IJST (2011) A1: 1-7
Iranian Journal of Science & Technology

http://www.shirazu.ac.ir/en

Marginal behavior of the quantum potential in Extended Phase Space

S. Bahrami[1]* and S. Nasiri[1, 2]

[1]*Department of Physics, Zanjan University, P. O. Box 45196-313, Zanjan, Iran*
[2]*Institute for Advanced Studies in Basic Sciences, IASBS, P. O. Box 49195, Zanjan, Iran*
Email: Bahrami.Samira@gmail.com, Nasiri@iasbs.ac.ir

Abstract

Here, the concept of quantum potential that has been illustrated in Extended Phase Space (EPS) in previous works is explored for its marginal behavior. Unlike in configuration space, different representations of the quantum mechanics can be found in EPS when exploiting appropriate canonical transformations. These canonical transformations revealed that there exist several representations in which the quantum potential could be removed not only for a particle in constant, linear and quadratic potentials, but also for a free particle. In these representations one still has the form-invariance of the ordinary Hamilton-Jacobi equation. The present work is an attempt to show that the Squeezed Wigner representation is a possible representation in which, for large values of squeezing parameter, the quantum potential disappears from the dynamical equation for a particle in the general potential

Keywords: Quantum potential; distribution function; canonical transformation

1. Introduction

Various authors have studied the concept of quantum potential from different perspectives. One such view has been discussed using Newton's Second Law. Taking this Law into account, Bohm [1] assumed the quantum potential on the same footing as a quantum force. Muga, Sala and Snider [2] and Takabayashi [3] have described quantum potential in terms of an intrinsic internal energy which is associated with the spatial localization and momentum dispersion emerging from the inherent extended nature of the quantum particles. Still, in another perspective, Holland [4] considered the quantum potential as a scalar potential that arises from the interaction of a point like particle with the Schrödinger field. Moreover, Shojaee and Shojaee [5] and Carroll [6], on the basis of Weyl's curvature, have explored the connection of the quantum potential with the quantum fluctuations and the quantum energy.

Given such prominent views on the quantum potential, the quantum potential, unlike the external potentials, is not a pre-assigned function of the system coordinates. It can only be derived from the wave function of the systems [7] or from the corresponding quantum distribution functions used to calculate the average values of the observables [8]. This representation-dependent property of the quantum potential allows one to find the appropriate representations where the quantum potential could be removed from the modified Hamilton-Jacobi equation. In this respect, using extended phase space formulation of the quantum mechanics, Nasiri [8] has shown that the quantum potential could be removed from the dynamical equation of a particle in constant, linear and quadratic potentials. This occurs in the Wigner representation [9] which keeps the Hamilton-Jacobi equation form invariant. Along a similar line, Bahrami and Nasiri [10] have shown that the Husimi representation [11] is another possible representation in which the quantum potential is removed from the Hamilton-Jacobi equation for a particle in the quadratic potential. Here, it is shown that the squeezed Wigner representation [12] is a possible representation to release the quantum potential marginally from the Hamilton-Jacobi equation for a particle in the general potential in EPS.

In section 2, a brief review of EPS formalism is presented. In section 3, the extended canonical transformations and the way they are used to obtain the squeezed Wigner representation and the corresponding dynamical equations are proposed. In section 4, an expression for the quantum potential is obtained in EPS. In section 5, an appropriate phase space representation in which the

*Corresponding author
Received: 4 December 2010 / Accepted: 9 April 2011

quantum potential is marginally removed for the general potential is introduced. Finally, important concluding remarks are given in the last section.

2. Review of EPS formalism

Regarding the immediate relevance of the previous works to the focus of this study, the subject discussed in this section and the one that follows needs to encompass a brief review of the EPS formulation put into words by Nasiri, Sobouti and Taati [13]. However, more details for the interested readers have been provided in the original works by these researchers [see 13, 14]. The findings of Sobouti et al. [13] as well as Sobouti and Nasiri [14] give an indication of combining the configuration and the momentum space Lagrangians in order to define an extended Lagrangian in the phase space,

$$\mathcal{L}\left(p,q,\dot{p},\dot{q}\right)=-\dot{q}p-q\dot{p}+L^{q}\left(q,\dot{q}\right)+L^{p}\left(p,\dot{p}\right), \quad (1)$$

where $L^q(q,\dot{q})$ and $L^p(p,\dot{p})$ are q and p space Lagrangians and the first two terms constitute a total time derivative. Assuming the phase space coordinate p and q as independent variables on virtual trajectories allows one to define momenta π_p and π_q, conjugating to p and q, respectively,

$$\pi_p=\frac{\partial\mathcal{L}}{\partial\dot{p}}=\frac{\partial L^p}{\partial\dot{p}}-q, \quad (2a)$$

$$\pi_q=\frac{\partial\mathcal{L}}{\partial\dot{q}}=\frac{\partial L^q}{\partial\dot{q}}-p. \quad (2b)$$

