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Fuzzy soft Γ-ring

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Abstract

The concept of fuzzy soft Γ-ring is introduced; and some properties of fuzzy soft Γ-rings are given. Then the definitions of fuzzy soft Γ-ideals are proposed and some of their theories are considered.

Keywords: $\Gamma -$ ring; fuzzy soft ring; soft ring

1. Introduction

Since the concept of soft sets was introduced by Molodtsov [1] in 1999, soft sets theory has been extensively studied by many authors. This theory has been applied to many different fields, such as function smoothness, Riemann and Perron integration, measurement and game theory, decision making. Maji et al [2, 3] pointed out several directions or applications of soft sets. They also studied several operations on the theory of soft sets. Chen et al. [4] introduced a new definition of soft set parameterizations reduction, and compared this definition to the related concept of attributes reduction in rough set theory. Aktas et al.[5] studied the basic concept of soft set theory, and compared soft sets to fuzzy and rough sets, providing some examples to clarify their differences.

The algebraic structure of set theories dealing with uncertainties has been studied by some authors. Aktas et al. [5] applied the notion of set to the theory groups. Jun [6] introduced the notions of soft BCK/BCI-algebras, and then investigated their basic properties [7] Öztürk et al. [8] discussed a new view of fuzzy Gamma rings.

It is well known that the concept of fuzzy sets, introduced by Zadeh [9], has been extensively applied to many scientific fields. In 1971, Rosenfeld [10] applied the concept to the theory of groupoids and groups. In 1982, Liu [11] defined and studied fuzzy subrings as well as fuzzy ideals. Since then many papers concerning various fuzzy algebraic structures have appeared in the literature. The various constructions of fuzzy quotients rings and fuzzy isomorphism have been investigated respectively by several researchers (see e.g. [12, 13, and 14]).

Also, Maji et al. presented the definition of fuzzy soft set, and Roy et al. presented some applications of this notion to the decision- making problems in [2]. Inan et al. have already introduced the definition of fuzzy soft rings and studied some of their basic properties.

In this paper, we attempt to study fuzzy soft $\Gamma -$ ring theory by using fuzzy soft sets. We first introduce fuzzy soft $\Gamma -$rings generated by fuzzy soft sets, and give their properties. Consequently we study the definition of fuzzy soft $\Gamma -$ideal and derive some results from them, respectively.

2. Preliminaries

In this section, for the sake of completeness, we first cite some useful definitions and results.

Definition 2.1. [9] A fuzzy subset μ in a set X is a function $\mu : X \to [0,1]$.

Definition 2.2. [1] Let U be an initial universe. E a set of parameters and $A \subset E$. Let $P(U)$ denote the fuzzy power set of U. A pair (F, A) is called a fuzzy soft set over U, where F is a mapping given by $F : A \to P(U)$.

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Definition 2.3. [1] Let (F,A) and (G,B) be two fuzzy soft sets over U. Then (F,A) is said to be a fuzzy soft subset of (G,B) if

i. $A \subset B$

ii. $F(x)$ is a fuzzy subset of $G(x)$ for all $x \in A$.

We denote the above inclusion relationship by $(F,A) \subset (G,B)$. Similarly, (F,A) is called a fuzzy soft superset of (G,B) if (G,B) is a fuzzy soft subset of (F,A). The above relationship can be denoted by $(F,A) \supset (G,B)$.

Definition 2.4. [5] The intersection of two fuzzy soft sets (F,A) and (G,B) over U is the soft set (H,C), where $C = A \cap B$ and for all $x \in C$, either $H(x) = F(x)$ or $H(x) = G(x)$. This intersection is denoted by $(F,A) \tilde{\cap} (G,B) = (H,C)$.

Definition 2.5. [5] The union of two fuzzy soft sets (F,A) and (G,B) over U is the soft set (H,C), where $C = A \cap B$ and for all $x \in C$, define by:

$$H(x) = \begin{cases} F(x) & \text{if } x \in A - B \\ G(x) & \text{if } x \in B - A \\ F(x) \cup G(x) & \text{Otherwise} \end{cases}$$

The above relationship is denoted by $(F,A) \tilde{\cup} (G,B) = (H,C)$.

