IJST (2012) A3 (Special issue-Mathematics): 411-416
Iranian Journal of Science & Technology

http://www.shirazu.ac.ir/en

Common fixed point theorems for sequences of mappings with some weaker conditions

Kh. Abd-Rabou

Department of Mathematics, Faculty of Science, Zagazig University, Egypt
Department of Mathematics, Community College, Shaqra University, Al-qawwiya, K. S. A
E-mail: k_abdrabo@yahoo.com

Abstract

In this paper, we prove a common fixed point theorem for six mappings (two set valued and four single valued mappings) without assuming compatibility and continuity of any mapping on non complete metric spaces. To prove the theorem, we use a non compatible condition, that is, weak commutativity of type (KB). We show that completeness of the whole space is not necessary for the existence and uniqueness of common fixed point, and give an example to support our theorem. Also, we prove a common fixed point theorem for two self mappings and two sequences set-valued mappings by the same weaker conditions. Our results improve, extend and generalizes the corresponding results given by many authors.

Keywords: Common fixed point; single and set-valued mappings; weak commutativity of type (KB)

1. Introduction

Fixed point theorems for hybrid pair of set and single valued mappings have numerous applications in science and engineering (e.g. [1-6]). Sessa [7] introduced the concept of weakly commuting maps. Jungck [8] defined the notion of compatible maps in order to generalize the concept of weak commutativity and showed that weakly commuting mappings are compatible but the converse is not true. On the other hand, Jungck and Rhoades [9, 10] defined the concept of compatibility and weak compatibility between a set valued mapping and a single valued mapping. Most of the fixed point theorems existing in the mathematical literature deal with compatible and continuous mappings. So it would be natural to ask: what about the mappings which are not compatible and continuous? Banach fixed point theorem has many applications but suffers from one drawback, the definition requires the continuity of the function. It has been known from the paper of Kannan [11] that there exist maps that have a discontinuity in the domain but have a fixed point. These observations motivated several authors to prove fixed point theorems for non compatible, discontinuous mappings. Pant [12-15] initiated the study of non compatible maps and introduced point wise R- weak commutativity of mappings in [12]. He also showed that point wise R-weak commutativity is a necessity, hence minimal condition for the existence of a common fixed point of contractive type maps [13]. Pathak, Cho and Kang [16] introduced the concept of R- weakly commuting mappings of type A and showed that they are not compatible. Recently, Kubiaczyk and Deshpande [17] extended the notion of R-weakly commuting mappings of type A in the settings of hybrid pair of mappings and defined weakly commuting mappings of type (KB). Some common fixed point theorems have been proved by using this new concept of weakly commuting mappings of type (KB) ([17-19]). In this paper, we prove common fixed point theorems for hybrid pairs of set and single valued mappings by using a non compatible condition, that is, weak commutativity of type (KB) on metric spaces. We show that the completeness of the whole space can be replaced by a weaker condition. Our results improve, extend and generalize the results of Fisher [20], Sastry and Naidu [21], Ahmed [22], Sharma, Deshpande and Pathak [19].

Received: 5 November 2011 / Accepted: 18 February 2012

2. Basic Preliminaries

In the sequel, (X,d) denotes a metric space and $B(X)$ is the set of all nonempty bounded subsets of X. As in [22, 23] we define.

$$\delta(A,B) = sup\{d(a,b): a \in A, b \in B\},$$
$$D(A,B) = inf\{d(a,b): a \in A, b \in B\}.$$

If $A=\{a\}$ we denote $\delta(a,B), D(a,B)$ for $\delta(A,B)$ and $D(A,B)$ respectively. If $A=\{a\}$ and $B=\{b\}$, one can deduce that $\delta(A,B)=D(A,B)=d(a,b)$. It follows immediately from the definition of $\delta(A,B)$ that, $\delta(A,B)=\delta(B,A)\geq 0, \delta(A,B)=0$ iff $A=B=\{a\}, \delta(A,B)\leq \delta(A,C)+\delta(C,B), \delta(A,A)=diamA$ for all $A,B,C\in B(X)$.

Definition 2.1. [23] A sequence $\{A_n\}$ of non empty subset of X is said to be convergent to a subset A of X if

(i) Each point $a\in A$ is the limit of a convergent sequence $\{a_n\}$ where $\{a_n\}\in A$ for all $n\in N$,

(ii) for arbitrary $\varepsilon>0$, there exists an integral $m>0$ such that $A_n\subseteq A_\varepsilon$ for $n>m$ where A_ε denotes the set of all points $x\in X$ for which there exists a point $a\in A$ depending on x, such that $d(x,a)<\varepsilon$. A is said to be the limit of the sequence $\{A_n\}$.