Evidently, π_p and π_q vanish on actual trajectories and remain non zero on virtual ones. By means of these extended momenta and using Legendre transformation, one defines an extended Hamiltonian

$$\begin{aligned}&\mathcal{H}\left(p,q,\pi_p,\pi_q\right)=\dot{p}\pi_p+\dot{q}\pi_q-\mathcal{L}\left(p,q,\dot{p},\dot{q}\right)=\\&H\left(p+\pi_q,q\right)-H\left(p,q+\pi_p\right).\end{aligned} \quad (3)$$

In the above equation $\mathcal{H}\left(p,q,\pi_p,\pi_q\right)$ is the extended Hamiltonian and $H(p,q)$ is the ordinary Hamiltonian. Through using canonical quantization rule, the following postulates are outlined:

1) Let p, q, π_p and π_q be operators in a Hilbert space $\aleph$ of all complex functions, satisfying the following commutation relations:

$$\left[\pi_p,p\right]=-i\hbar,\quad \pi_p=-i\hbar\frac{\partial}{\partial p},\quad \left[\pi_q,q\right]=-i\hbar,\quad \pi_q=-i\hbar\frac{\partial}{\partial q}, \quad (4a)$$

$$[p,q]=\left[\pi_p,\pi_q\right]=\left[p,\pi_q\right]=\left[q,\pi_p\right]=0. \quad (4b)$$

By virtue of the above equations, the extended Hamiltonian, $\mathcal{H}\left(p,q,\pi_p,\pi_q\right)$ will also be an operator in $\aleph$.

2) A state function $\chi\left(p,q,t\right)\epsilon\aleph$ is assumed to satisfy the following dynamical equation:

$$\begin{aligned}i\hbar\frac{\partial\chi}{\partial t}&=\mathcal{H}\chi=\left[H\left(p-i\hbar\frac{\partial}{\partial q},q\right)-H\left(p,q-i\hbar\frac{\partial}{\partial p}\right)\right]\chi \quad (5)\\&=\sum\frac{\left(-i\hbar\right)^n}{n!}\left\{\frac{\partial^n H}{\partial p^n}\frac{\partial^n}{\partial q^n}-\frac{\partial^n H}{\partial q^n}\frac{\partial^n}{\partial p^n}\right\}\chi.\end{aligned}$$

3) The averaging rule for an observable, $O(p,q)$ a c-number operator in this formalism, is given as

$$\left\langle\hat{O}(p,q)\right\rangle=\int dp\,dq\,O\left(p,q\right)\chi^*\left(p,q\right). \quad (6)$$

To find the solutions for Eq. (5) one may assume

$$\chi\left(p,q,t\right)=F\left(p,q,t\right)e^{-\frac{ipq}{\hbar}}. \quad (7)$$

The phase factor comes out due to the total derivates, $-\frac{d}{dt}(pq)$, in the Lagrangian of Eq. (1). Substituting Eq. (7) into Eq. (5) and eliminating the exponential factor gives

$$\left[H\left(p-i\hbar\frac{\partial}{\partial q},q\right)-H\left(p,q-i\hbar\frac{\partial}{\partial p}\right)\right]F=i\hbar\frac{\partial F}{\partial t}. \quad (8)$$

Equation (8) has separable solutions of the form

$$F(p,q,t)=\psi(q,t)\varphi^*(p,t), \quad (9)$$

where $\psi(q,t)$ and $\varphi^*(p,t)$ are the solutions of the Schrödinger equation in the p and q representations, respectively. $\chi\left(p,q,t\right)$ is the distribution function in EPS and could be written in the following forms:

$$\chi(p,q,t)=\psi(q,t)\varphi^*(p,t)e^{(-ipq/\hbar)}, \quad (10a)$$

$$\chi\left(p,q,t\right)=\frac{1}{2\pi\hbar}\int dye^{\frac{ipy}{\hbar}}\psi\left(q,t\right)\psi\left(q+y,t\right). \quad (10b)$$