Definition 2.6. [5] Let (F,A) and (G,B) be two fuzzy soft sets. Then we denote (F,A) *AND* (G,B) by $(F,A) \tilde{\wedge} (G,B)$. The soft set $(F,A) \tilde{\wedge} (G,B)$ is defined by $(H, A \times B)$, where $H(\alpha,\beta) = F(\alpha) \cap G(\beta)$, for all $(\alpha,\beta) \in A \times B$.

Definition 2.7. [5] Let (F,A) and (G,B) be two fuzzy soft sets. Then (F,A) *OR* (G,B) denoted by $(F,A) \tilde{\vee} (G,B)$ is defined by $(H, A \times B)$, where $H(\alpha,\beta) = F(\alpha) \cup G(\beta)$, for all $(\alpha,\beta) \in A \times B$.

Now, we show the definition and ideal of $\Gamma-$ring and of a $\Gamma-$ ring.

Definition 2.8. [15] Let S and Γ be two additive abelian groups. S is called a $\Gamma-$ring if there exist a mapping $S \times \Gamma \times S \to S$ by $(a,\alpha,b) \mapsto a\alpha b$ satisfying the following conditions:

1. $(a+b)\alpha c = a\alpha c + b\alpha c,$
2. $a\alpha(b+c) = a\alpha b + a\alpha c,$
3. $a(\alpha+\beta)b = a\alpha b + a\beta b,$
4. $a\alpha(b\beta c) = (a\alpha b)\beta c$

for all $a,b,c \in S$ and for all $\alpha,\beta \in \Gamma$.

A left (resp. right) ideal of a $\Gamma-$ring S is a subset A of S which is an additive subgroup of S and $S\Gamma A \subset A$ (resp, $S\Gamma A \subset A$) where, $S\Gamma A = \{x\alpha y \mid x,y \in S, \alpha \in \Gamma\}$. If A is both a left and right ideal, then A is called a gamma ideal of S.

Definition 2.8. [16] Let S and K be two $\Gamma-$ rings, and f be a mapping of S into K. Then f is called $\Gamma-$homomorphism if $f(a+b) = f(a) + f(b)$ and $f(a\alpha b) = f(a)\alpha f(b)$ for all $a,b \in S$ and $\alpha \in \Gamma$.

Definition 2.8. [15] A fuzzy set μ in $\Gamma-$ring S is called a fuzzy ideal of S, if for all $x.y \in S$ and $\alpha \in \Gamma$, the following requirements are satisfied:

1. $\mu(x-y) \geq \min\{\mu(x), \mu(y)\}$
2. $\mu(x\alpha y) \geq \max\{\mu(x), \mu(y)\}$.

3. Fuzzy soft $\Gamma-$ring

In what follows let A be $\Gamma-$ring and nonempty set and R will refer to an arbitrary ternary relation among one element of A, an element of Γ and an element of A, that is, R is a subset of $A \times \Gamma \times A$, unless otherwise specified. A set valued function $f: N \to P(A)$ can be defined $f(a) = \{b \in A \, \hat{u} R(a\alpha b); \forall \alpha \in \Gamma\}$ as for all

$a \in N$. The pair (f, N) is then a soft set over A.

Definition 3.1. Let (f, N) be a soft set over $\Gamma-$ ring A. Then (f, N) is called a soft $\Gamma-$ring over A iff $f(a)$ is a sub $\Gamma-$ring of A, for all $a \in N \subset A$.

Definition 3.2. Let $(R,+,.)$ be a ring and E be a parameter set and $A \subset E$. Let f be a mapping given by $f: A\times\Gamma\times A \to [0,1]^R$, where $[0,1]^R$ denotes the collection of all fuzzy subsets of R,

$$f: A\times\Gamma\times A \to [0,1]^R$$
$$(a,\alpha,b) \mapsto f(a\alpha b) = f_{a\alpha b}$$
$$f_{a\alpha b} = f_\mu : R \to [0,1] \; \{\mu = a\alpha b\}$$

is called a fuzzy soft $\Gamma-$ring over R if and only if for each $a,b \in A$ and $\alpha \in \Gamma$ the corresponding fuzzy subset f_μ of f is fuzzy soft $\Gamma-$subring of R, i.e. $\forall x,y \in R$

i. $f_\mu(x+y) \geq f_\mu(x) \wedge f_\mu(y)$

ii. $f_\mu(-x) \geq f_\mu(x)$

iii. $f_\mu(x\alpha y) \geq f_\mu(x) \wedge f_\mu(y)$.