Lemma 2.1. [23] If $\{A_n\}$ and $\{B_n\}$ are sequence in $B(X)$ converging to A and B respectively in $B(X)$, then the sequence $\{\delta(A_n,B_n)\}$ converges to $\delta(A,B)$.

Lemma 2.2. [23] Let $\{A_n\}$ be a sequence in $B(X)$ and $y\in X$ such that $\delta(A_n,y)\to 0$. Then the sequence $\{A_n\}$ converges to the set $\{y\}$ in $B(X)$.

Definition 2.2. [23] The mappings $I:X\to X$ and $F:X\to B(X)$ are weakly commuting if $IFx\in B(X)$ and $\delta(FIx,IFx)\leq max\{\delta(Ix,Fx), diamIFx\}$ for all $x\in X$.

Note that, if F is a single valued mapping then the set $\{IFx\}$ consists of a single point. Therefore, $diamIFx=0$ for all $x\in X$ and above inequality reduces to the well known condition given by Sessa [7]. Two commuting mappings F and I are weakly commuting but the converse is not true as shown in [23].

Definition 2.3. [23] The mappings $I:X\to X$ and $F:X\to B(X)$ are δ-compatible if $lim_{n\to\infty}\delta(FIx_n, IFx_n)=0$ whenever $\{x_n\}$ is a sequence in X such that $IFx_n\in B(X), Fx_n\to\{t\}, Ix_n\to t$, for some t in X.

Definition 2.4. [10] The mappings $I:X\to X$ and $F:X\to B(X)$ are weakly compatible if they commute at coincidence points. i.e. for each point $u\in X$ such that $Fu=\{Iu\}$, we have $FIu=IFu$. Not that the equation $Fu=\{Iu\}$ implies that Fu is singleton.

Definition 2.5. [17] The mappings $I:X\to X$ and $F:X\to B(X)$ are said to be weakly commuting of type (KB) at x if there exists some positive real number R such that $\delta(IIx,FIx)\leq R\delta(Ix,Fx)$. Here, I and F are weakly commuting of type (KB) on X if the above inequality holds for all x. If F is single valued self mapping of X, this definition of weak commutativity reduces to that of Pathak, Cho and Kang [16].

Every δ-compatible pair of hybrid maps is weakly commuting of type (KB) but the converse is not necessarily true. For examples, it can be seen in [17, 19, 24].

3. Main Results

Now we can introduce our main theorems.

Theorem 3.1. Let S,R,H and T be four self mappings of a metric space (X,d) and $A,B:X\to B(X)$ set-valued mappings satisfying the following conditions:

(1) $\bigcup A(X)\subseteq SR(X)$ and $\bigcup B(X)\subseteq TH(X)$,

(2) the pairs $\{A,TH\}$ and $\{B,SR\}$ are weakly commuting of type (KB) at coincidence points in X,

(3) $\begin{array}{l}\delta(Ax,By)\leq \max\{cd(THx,SRy), c\delta(THx,Ax),\\ c\delta(SRy,By), aD(THx,By)+bD(SRy,Ax)\}\end{array}$,

for all $x,y\in X$, where

$0\leq c<1, a,b\geq 0, a+b<1, cmax\{\frac{a}{1-a},\frac{b}{1-b}\}<1$.

Suppose that one of the mappings $SR(X)$ and $TH(X)$ is complete subspace of X. Then A,B,S,H,R and T have a unique common fixed point.

Proof: Let $x_0 \in X$ be an arbitrary point in X. By (1), there exists a point $x_1 \in X$ such that $SRx_1 \in Ax_0 = Z_0$ and for this point x_1 there exists a point $x_2 \in X$ such that $THx_2 \in Bx_1 = Z_1$ and so on. Continuing in this manner, we can define a sequence as follows:

$$\mathrm{SRx}_{2n+1} \in \mathrm{Ax}_{2n} = \mathrm{Z}_{2n}, \mathrm{THx}_{2n+2} \in \mathrm{Bx}_{2n+1} = \mathrm{Z}_{2n+1}, \forall \quad n = 0,1,2,... .$$

For simplicity, we put $V_n = \delta(Z_n, Z_{n+1})$ for $n = 0,1,2,...$. By (3), we have

$$V_{2n} = \delta(Z_{2n}, Z_{2n+1}) = \delta(Ax_{2n}, Bx_{2n+1})$$
$$\leq \max\{cd(THx_{2n}, SRx_{2n+1}), c\delta(THx_{2n}, Ax_{2n}), c\delta(SRx_{2n+1}, Bx_{2n+1}),$$
$$aD(THx_{2n}, Bx_{2n+1}) + bD(SRx_{2n+1}, Ax_{2n})\}$$
$$\leq \max\{cd(Z_{2n-1}, Z_{2n}), c\delta(Z_{2n-1}, Z_{2n}), c\delta(Z_{2n}, Z_{2n+1}),$$
$$aD(Z_{2n-1}, Z_{2n+1}) + bD(Z_{2n}, Z_{2n})\}$$
$$\leq max\{cV_{2n-1}, cV_{2n}, a(V_{2n-1} + V_{2n})\}$$
$$\leq max\{c, \frac{a}{1-a}\}V_{2n-1} \text{ for } n \in N.$$