3. The extended canonical transformation

The canonical transformations that leave the extended Hamilton equations form invariant are defined as extended canonical transformations [14]. A simple infinitesimal extended canonical transformation on p, q, π_p and π_q gives the Wigner distribution function and the corresponding evolution equation [13, 14]. Let

$$p \to p + \delta\lambda \pi_q, \qquad \pi_p \to \pi_p,$$
$$q \to q + \delta\lambda \pi_p, \qquad \pi_q \to \pi_q, \tag{11}$$

where $\delta\lambda \prec\prec 1$. The generator of this infinitesimal transformation is simply

$$G\left(p,q,\pi_p,\pi_q\right) = \pi_p \pi_q. \tag{12}$$

The corresponding unitary transformation for finite λ is

$$\hat{U} = e^{\frac{i\lambda\hat{G}}{\hbar}} = e^{i\lambda\frac{\partial^2}{\partial p \partial q}}, \;\; \hat{U}\hat{U}^+ = 1. \tag{13}$$

The above transformation, for $\lambda = -\frac{1}{2}$, transforms the distribution $\chi(p,q,t)$ to those of the Wigner [13],

$$W(p,q,t) = \hat{U}\chi(p,q,t) = \frac{1}{2\pi\hbar}\int dy e^{\frac{ipy}{\hbar}} \psi\left(q - \frac{y}{2}\right)\psi^*\left(q + \frac{y}{2}\right), \tag{14}$$

and the evolution Eq. (5) changes into the Wigner equation

$$i\hbar\frac{\partial W}{\partial t} = \mathcal{H}_w W = \left[H\left(p + \frac{\pi_q}{2}, q - \frac{\pi_p}{2}\right) - H\left(p - \frac{\pi_q}{2}, q + \frac{\pi_p}{2}\right)\right]W, \tag{15a}$$

or

$$\frac{\partial W}{\partial t} = -p\frac{\partial W}{\partial q} + \sum_{n=0}\frac{1}{(2n+1)!}\left(\frac{\hbar}{2i}\right)^{2n}\frac{\partial^{2n+1}V}{\partial q^{2n+1}}\frac{\partial^{2n+1}W}{\partial p^{2n+1}}. \tag{15b}$$

The interested reader may consult the details in the given references [13, 14]. It should be mentioned that a similar technique could be used to obtain squeezed distribution functions and their corresponding evolution equations and ordinary rules. Let us consider the following infinitesimal linear transformation

$$p \to p + \delta\alpha p, \qquad \pi_p \to \pi_p - \delta\alpha \pi_p,$$
$$q \to q, \qquad \pi_q \to \pi_q, \tag{16}$$

where $\delta\alpha$ is a positive infinitesimal transformation parameter. It can be easily shown that the above transformation which exhibits the squeezing operation in (p,π_p) space is a canonical transformation. The corresponding generator is $G_1'(p,q,\pi_p,\pi_q) = p\pi_p$.

The squeezing transformation in (q,π_q) space is

$$p \to p, \qquad \pi_p \to \pi_p,$$
$$q \to q - \delta\beta q, \qquad \pi_q \to \pi_q + \delta\beta\pi_q, \tag{17}$$

where $\delta\beta$ is also the positive infinitesimal transformation parameter. Transformation (17) is again a canonical transformation and the corresponding non-hermition generator is $G_2'\left(p,q,\pi_p,\pi_q\right) = -q\pi_q$. At the operator level, the non-hermitian operators G_1' and G_2' commute and their successive application will be done by a hermition operator

$$G' = G_1' + G_2'.$$
$$G' = G_1' + G_2' = p\pi_p - q\pi_q. \tag{18}$$

Thus the corresponding unitary transformation which squeezes the (p,q,π_p,π_q) space becomes

$$\hat{U}' = e^{\frac{i\hat{G}'}{\hbar}} = e^{\alpha p\frac{\partial}{\partial p} - \beta q\frac{\partial}{\partial q}}, \tag{19}$$

The above operator transforms the Wigner distribution function into the squeezed Wigner distribution function [12] as

$$W_s(p,q,t) = \hat{U}'W(p,q,t)$$
$$= \frac{1}{2\pi\hbar}\int dy e^{\frac{ie^{\alpha}py}{\hbar}}\psi(e^{-\beta}q - \frac{1}{2}y)\psi^*(e^{-\beta}q + \frac{1}{2}y). \tag{20}$$