Theorem 3.3. Let (f, N) be a fuzzy soft $\Gamma-$set over R, then (f, N) be a fuzzy soft $\Gamma-$ring over R iff for each $\mu \in N$ and $x,y \in R$ the following conditions hold

i. $f_\mu(x-y) \geq f_\mu(x) \wedge f_\mu(y)$

ii. $f_\mu(x\alpha y) \geq f_\mu(x) \wedge f_\mu(y)$

Proof: First we suppose that (f, N) be a fuzzy soft $\Gamma-$ring over R. Then for each $\mu \in N$ and $x, y \in R$

$$\begin{aligned} f_\mu(x-y) &= f_\mu(x+(-y)) \\ &\geq f_\mu(x) \wedge f_\mu(-y) \\ &\geq f_\mu(x) \wedge f_\mu(y) \end{aligned}$$

conversely, suppose that the given conditions hold. Then for each $\mu \in N, x, y \in R$

$$\begin{aligned} f_\mu(0) &= f_\mu(x-x) \\ &\geq f_\mu(x) \wedge f_\mu(x) \\ &= f_\mu(x) \end{aligned}$$

where 0 is the additive identity of R

$$\begin{aligned} f_\mu(-x) &= f_\mu(0-x) \\ &\geq f_\mu(0) \wedge f_\mu(x) \\ &\geq f_\mu(x) \wedge f_\mu(x) \\ &= f_\mu(x) \end{aligned}$$

Now,

$$\begin{aligned} f_\mu(x+y) &= f_\mu(x-(-y)) \\ &\geq f_\mu(x) \wedge f_\mu(-y) \\ &\geq f_\mu(x) \wedge f_\mu(y). \end{aligned}$$

$$f_\mu(x\alpha y) \geq f_\mu(x) \wedge f_\mu(y)$$

so this completes proof.

Example 3.4. $R = Z_6 = \{\overline{0}, \overline{1}, \overline{2}, \overline{3}, \overline{4}, \overline{5}\}$ define $\overline{x}\alpha\overline{y} = \overline{x\alpha y}$ for all $\overline{x}, \overline{y} \in R, \alpha \in \Gamma$. Then R is a $\Gamma-$ring and let $N = A\times\Gamma\times A.$ $f: N \to P(R)$ be a set valued function defined by $f_a = \left\{ \begin{array}{l} (\overline{0},0.8),(\overline{1},0.2),(\overline{2},0,4), \\ (\overline{3},0.2),(\overline{4},0,4),(\overline{5},0.2) \end{array} \right\}$

$$f_b = \left\{ \begin{array}{l} (\overline{0},0,9),(\overline{1},0.5),(\overline{2},0.5), \\ (\overline{3},0,1),(\overline{4},0.5),(\overline{5},0.5) \end{array} \right\}$$

and

$$f_c = \left\{ \begin{array}{l} (\overline{0},0,7),(\overline{1},0.3),(\overline{2},0.3), \\ (\overline{3},0,3),(\overline{4},0.3),(\overline{5},0.3) \end{array} \right\}.$$

Obviously (f, N) is a fuzzy soft set over R. Also, we see that f_μ is a fuzzy ideal of R for all $\mu \in N$, thus (f, N) is a fuzzy soft $\Gamma-$ring over R.

Definition 3.5. Let (f, N) and (g, M) be two fuzzy soft $\Gamma-$rings over R. "$(f,N) AND (g,M)$", denoted by $(f,N)\tilde{\wedge}(g,M)$, is defined by $(f,N)\tilde{\wedge}(g,M) = (h,S)$ where $S = N \cap M$, $h_s = f_s \cap g_s$ for all $s \in S$.

Theorem 3.6. If (f,N) and (g,M) be two fuzzy soft $\Gamma-$ rings over R, then $(f,N)\tilde{\wedge}(g,M)$ is a fuzzy soft $\Gamma-$ ring over R.