Similarly, one can show that

$$V_{2n+1} \leq max\{c, \frac{b}{1-b}\}V_{2n} \text{ for } n \in N.$$

If we put $\beta = max\{c, \frac{b}{1-b}\}.max\{c, \frac{a}{1-a}\}$, then by hypothesis it can be easily seen that $0 \leq \beta < 1$. So we deduce that

$$V_{2n} \leq \beta V_{2n-2} \leq ... \leq \beta^n V_0, V_{2n+1} \leq \beta V_{2n-1} \leq ... \leq \beta^n V_1$$

for $n \in N$.

Put $M = max\{V_0, V_1\}$. It follows from the above inequality that if z_n is an arbitrary point in the set Z_n for $n \in N$, then we obtain

$$d(z_{2n}, z_{2n+1}) \leq \delta(Z_{2n}, Z_{2n+1}) \leq \beta^n M,$$
$$d(z_{2n+1}, z_{2n+2}) \leq \delta(Z_{2n+1}, Z_{2n+2}) \leq \beta^n M.$$

This implies that $\{z_n\}$ and any subsequence thereof is a Cauchy sequence in X.

Now suppose that $SR(X)$ is complete.

$$d(SRx_{2m+1}, SRx_{2n+1} \leq \delta(Z_{2m}, Z_{2n}) < \varepsilon \text{ for } m, n > n_0, n_0 = 1,2,3....$$

Therefore $\{SRx_{2n+1}\}$ is a Cauchy sequence and hence $\{SRx_{2n+1}\} \to z = SRv \in SR(X)$. But $THx_{2n} \in Bx_{2n-1} = Z_{2n-1}$ and hence, we have

$$d(THx_{2n}, THx_{2n+1}) \leq \delta(Z_{2n-1}, Z_{2n}) = V_{2n-1} \to 0.$$

Consequently, $TH_{2n} \to z$. Moreover, we have for $n = 1,2,3...$

$$\delta(Ax_{2n}, z) \leq \delta(Ax_{2n}, THx_{2n}) + \delta(THx_{2n}, z) \leq \delta(Z_{2n}, Z_{2n-1}) + d(THx_{2n}, z).$$

Therefore, $\delta(Ax_{2n}, z) \to 0$. Similarly, it follows that $\delta(Bx_{2n}, z) \to 0$.

By (3), we have for $n = 1,2,3...$.

$$\delta(Ax_{2n}, Bv) \leq \max\{cd(THx_{2n}, SRv), c\delta(THx_{2n}, Ax_{2n}), c\delta(SRv, Bv),$$
$$aD(THx_{2n}, Bv) + bD(SRv, Ax_{2n})\}.$$

Since $\delta(THx_{2n}, Bv) \to \delta(z, Bv)$, when $THx_{2n} \to z$, we get as $n \to \infty$

$$\delta(z, Bv) \leq max\{c, a\}\delta(z, Bv),$$

which is a contradiction. Thus $Bv = \{z\} = \{SRv\}$. But $\bigcup B(X) \subseteq TH(X)$, there exists $u \in X$ such that $\{THu\} = Bv = \{z\} = \{SRv\}$. Now if $Au \neq Bv, \delta(Au, Bv) \neq 0$, then by (3), we obtain

$$\delta(Au, Bv) \leq \max\{cd(THu, SRv), c\delta(THu, Au), c\delta(SRv, Bv), aD(THu, Bv) + bD(SRv, Au)\}.$$

As $n \to \infty$, we have

$\delta(Au, z) \leq max\{c, b\}\delta(Au, z)$. This is a contradiction. Thus we have

$$Au = \{THu\} = Bv = \{z\} = \{SRv\}.$$

Since $Au = \{THu\}$ and the pair $\{A, TH\}$ is weakly commuting of type (KB) at coincidence points in X, we obtain $\delta(THTHu, ATHu) \leq R\delta(THu, Au)$ which gives $Az = \{THz\}$.

By (3), we get

$$\delta(Az, z) \leq \delta(Az, Bv)$$
$$\leq \max\{cd(THz, SRv), c\delta(THz, Az), c\delta(SRv, Bv), aD(THz, Bv) + bD(SRv, Az)\}$$
$$\leq max\{c, a+b\}\delta(Az, z).$$

Here we reach a contradiction. Thus $Az = \{z\}$. Consequently, we have $Az = \{z\} = \{THz\}$.