To keep $W_s(p,q,t)$ as a normalized distribution function, one should set $\alpha = \beta$. Thus

$$\hat{U}' = e^{\frac{i\hat{G}'}{\hbar}} = e^{\alpha(p\frac{\partial}{\partial p} - q\frac{\partial}{\partial q})}. \tag{21}$$

The corresponding evolution equation transforms to

$$\hat{U}'\left(i\hbar\frac{\partial W}{\partial t}\right) = \hat{U}'\left(\mathcal{H}_W W\right). \tag{22}$$

Using $\hat{U}\hat{U}'^{+} = 1$ one gets

$$i\hbar\frac{\partial W_s}{\partial t} = \mathcal{H}_{W_s} W_s, \qquad (23)$$

where $\mathcal{H}_{W_s} = \hat{U}'\mathcal{H}_W \hat{U}^{+\prime}$.

4. Quantum potential and generalization to EPS

The concept of the quantum potential emerges from the Bohmic interpretation of quantum mechanics. In this picture the correspondence between quantum mechanics and the Hamilton-Jacobi theory of classical mechanics is more pronounced. Following the literature in this specific area, an additional term, known as the quantum potential, appears in the Hamilton-Jacobi equation where a complex function is assumed for the Schrödinger wave function in quantum level. However, the physical origin of this term is not well known yet. Some people assume that the quantum potential is an inherent property of the matter and has quantum mechanical origin. Others, on the other hand, show that it is a representation dependent quantity and may be removed from the Hamilton-Jacobi equation in an appropriate representation. To move along both the latter view and the previous works [see 8, 10], we show here that the quantum potential could be removed from the Hamilton-Jacobi equation in an appropriate representation in EPS.

4.1. Quantum Potential in q Space

Let the Schrödinger wave function in q space be written as follows [4]

$$\psi(q,t) = R^q(q,t)e^{\frac{iS^q(q,t)}{\hbar}}, \qquad (24)$$

where $R^q(q,t)$ is the amplitude and $S^q(q,t)$ is the action that serves as the phase function of ψ in q space given by

$$S^q(q,t) = \int_0^t dt' L^q(q,\dot{q},t') \qquad (25)$$

Using Eq. (24) in the Schrödinger equation, one gets

$$\begin{aligned} &i\hbar\left(\frac{\partial R^q}{\partial t} + \frac{i}{\hbar}R^q\frac{\partial S^q}{\partial t}\right) \\ &= \frac{\hbar^2}{2m}\Big[\frac{\partial^2 R^q}{\partial q^2} + \frac{2i}{\hbar}\frac{\partial R^q}{\partial q}\frac{\partial S^q}{\partial q} \\ &+ \frac{i}{\hbar}R^q\frac{\partial^2 S^q}{\partial q^2} - \frac{1}{\hbar^2}R^q\left(\frac{\partial S^q}{\partial q}\right)^2\Big] + V(q)R^q. \end{aligned} \qquad (26)$$

The real part of Eq. (26) gives

$$\frac{\partial S^q}{\partial t} - \frac{\hbar^2}{2m}\frac{1}{R^q}\frac{\partial^2 R^q}{\partial q^2} + \frac{1}{2m}\left(\frac{\partial S^q}{\partial q}\right)^2 + V(q) = 0. \qquad (27)$$

Assuming $p = \frac{\partial S^q}{\partial q}$, one gets

$$\frac{\partial S^q}{\partial t} - \frac{\hbar^2}{2m}\frac{1}{R^q}\frac{\partial^2 R^q}{\partial q^2} + \frac{1}{2m}p^2 + V(q) = 0. \qquad (28)$$

Apart from the term $-\frac{\hbar^2}{2m}\frac{1}{R^q}\frac{\partial^2 R^q}{\partial q^2}$, known as the quantum potential, Eq. (28) is the well known Hamilton-Jacobi equation [4]. The imaginary part of Eq. (26) yields a continuity equation for $|R|^2$, which is not our interest here.