Proof: $(f,N)\tilde{\wedge}(g,M)=(h,S)$ where $S=N\cap M$, for all $s\in S, h_s=f_s\cap g_s$ $\forall x,y\in R$ and $\alpha\in\Gamma$,

$$\begin{aligned}h_s(x-y)&=f_s(x-y)\cap g_s(x-y)\\&\geq(f_s(x)\wedge f_s(y))\wedge(g_s(x)\wedge g_s(y))\\&=(f_s(x)\wedge g_s(x))\wedge(f_s(y)\wedge g_s(y))\\&=(f_s\cap g_s)(x)\wedge(f_s\cap g_s)(y)\\&=h_s(x)\wedge h_s(y)\\&\Rightarrow h_s(x-y)\geq h_s(x)\wedge h_s(y)\end{aligned}$$

$$\begin{aligned}h_s(x\alpha y)&=f_s(x\alpha y)\cap g_s(x\alpha y)\\&\geq f_s(x)\wedge f_s(y))\wedge(g_s(x)\wedge g_s(y))\\&=(f_s(x)\wedge g_s(x))\wedge(f_s(y)\wedge g_s(y))\\&=(f_s\cap g_s)(x)\wedge(f_s\cap g_s)(y)\\&=h_s(x)\wedge h_s(y)\\&\Rightarrow h_s(x\alpha y)\geq h_s(x)\wedge h_s(y)\end{aligned}$$

$\Rightarrow$ Thus h_s is a fuzzy soft sub$\Gamma-$ring of R for all $s\in S=N\cap M$ and so $(f,N)\tilde{\wedge}(g,M)$ is a fuzzy soft$\Gamma-$ring over R.

Definition 3.7. Let (f,N) and (g,M) be two fuzzy soft$\Gamma-$ rings over R. "$(f,N)OR(g,M)$", denoted by $(f,N)\tilde{\vee}(g,M)$, is defined by $(f,N)\tilde{\vee}(g,M)=(h,S)$ where $S=N\cap M$, $h_s=f_s\cup g_s$ for all $s\in S$.

Theorem 3.8. If (f,N) and (g,M) be two fuzzy soft $\Gamma-$rings over R, Then $(f,N)\tilde{\vee}(g,M)$ is a fuzzy soft $\Gamma-$ring over R.

Proof: It can be similar proof of Theorem 3.6.

Definition 3.9. The intersection of two fuzzy soft $\Gamma-$rings (f,N) and (g,M) over R is the fuzzy soft $\Gamma-$ring (h,S), where $S=N\cap M$, and for all $\mu\in S$, either $f_\mu=h_\mu$ or $g_\mu=h_\mu$. This intersection is denoted by $(f,N)\tilde{\cap}(g,M)=(h,S)$.

Theorem 3.10. Let (f,N) and (g,M) be two fuzzy soft $\Gamma-$rings over R. Then $(f,N)\tilde{\cap}(g,M)$ is a fuzzy soft $\Gamma-$ring over R.

Proof: Let $(f,N)\tilde{\cap}(g,M)=(h,S)$, where $S=N\cap M$, $\mu\in S$, then $x,y\in R,\alpha\in\Gamma$

$$\begin{aligned}h_\mu(x-y)&=f_\mu(x-y)\wedge g_\mu(x-y)\\&\geq f_\mu(x)\wedge f_\mu(y)\wedge g_\mu(x)\wedge g_\mu(y)\\&=(f_\mu(x)\wedge g_\mu(x))\wedge(f_\mu(y)\wedge g_\mu(y))\\&=(f_\mu\wedge g_\mu)(x)\wedge(f_\mu\wedge g_\mu)(y)\\&=h_\mu(x)\wedge h_\mu(y)\end{aligned}$$

where f_μ,g_μ,h_μ are the fuzzy soft$\Gamma-$subsets of R corresponding to the parameter $\mu\in S$.

$$\begin{aligned}h_\mu(x\alpha y)&=f_\mu(x\alpha y)\wedge g_\mu(x\alpha y)\\&\geq f_\mu(x)\wedge f_\mu(y)\wedge g_\mu(x)\wedge g_\mu(y)\\&=(f_\mu(x)\wedge g_\mu(x))\wedge(f_\mu(y)\wedge g_\mu(y))\\&=(f_\mu\wedge g_\mu)(x)\wedge(f_\mu\wedge g_\mu)(y)\\&=h_\mu(x)\wedge h_\mu(y)\end{aligned}$$

this completes the proof.

