.504828	841.5
Pd	46	46	762.1	650.800502	881.1
Cd	48	48	793.4	679.096176	920.7
Sn	50	50	824.9	707.39185	960.3

References

[1] Goeppert-Mayer, M. (1949). On Closed Shells in Nuclei II. *Phys. Rev., 75,* 1969.

[2] Haxel, O., Jensen, J. H. D. & Suess, H. E. (1949). On the Magic Numbers in Nuclear Structure. *Phys. Rev., 75,* 1766.

[3] Bohr, N. & Wheeler, J. A. (1939). The Mechanism of Nuclear Fission. *Phys. Rev., 56*, 426.

[4] Battye, R., Manton, N. & Sutcliffe, P. (2007). Skyrmions and the Alpha-Particle Model of Nuclei. *Proc. Roy. Soc. Lond., A463*, 261.

[5] Ludu, A., Sandulescu, A. & Greiner, W. (1997). Nonlinear Liquid Drop Model. Cnoidal Waves. *J. Phys., G23*, 343.

[6] Borg, J., Mishustin, I. N. & Bondorf, J. P. (1999). Nuclear Liquid-Gas Phase Transition within the Lattice Gas Model. *Phys. Lett., B470*, 13.

[7] Ray, S., Shamanna, J. & Bondorf, J. B. (1997). Isospin Lattice-Gas Model and Liquid-Gas Phase Transition in Asymmetric Nuclear Matter. *Phys. Lett., B392*, 7.

[8] Hora, H. (2006). From Laser Produced Debye Layers in Plasma to a Theory of Nuclear Forces and Quark-Gluon Plasmas. *Laser and Particle Beams, 24,* 35.

[9] Hora, H. (1991). Plasma model for the surface tension of nuclei and the phase transition to the quark plasma: in Report CERN-PS/DI-note 91/05.

[10] Ghahramany, N., Hora, H., Miley, G. H., Ghanaatian, M., Hooshmand, M., Philberth, K. & Osman, F. (2008). Nuclear Magic Numbers based on a Quark-like Model is Compared with the Boltzmann Distribution Model from Nuclear Abundance in the Universe and Low Energy Nuclear Reactions. *PHYSICS ESSAYS, 21-3*, 200.

[11] Ghahramany, N., Ghanaatian, M. & Hooshmand, M. (2007). Quark-Gluon Plasma Model and the Origin of Magic Numbers. *Iranian Physical Journal, 1-2*, 35.

[12] Hora, H. & Miley, G. H. (2007). Maruhm-Greiner-Maxinum of Uranium Fission for Confirmation of Low Energy Nuclear Reactions LENR via a Compound Nuceus with Double Layer Magic Numbers. *J. Fusion Energy, 26,* 349.

[13] Maruhn, J. & Greiner, W. (1974). Theory of Fission-Mass Distributions Demonstrated for ^{226}Ra, ^{236}U, ^{258}Fm. *Phys. Rev. Lett., 32*, 548.

[14] Miley, G. H. (1997). Possible Evidence of Anomalous Energy Effects in H/D-Loaded Solids-Low Energy Nuclear Reactions (LENRs). *J. New Energy, 2, 3-4*, 6.

[15] Poenaru, D. N., Greiner, W., Hourany, E. & Gherghescu, R. A. (1997). Even-Even-Like Alpha and Cluster Emitters. *Il Nuovo Cimento, Vol. 110 A, N. 9-10,* 1049.

[16] Osman, F., Ghahramany, N. & Hora, H. (2005). Debye Sheath Mechanism at Laser Plasma Interaction and Generalization to Nuclear Forces and Quark-Gluon Plasma. *Laser Part. Beams, 23,* 461.

[17] Hora, H. & Miley, G. H. (2000). Heavy Nuclide Synthesis by Neutrons in Astrophysics and by Screened Protons in Host Metals. *Czech. J. Phys., 50,* 433.

[18] Hora, H., Miley, G. H. & Osman, F. (2005). Boltzmann Equation of Endothermic Heavy Nuclear Synthesis in the Universe and Quark Relation of the Magic Numbers. *Astrophys. Space Sci., 298,* 247.

[19] Ghahramany, N., Ghanaatian, M. & Hora, H. (2007). Debye Layer Confinement of Nucleons in Nuclei for Heavy Nuclear Synthesis. *Iranian Physical Journal, 1-3*, 21.

[20] Rujula, A. De, Georgi, H. & Glashow, S. L. (1975). Hadron masses in a gauge theory. *Phys. Rev., D 12,* 147.

[21] Javahery, R. (2008). A new explanation of Quark causing Binding energy between Nucleons in the nuclide. *The General Science Journal*, http://www.wbabin.net/science/javaheri.pdf.