4.2. Quantum Potential in p Space

Unlike in q space, the quantum potential does not have a simple form for the general potential $V(q)$ in p space. As an example, we consider the harmonic potential and obtain the Hamilton-Jacobi equation and the corresponding quantum potential term. The Schrödinger equation for a particle in a harmonic potential, $V(q) = \frac{1}{2}kq^2$, in p representation is

$$i\hbar\frac{\partial\varphi(p,t)}{\partial t} = \frac{p^2}{2m}\varphi(p,t) - \frac{\hbar^2 k}{2}\frac{\partial^2\varphi(p,t)}{\partial p^2}, \qquad (29)$$

where $\varphi(p,t)$ is the Schrödinger wave function in p space and is assumed to be:

$$\varphi(p,t) = R^p(p,t)e^{\frac{-iS^p(p,t)}{\hbar}}, \qquad (30)$$

$\varphi(p,t)$ is the amplitude and $S^p(p,t)$ is defined as

$$S^p(p,t) = \int_0^t dt' L^p(p,\dot{p},t'). \qquad (31)$$

Here again, like the equation substitution performed in the previous section, by substituting Eq. (30) into Eq. (29) and then considering the real part, one can obtain

$$\frac{\partial S^p}{\partial t} - \frac{\hbar^2 k}{2R^p}\frac{\partial^2 R^p}{\partial p^2} + \frac{p^2}{2m} + \frac{k}{2}\left(\frac{\partial S^p}{\partial p}\right)^2 = 0. \qquad (32)$$

Considering $q=\frac{\partial S^p}{\partial p}$, Eq. (32) is the modified Hamilton–Jacobi equation for the harmonic potential in the p space and contains an additional term i.e., $-\frac{\hbar^2 k}{2R^p}\frac{\partial^2 R^p}{\partial p^2}$, as the p space version of the quantum potential.

4.3. Quantum Potential in EPS

To generalize the concept of the quantum potential into EPS, we again consider the case of the harmonic potential. According to equation (3), the extended Hamiltonian for the harmonic potential is

$$\mathcal{H}=\frac{\pi_q^2}{2m}+\frac{p\pi_q}{m}-\frac{k\pi_p^2}{2}-kq\pi_p. \tag{33}$$

First, assume that

$$\chi(p,q,t)=R(p,q,t)e^{\frac{iS(p,q,t)}{\hbar}}, \tag{34}$$

where $R(p,q,t)$ is amplitude and $S(p,q,t)$ is the extended action related to the extended Lagrangian as follows

$$S(p,q,t)=\int_0^t dt'\mathcal{L}(p,q,\dot{p},\dot{q},t'). \tag{35}$$

Next, substituting the evolution equation (5) with Eqs. (34) and (33) and then considering the real part, one gets

$$\begin{aligned}&\frac{\partial S}{\partial t}-\frac{\hbar^2}{2m}\frac{1}{R}\frac{\partial^2 R}{\partial q^2}+\frac{\hbar^2 k}{2}\frac{1}{R}\frac{\partial^2 R}{\partial p^2}\\&+\frac{1}{2m}(\frac{\partial S}{\partial q})^2+\frac{k}{2}(\frac{\partial S}{\partial p})^2=0.\end{aligned} \tag{36}$$

On the other hand, substituting Eq. (10a) with the corresponding $\psi(q,t)$ and $\varphi(q,t)$ of the Eqs. (24) and (30), one gets

$$\chi(p,q,t)=R^q(q,t)R^p(p,t)e^{\frac{i(S^q+S^p-pq)}{\hbar}}. \tag{37}$$

Also, comparing Eqs. (37) and (34) gives

$$S(p,q,t)=S^q(q,t)+S^p(p,t)-pq. \tag{38}$$

Using the above equation and Eqs. (2a, b), one gets

$$\pi_p=\frac{\partial S}{\partial p},\qquad \pi_q=\frac{\partial S}{\partial q}. \tag{39}$$

And finally, substituting Eq. (36) with Eqs. (39) and (33) gives

$$\frac{\partial S}{\partial t}-\frac{\hbar^2}{2m}\frac{1}{R}\frac{\partial^2 R}{\partial q^2}+\frac{\hbar^2 k}{2}\frac{1}{R}\frac{\partial^2 R}{\partial p^2}+\mathcal{H}=0. \tag{40}$$

Equation (40) is the modified Hamilton-Jacobi equation for the harmonic potential in EPS. The second and the third terms of the Eq. (40) together define the quantum potential in the extended phase space. Of note, according to Eqs. (28) and (32), the second term is the EPS counterpart of the quantum potential in q space and the third term is the same counterpart in p space.