Definition 3.11. The union of two fuzzy soft $\Gamma-$ rings (f,N) and (g,M) over R be denoted by $(f,N)\tilde{\cup}(g,M)$, we define $(f,N)\tilde{\cup}(g,M)=(h,C)$, where $C=N\cup M$ and $\forall c\in C$

$$h_c=\begin{cases}f_c & ,if\ \ c\in N\text{-}M\\ g_c & ,if\ \ c\in M\text{-}N\\ f_c\vee g_c & ,if\ \ c\in N\cap M\end{cases}$$

for all $x,y\in R$ and $\alpha\in\Gamma$.

Theorem 3.12. Let (f,N) and (g,M) be two fuzzy soft $\Gamma-$rings over R. Then $(f,N)\tilde{\cup}(g,M)$ is a fuzzy soft $\Gamma-$ring over R, if $N\cap M=\varnothing$.

Proof: We know that $(f,N)\tilde{\cup}(g,M)=(h,C)$, where $C=N\cup M$ and $\forall c\in C$

$$h_c=\begin{cases} f_c & ,if \;\; c\in N\text{-}M \\ g_c & ,if \;\; c\in M\text{-}N \\ f_c\vee g_c & ,if \;\; c\in N\cap M \end{cases}$$

for all $x,y\in R$ and $\alpha\in\Gamma$.

For $c\in N-M$,

$$\begin{aligned} h_c(x-y) &= f_c(x-y) \\ &\geq f_c(x)\wedge f_c(y) \end{aligned}$$

Since $c\in N-M$, we say that $h_c(x)=f_c(x)$ and $h_c(y)=f_c(y)$. $h_c(x-y)\geq h_c(x)\wedge h_c(y)$.

$$\begin{aligned} h_c(x\alpha y) &= f_c(x\alpha y) \\ &\geq f_c(x)\wedge f_c(y) \\ &= h_c(x)\wedge h_c(y) \\ &\Rightarrow h_c(x\alpha y)\geq h_c(x)\wedge h_c(y). \end{aligned}$$

for $c\in M-N$

$$\begin{aligned} h_c(x-y) &= g_c(x-y) \\ &\geq g_c(x)\wedge g_c(y) \\ &= h_c(x)\wedge h_c(y) \\ &\Rightarrow h_c(x-y)\geq h_c(x)\wedge h_c(y) \end{aligned}$$

$$\begin{aligned} h_c(x\alpha y) &= g_c(x\alpha y) \\ &\geq g_c(x)\wedge g_c(y) \\ &= h_c(x)\wedge h_c(y) \\ &\Rightarrow h_c(x\alpha y)\geq h_c(x)\wedge h_c(y). \end{aligned}$$

Definition 3.13. Let (f,N) and (g,M) be two fuzzy soft $\Gamma-$ rings over R. The fuzzy soft $\Gamma-$ ring (g,M) is called a fuzzy soft sub $\Gamma-$ring of (f,N), if it satisfies:

i. $M\subset N$

ii. g_μ is sub $\Gamma-$ring of f_μ, for all $\mu\in M$.

Definition 3.14. Let $(f_i,N_i)_{i\in I}$ be fuzzy soft $\Gamma-$ rings over R. Then the intersection of these fuzzy soft $\Gamma-$rings is defined as being the fuzzy soft $\Gamma-$ring (g,M) satisfying the following conditions:

1. $M=\bigcap_{i\in I} A_i$
2. For all $\mu\in M$, there exist an $i_0\in I$ such that $g_\mu=f_\mu(i_0)$. In this case, we write $\bigcap_{i\in I}(f_i,N_i)=(g,M)$.

Definition 3.15. Let $(f_i,N_i)_{i\in\iota}$ be fuzzy soft $\Gamma-$ rings over R. Then,

1. $(g,M)=\tilde{\bigwedge}_{i\in I}(f_i,N_i)$ is a fuzzy soft $\Gamma-$ ring such that $M=\prod_{i\in I}N_i$ and $g_\mu=\bigcap_{i\in I}f_{i,\mu_i}$ for all $\mu=(\mu_i)_{i\in I}\in M$.
2. $(g,M)=\tilde{\bigvee}_{i\in I}(f_i,N_i)$ is a fuzzy soft $\Gamma-$ ring such that $M=\prod_{i\in I}N_i$ and $g_\mu=\bigcup_{i\in I}f_{i,\mu_i}$ for all $\mu=(\mu_i)_{i\in I}\in M$.

Theorem 3.16. Let (f,N) be a fuzzy soft $\Gamma-$ ring over R. If $\{(f_i,N_i)\,|\,i\in I\}$ is a nonempty family of fuzzy soft sub $\Gamma-$ring of (f,N), where I is an index set, then,

1. $\tilde{\bigcap}_{i\in I}(f_i,N_i)$ is a fuzzy soft sub $\Gamma-$ring of (f,N)
2. $\tilde{\bigwedge}_{i\in I}(f_i,N_i)$ is a fuzzy soft sub $\Gamma-$ring of (f,N)
3. $\tilde{\bigvee}_{i\in I}(f_i,N_i)$ is a fuzzy soft sub $\Gamma-$ring of (f,N), where $N_i\cap N_j=\varnothing$ for all $i,j\in I$.

Proof: Using Definition 3.14 and since (f_i,N_i) is a fuzzy soft sub $\Gamma-$ring of (f,N) for all $i\in I$, we have $g:M\to P(R)$ by $g_\mu=f_{i,\mu}$ for all $\mu\in M=\bigcap_{i\in I}N_i\subset N$ and $i\in I$. In this case, $f_{i,\mu}$ is a sub $\Gamma-$ring of R for all $\mu\in M$ and $i\in I$, so $\bigcap_{i\in I}f_{i,\mu}$ is a sub $\Gamma-$ring of R. Thus (g,M) is a fuzzy soft $\Gamma-$ring over R. Hence $(g,M)=\tilde{\bigcap}_{i\in I}(f_i,N_i)$ is a fuzzy soft sub $\Gamma-$ring of (f,N) by Definition 3.14.

The proofs of 2. and 3. can be written similarly.

Definition 3.17. Let (f,N) be a fuzzy soft $\Gamma-$ ring over R. Then (f,N) be a fuzzy soft $\Gamma-$ ideal over R iff for each $\mu\in N$ and $x,y\in R,\alpha\in\Gamma$ the following conditions hold:

$$f_\mu(x-y)\geq f_\mu(x)\wedge f_\mu(y)$$

$$f_\mu(x\alpha y)\geq f_\mu(x)\vee f_\mu(y)$$

defined by $(f,N)\,\tilde{\trianglerighteq}_\Gamma\, R$.

Theorem 3.18. For any fuzzy soft $\Gamma-$ ideals (f,N) and (g,M) over R, where $N\cap M\neq\varnothing$, we have $(f,N)\,\tilde{\cap}\,(g,M)\,\tilde{\trianglerighteq}_\Gamma\, R$

Proof: Using Definition 3.9 we can write $(f,N)\,\tilde{\cap}\,(g,M)=(h,S)$, where $S=N\cap M$ and for all $s\in S=N\cap M, h_s=f_s\cap g_s$ $\forall x,y\in R,$

$$\begin{aligned} h_s(x-y)&=f_s(x-y)\cap g_s(x-y)\\ &\geq(f_s(x)\wedge f_s(y))\cap(g_s(x)\wedge g_s(y))\\ &=(f_s(x)\wedge g_s(x))\wedge(f_s(y)\wedge g_s(y)) \end{aligned}$$

since $(f,N)\,\tilde{\trianglerighteq}_\Gamma\, R$ and $(g,M)\,\tilde{\trianglerighteq}_\Gamma\, R$, we know that f_s and g_s are fuzzy soft $\Gamma-$ideal of R for all $s\in S$. Therefore,

$h_s(x-y)\geq h_s(x)\wedge h_s(y)$. And,

$$\begin{aligned} h_s(x\alpha y)&=f_s(x\alpha y)\cap g_s(x\alpha y)\\ &\geq(f_s(x)\vee f_s(y))\wedge(g_s(x)\vee g_s(y))\\ &=(f_s(x)\wedge(g_s(x))\vee(f_s(y)\wedge g_s(y))\\ &=f_s\cap g_s(x)\vee f_s\cap g_s(y)\\ &=h_s(x)\vee h_s(y). \end{aligned}$$

Hence (h,S) is fuzzy soft $\Gamma-$ideal and therefore $(f,N)\,\tilde{\cap}\,(g,M)=(h,S)\,\tilde{\trianglerighteq}_\Gamma\, R$.

Theorem 3.19. For any fuzzy soft $\Gamma-$ideals (f,I) and (g,J) over R, in which I and J are disjoint, we have $(f,I)\,\tilde{\cup}\,(g,J)\,\tilde{\trianglerighteq}_\Gamma\, R$.