5. Quantum potential free representation

Evidence shows that one can remove the quantum potential from the modified Hamilton-Jacobi equation using the EPS method for the constant, linear and quadratic potentials [8, 10]. In fact, by means of a suitable extended canonical transformation, the second and third terms in Eq. (40) (i.e., the quantum potential in EPS) can cancel each other out. As such, a question that arises is "is it possible to do this for any given potential?". To answer this question we consider the general potential $V(q)=\sum_{n=0} b_n q^n$, where b_n is a constant. In previous works, $V(q)=b_n q^n$ was considered the potential, and the specific cases of $n=0$ (free particle), $n=1$ (linear potential) and $n=2$ (harmonic potential) were worked out. We have shown that for these cases the quantum potential could be removed in the Husimi and the Wigner representations [8, 10]. Now we are looking for the possible representation in which one can remove the quantum potential for the general case. Using the corresponding extended Hamiltonian for the general potential in the Wigner representation (see Eq. (15a)), binomial expansion and the commutations (4a), one gets [8]

$$\begin{aligned}\mathcal{H}_w=\frac{p\pi_q}{m}-2\sum_{n=0}^{n}\sum_{m=0}^{n} b_n\binom{n}{m}q^{n-m}\left(\frac{\pi_p}{2}\right)^m=\frac{p}{m}\left(-i\hbar\frac{\partial}{\partial q}\right)\\-2\sum_{n=0}^{n}\sum_{m=0}^{n} b_n\binom{n}{m}q^{n-m}\left(\frac{-i\hbar}{2}\right)^m(\frac{\partial}{\partial p})^m,\end{aligned} \tag{41}$$

where m gives even values. Now assume

$$W(p,q,t) = R'(p,q,t)e^{\frac{iS'(p,q,t)}{\hbar}}, \tag{42}$$

where $R'(p,q,t)$ is amplitude and $S'(p,q,t)$ is the corresponding extended action. By a rather lengthy manipulation on Eqs. (41), (42) and (15a), one gets

$$\begin{aligned} & i\hbar\left(\frac{\partial R'}{\partial t}+\frac{iR'}{\hbar}\frac{\partial S'}{\partial t}\right) = -i\hbar\frac{p}{m}\left(\frac{\partial R'}{\partial q}+\frac{iR'}{\hbar}\frac{\partial S'}{\partial q}\right) \\ & -2\sum_{n=0}^{}\sum_{m=0}^{n}\sum_{m'=0}^{m}\sum_{m''=0}^{m'} b_n \binom{n}{m}\binom{m}{m'}\binom{m'}{m''}\left(\frac{-i\hbar}{2}\right)^m\left(\frac{i}{\hbar}\right)^{m''} \\ & \times q^{n-m}\frac{\partial^{m-m'}R'}{\partial p^{m-m'}}\frac{\partial^{m'-m''}}{\partial p^{m'-m''}}\left(\frac{\partial s'}{\partial p}\right)^{m''}. \end{aligned} \tag{43}$$

The real part of the above equation gives the modified Hamilton-Jacobi equation in phase space. With Eq. (41), the even values of m have no contribution in the quantum potential. Thus, we consider only the odd values of m. If m'' takes odd values, then the second term in the RHS becomes real. Finally, for the real part of Eq. (43) one gets

$$\begin{aligned} & \frac{\partial S'}{\partial t}+\frac{p}{m}\frac{\partial S'}{\partial q} \\ & -\frac{2}{R'}\sum_{n=0}^{}\sum_{m=0}^{n}\sum_{m'=0}^{m}\sum_{m''=0}^{m'} b_n \binom{n}{m}\binom{m}{m'}\binom{m'}{m''}\left(\frac{-i\hbar}{2}\right)^m\left(\frac{i}{\hbar}\right)^{m''} \\ & \times q^{n-m}\frac{\partial^{m-m'}R'}{\partial p^{m-m'}}\frac{\partial^{m'-m''}}{\partial p^{m'-m''}}\left(\frac{\partial s'}{\partial p}\right)^{m''} = 0. \end{aligned} \tag{44}$$

Equation (44) shows the extended form of the modified Hamilton-Jacobi equation in EPS for the general potential. This equation generates the quantum potential terms, except when $m = m' = m''$. To show how the equation works, we make the specific cases for $n = 2$ and 3

a) $n = 2$

For this case, as argued before, the only possibility is $m = m' = m'' = 1$. Thus one gets

$$\frac{\partial S'}{\partial t}+\frac{p}{m}\frac{\partial S'}{\partial q}-\frac{2b_2}{R'}\binom{2}{1}\binom{1}{1}\binom{1}{1}\left(\frac{-i\hbar}{2}\right)\left(\frac{i}{\hbar}\right)qR'\left(\frac{\partial S'}{\partial p}\right) = 0. \tag{45}$$