Proof: Assume that $(f,I)\,\tilde{\trianglerighteq}_\Gamma\, R$ and $(g,J)\,\tilde{\trianglerighteq}_\Gamma\, R$. By means of definition we can write $(f,I)\,\tilde{\cup}\,(g,J)=(h,K)$ where $K=I\cup J$ and for every $\mu\in K$

$$h_\mu(c)=\begin{cases} f_\mu & ,if\quad c\in I\text{ - }J\\ g_\mu & ,if\quad c\in J\text{ - }I\\ f_\mu\vee g_\mu & ,if\quad c\in I\cap J \end{cases}$$

since $I\cap J=\varnothing$ is either $c\in I-J$ or $c\in J-I$ for all $c\in K$. If $c\in I-J$, then $h_\mu=f_\mu$ is the ideal of R since $(f,I)\,\tilde{\trianglerighteq}_\Gamma\, R$. If $c\in J-I$, then $h_\mu=g_\mu$ is the ideal of R since $(g,J)\,\tilde{\trianglerighteq}_\Gamma\, R$. Thus h_μ is the ideal of R for all $c\in K$, and so $(f,I)\,\tilde{\cup}\,(g,J)=(h,K)\,\tilde{\trianglerighteq}_\Gamma\, R.$

Theorem 3.20. Let (f,N) be a fuzzy soft $\Gamma-$ ideal over R. If $\{(f_j,I_j)\mid j\in J\}$ is a nonempty family of fuzzy soft ideals of (f,N), where J is an index set, then,

1. $\underset{j\in J}{\cap}(f_j,I_j)\,\tilde{\trianglerighteq}_\Gamma\, R$
2. $\underset{j\in J}{\wedge}(f_j,I_j)\,\tilde{\trianglerighteq}_\Gamma\, R$
3. $\underset{j\in J}{\vee}(f_j,I_j)\,\tilde{\trianglerighteq}_\Gamma\, R$ where $I_j\cap I_k=\{0\}$ for all $j,k\in J$.

Definition 3.21. Let (f,N) and (g,M) be two fuzzy soft $\Gamma-$rings over A and B, respectively. Let $F:A\to B$ and $G:N\to M$ be two functions. Then the pair (F,G) is called a fuzzy soft $\Gamma-$ring homomorphism if it satisfies the following conditions:

F is an onto ring homomorphism
G is an onto ring homomorphism
$F(f_\mu)=(g_\mu)G$ for all $\mu\in N$.

If there exist a fuzzy soft $\Gamma-$ring homomorphism between (f,N) and (g,M), we say that (f,N) is fuzzy soft homomorphic to (g,M), and is denoted by $(f,N)\sim_\Gamma(g,M)$. Moreover, F is an isomorphism and (f,N) is fuzzy soft homomorphic to (g,M), which is denoted by $(f,N)\simeq_\Gamma(g,M)$.

Now, we show that the homomorphic image and preimage of a fuzzy soft $\Gamma-$ring are also fuzzy soft $\Gamma-$ring.

Definition 3.22. Let $\varphi : X \to Y$ and $\psi : N \to M$ be two functions, where N and M are parameter sets for fuzzy soft $\Gamma-$sets X and Y, respectively. Then the pair (φ,ψ) is called a fuzzy soft $\Gamma-$function from X to Y.

Definition 3.23. Let (f,N) and (g,M) be two fuzzy soft $\Gamma-$rings over X and Y. Let (φ,ψ) be fuzzy soft $\Gamma-$function from X to Y.

1. The image of (f,N) under the fuzzy soft $\Gamma-$function (φ,ψ) denoted by $(\varphi,\psi)(f,N)$ is the fuzzy soft $\Gamma-$ring over Y defined by $(\varphi,\psi)(f,N) = (\varphi(f),\psi(N))$ where,
$$\varphi(f)_k(y) = \begin{cases} \underset{\varphi(x)=y}{\vee}\ \underset{\varphi(a)=k}{\vee} f_a(x) & ,if\ x \in \varphi^{-1}(y) \\ 0 & ,otherwise \end{cases}$$
$\forall k \in \varphi(N), \forall y \in Y$.
2. The preimage of (g,M) under the fuzzy soft $\Gamma-$function (φ,ψ) denoted by $(\varphi,\psi)^{-1}(g,M)$ is the fuzzy soft $\Gamma-$ring over X defined by $(\varphi,\psi)^{-1}(g,M) = (\varphi^{-1}(g),\psi^{-1}(M))$ where, $\forall x \in X$ $\varphi^{-1}(g)_a(x) = g_{\psi(a)}(\varphi(x))\ \forall a \in \psi^{-1}(M)$.

If φ and ψ are injective (surjective), then (φ,ψ) is said to be injective (surjective).

Definition 3.24. Let (φ,ψ) be a fuzzy soft $\Gamma-$function from X to Y. If φ is a homomorphism function from X to Y, then (φ,ψ) is said to be fuzzy soft $\Gamma-$homomorphism. If φ is a isomorphism function from X to Y and ψ is one to one mapping from N onto M, then (φ,ψ) is said to be fuzzy soft $\Gamma-$isomorphism.

Theorem 3.25. Let (f,N) be a fuzzy soft $\Gamma-$ring over R and (φ,ψ) be a fuzzy soft $\Gamma-$homomorphism from R to S. Then $(\varphi,\psi)(f,N)$ is a fuzzy soft $\Gamma-$ring over S.

Proof: Let $k \in \psi(N)$ and $y_1, y_2 \in Y$. If $\varphi^{-1}(y_1) = \varnothing$ or $\varphi^{-1}(y_2) = \varnothing$ the proof is clear. Let us assume that $\varphi(x_1) = y_1, \varphi(x_2) = y_2$.
$$\begin{aligned} \varphi(f)_k(y_1 - y_2) &= \underset{\varphi(x_1-x_2)=y_1-y_2}{\vee}\ \underset{\psi(a)=k}{\vee} f_a(k) \\ &\geq \underset{\psi(a)=k}{\vee} f_a(x_1 - x_2) \\ &\geq \underset{\psi(a)=k}{\vee} (f_a(x_1) \wedge f_a(x_2)) \\ &= \underset{\psi(a)=k}{\vee} f_a(x_1) \wedge \underset{\psi(a)=k}{\vee} f_a(x_2) \end{aligned}$$
this inequality is satisfied for each $x_1, x_2 \in X$, where $\varphi(x_1) = y_1$, is satisfied $\varphi(x_2) = y_2$. Then we have:
$$\begin{aligned} &\varphi(f)_k(y_1 - y_2) \geq \\ &\left(\underset{\varphi(x_1)=y_1}{\vee}\ \underset{\psi(a)=k}{\vee} f_a(x_1)\right) \wedge \left(\underset{\varphi(x_2)=y_2}{\vee}\ \underset{\psi(a)=k}{\vee} f_a(x_2)\right) \\ &= \varphi(f)_k(y_1) \wedge \varphi(f)_k(y_2) \end{aligned}$$
and similarly we have $\varphi(f)_k(y_1 \alpha y_2) \geq \varphi(f)_k(y_1) \wedge \varphi(f)_k(y_2)$.

Theorem 3.26. Let (g,M) be a fuzzy soft $\Gamma-$ring over S and (φ,ψ) be a fuzzy soft $\Gamma-$homomorphism from R to S. Then $(\varphi,\psi)^{-1}(g,M)$ is a fuzzy soft $\Gamma-$ring over R.

Proof: Let $a \in \psi^{-1}(B)$ and $x_1, x_2 \in X$, $\alpha \in \Gamma$.
$$\begin{aligned} \varphi^{-1}(g)_a(x_1 \alpha x_2) &= g_{\psi(a)}(\varphi(x_1 \alpha x_2)) \\ &= g_{\psi(a)}(\varphi(x_1)\alpha\varphi(x_2)) \\ &\geq g_{\psi(a)}(\varphi(x_1) \wedge g_{\psi(a)}(\varphi(x_2)) \\ &= \varphi^{-1}(g)_a(x_1) \wedge \varphi^{-1}(g)_a(x_2) \end{aligned}$$
and similarly we have $\varphi^{-1}(g)_a(x_1 - x_2) \geq \varphi^{-1}(g)_a(x_1) \wedge \varphi^{-1}(g)_a(x_2)$.

4. Conclusion

In this work the theoretical point of view of fuzzy sets in ring and ideal are discussed. The work is focused on fuzzy soft rings and fuzzy soft ideals. These concepts are basic structures for improvement of soft set theory. One can extend this work by studying other algebraic structures.

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