Considering $\pi_q = \frac{\partial S'}{\partial q}, \pi_p = \frac{\partial S'}{\partial p}$ Eq. (45) changes to

$$\frac{\partial S'}{\partial t}+\frac{p}{m}\pi_q + b_2 q\pi_p = 0. \tag{46}$$

The second and third terms constitutes extended Wigner Hamiltonian for the harmonic oscillator in EPS. Thus Eq. (46), as shown before [8], is the well-known classical Hamilton-Jacobi equation in the Wigner representation.

b) $n = 3$

For $V(q) = b_3 q^3, m, m', m''$ takes the values $m = 1$ and 3, $m' = 1, 2$ and 3, $m'' = 1$ and 3. Thus Eq. (44) becomes

$$\begin{aligned} & \frac{\partial S'}{\partial t}+\frac{p}{m}\frac{\partial S'}{\partial q} \\ & -\frac{2b_3}{R'}\Big[-\frac{3}{2}q^2R'\frac{\partial S'}{\partial p}-\frac{3}{8}\hbar^2\frac{\partial^2 R'}{\partial p^2}\frac{\partial S'}{\partial p} \\ & -\frac{3}{4}\hbar^2\frac{\partial R'}{\partial p}\frac{\partial^2 S'}{\partial p^2}-\frac{3}{8}\hbar^2 R'\frac{\partial^3 S'}{\partial p^3}-\frac{1}{8}R'\left(\frac{\partial S'}{\partial p}\right)^3\Big] = 0. \end{aligned} \tag{47}$$

Substituting π_p, π_q and extended Wigner Hamiltonian for this potential one gets

$$\begin{aligned} & \frac{\partial S'}{\partial t}+\mathcal{H}_w + b_3\Big(\frac{3\hbar^2}{4R'}\frac{\partial^2 R'}{\partial p^2}\frac{\partial S'}{\partial p} \\ & +\frac{3\hbar^2}{4R'}\frac{\partial R'}{\partial p}\frac{\partial^2 S'}{\partial p^2}+\frac{3\hbar^2}{4}\frac{\partial^3 S'}{\partial p^3}\Big) = 0, \end{aligned} \tag{48}$$

where $\mathcal{H}_w$ is obtained using (15a) for n=3. Equation (48) is the modified Hamilton-Jacobi equation. To remove the quantum potential from the modified Hamilton- Jacobi Eq. (48) or generally speaking, Eq. (44), one may use the extended unitary transformation (21) to get the extended squeezed Wigner Hamiltonian as follows:

$$\mathcal{H}_{sw} = \hat{U}'^{+}\mathcal{H}_w\hat{U}' = e^{-\frac{i\alpha\hat{G}'}{\hbar}}\mathcal{H}_w e^{\frac{i\alpha\hat{G}'}{\hbar}}, \tag{49}$$

where $\mathcal{H}_{sw}$ is the extended squeezed Wigner Hamiltonian. By means of applying the Baker-Campbell-Hausdorff relation [15], Eq. (49) becomes

$$\begin{aligned} \mathcal{H}_{sw} &= \mathcal{H}_w + \frac{i\alpha}{\hbar}[\mathcal{H}_w, G'] + \left(\frac{i\alpha}{\hbar}\right)^2[\mathcal{H}_w,[\mathcal{H}_w,G']] \\ & +\ldots+\left(\frac{i\alpha}{\hbar}\right)^n[\mathcal{H}_w,..[\mathcal{H}_w,[\mathcal{H}_w,G']]]. \end{aligned} \tag{50}$$

Using Eqs. (15a) for the general case and Eq. (18), one has

$$\begin{aligned} [\mathcal{H}_w, G'] &= \Big[\frac{p\pi_q}{m} - 2\sum_{n=0}^{}\sum_{m=0}^{n} b_n \binom{n}{m} q^{n-m}\left(\frac{\pi_p}{2}\right)^m, p\pi_p - q\pi_q\Big] \\ &= i\hbar\Big(\frac{2p\pi_q}{m} + 2\sum_{n=0}^{}\sum_{m=0}^{n} b_n\binom{n}{m} n q^{n-m}\left(\frac{\pi_p}{2}\right)^m\Big). \end{aligned} \tag{51}$$

Further, by Eq. (51) one may easily show that

$$\mathcal{H}_{sw} = \frac{e^{2\alpha} p\pi_q}{m} - 2\sum_{n=0}^{n}\sum_{m=0} b_n \binom{n}{m} e^{-n\alpha} q^{n-m} \left(\frac{\pi_p}{2}\right)^m. \quad (52)$$

For a large α, the terms that generate the quantum potential in the modified Hamilton-Jacobi equation vanishes. Note that the exponent is positive for the first term and negative for the terms in the summation. Thus as α becomes larger, the terms in the summation, which generates the quantum potential becomes smaller corresponding to the first term, which has no contribution to generating the quantum potential. Thus, in the squeezed Wigner representation for large squeezing parameter, the quantum potential could be removed marginally for the general potential.

6. Conclusion

According to the evidential support given previously, EPS formulation of the quantum mechanics provides a reasonable way to the definition of the extended Hamiltonians and the extended Lagrangians. On the basis of this enterprise, what gives a dynamical equation for the phase space distribution function is a canonical quantization in the EPS. Within such an important formalism, several different distribution functions and the corresponding representations can be obtained by means of the extended canonical transformations. The concept of quantum potential is thus generalized into EPS. The representation-dependent property of such intended quantum potential allows one to find those appropriate representations in which the quantum potential can be removed. In the previous works we presented that for constant, linear and quadratic potentials it becomes possible to remove the quantum potential from the modified Hamilton-Jacobi equation through using the Wigner and the Husimi representations. Here in this work, we attempted to set in a motion how the squeezed Wigner representation functions as a representation in which the quantum potential could be removed marginally for the general potential at the large squeezing parameter limit.

References

[1] Bohm, D. & Hiley, B. J. (1985). Unbroken Quantum Realism, from Microscopic to Macroscopic Levels. *Phys. Rev. Lett.*, *55*, 2511-2514.

[2] Muga, J. G., Sala, R. & Snider, R. F. (1993). Comparison of Classical and Quantal Evolution of Phase Space Distribution Functions. *Phys. Scripta*, *47*, 732-739.

[3] Takabayashi, T. (1952). The formulation of Quantum Mechanics in Terms of Ensemble in Phase Space. *Prog. Theoret. Phys.*, *11*, 341-373.

[4] Holland, P. R. (1993). *The Quantum Theory of Motion: Vol. 89*. Cambridge, UK, Cambridge University Press.

[5] Shojai, F. & Shojai, A. (2004). Constraints Algebra and Equation of Motion in Bohmian Interpretation of Quantum Gravity. *Classical Quantum Gravity*, *21*, 1-9.

[6] Carroll, R. W. (2006). *Fluctuations, Information, Gravity and the Quantum Potential*. Springer.

[7] Brown, M. R. (2007). The Quantum Potential: The Breakdown of Classical Symplectic Symmetry and the Energy of the Localization and Dispersion, quanta-ph/9703007.

[8] Nasiri, S. (2006). Quantum Potential and Symmetries in Extended Phase Space, SIGMA: Methods and Applications, *2*, 62-74.

[9] Wigner, E. P. (1932). On the Quantum Correction for Thermodynamic Equilibrium. *Physical Review*, *40*, 749-759.

[10] Bahrami, S. & Nasiri, S. (2008). Symmetry Transformation in Extended Phase Space: the Harmonic Oscillator in Husimi Representation, SIGMA: Methods and Applications, *4*, 14-21.

[11] Husimi, K. (1940). Some Formal Properties of the Density Matrix. *Proceedings of the Physic Mathematical Society of Japan*, 264-314.

[12] Kim, Y. S. & Noz, M. E. (1990). Phase Space Picture of Quantum Mechanics. *World Scientific Publishing Co. Pte. Ltd.*, *77*.

[13] Nasiri, S., Sobouti, Y. & Taati, F. (2006). Phase Space Quantum Mechanics- Direct, JMP 47, 092106.

[14] Sobouti, Y. & Nasiri, S. (1993). A Phase Space Formulation of Quantum State Functions. *International Journal of Modern Physics*, *B7*, 3255-3272.

[15] Kim, Y. S. & Noz, M. E. (1991). Phase Space Picture of Quantum Mechanics. *World Scientific*, p. 80.