Similarly $Bz = \{z\} = \{SRz\}$. Therefore, we have $Az = \{THz\} = \{z\} = Bz = \{SRz\}$.

Now, we prove that $Rz = z$. In fact, by (3) it follows that

$$\delta(Az, BRz) \le \max\{cd(THz, SRRz), c\delta(THz, Az), c\delta(SRRz, BRz), aD(THz, BRz) + bD(SRRz, Az)\}.$$

Since $Bz = \{z\} = \{SRz\}$ and $R : X \to X$, thus $BRz = \{Rz\}, SRRz = Rz$.

Then, the above inequality becomes $d(z, Rz) \le max\{c, a + b\}d(z, Rz)$. This is a contradiction. Thus we have $Rz = z$. Hence $Sz = SRz = z$. Similarly, we get $Tz = Hz = z$. Thus $Az = \{Tz\} = \{Hz\} = \{z\} = \{Sz\} = \{Rz\} = Bz$.

To prove uniqueness, let p another common fixed point of A, B, S, H, R and T.

$$d(z, p) \le \delta(Az, Bp) \le \max\{cd(THz, SRp), c\delta(THz, Az), c\delta(SRp, Bp), aD(THz, Bp) + bD(SRp, Az)\} \le max\{c, a + b\}d(z, p),$$

which is a contradiction, therefore $z = p$. Then A, B, S, H, R and T have a unique common fixed point.

The following examples illustrate Theorem 3.1

Example 3.1. Let $X = [0,\infty]$ be endowed with the Euclidean metric d. Define $S, H, R,T : X \to X$ and $A, B : X \to B(X)$ by

$$Ax = [0, \frac{x^6}{6}],\ Bx = [0, \frac{x^3}{6}],$$

$$Sx = \frac{x^4}{2} + x^2 + \frac{x}{2},\ Rx = x^3,$$

$$Tx = x^4 + 6x^2, Hx = x^{\frac{3}{2}}.$$

Then $THx = x^6 + 6x^3$, $SRx = \frac{x^{12}}{2} + x^6 + \frac{x^3}{2}$ and

$$\bigcup A(X) = TH(X) = \bigcup B(X) = SR(X) = X.$$

For any sequence x_n in X, we have $THx_n \to 0$ as $x_n \to 0$, $Ax_n \to 0$ as $x_n \to 0$,

$$\delta(ATHx_n, THAx_n) = \max\{\frac{(x_n^6 + x_n^3)^6}{6}, \left(\frac{x_n^6}{6}\right)^6 + 6\left(\frac{x_n^6}{6}\right)^3\} \to 0$$

as $x_n \to 0$, $THAx_n \in B(X)$, thus A and TH are δ compatible and so they are weakly commuting of type (KB). Similarly, B and SR are δ compatible and so they are weakly commuting of type (KB).

From the above, we have that

$$\delta(Ax, By) = max\{\frac{x^6}{6}, \frac{y^3}{6}\}$$

$$= max\{\frac{1}{3}\frac{x^6}{2}, \frac{1}{3}\frac{y^3}{2}\}$$

$$\le max\{\frac{1}{3}(x^6 + 6x^3), \frac{1}{3}(\frac{y^{12}}{2} + y^6 + \frac{y^3}{2})\}.$$

$$\le \frac{1}{3}\max\{\left(x^6 + 6x^3\right), \left(\frac{y^{12}}{2} + y^6 + \frac{y^3}{2}\right), \left|\left(x^6 + 6x^3\right) - \left(\frac{y^{12}}{2} + y^6 + \frac{y^3}{2}\right)\right|\}$$

$$= max\{\frac{1}{3}d(THx, SRy), \frac{1}{3}\delta(THx, Ax), \frac{1}{3}\delta(SRy, By)\}$$

$$\le \max\{\frac{1}{3}d(THx, SRy), \frac{1}{3}\delta(THx, Ax), \frac{1}{3}\delta(SRy, By), \frac{1}{6}D(THx, By) + \frac{1}{5}D(SRy, Ax)\}.$$

We see that Theorem 3.1 holds with $c = \frac{1}{3}, a = \frac{1}{6}, b = \frac{1}{5}$ and 0 is the unique common fixed point for A, B, S, H, R and T.

If we put $R = H = I$ (:the identity mapping)in Theorem 3.1, we get the following:

Theorem 3.2. Let S and T be self mappings of a metric space (X, d) and $A, B : X \to B(X)$ set-valued mappings satisfying the following conditions:

(1) $\bigcup A(X) \subseteq S(X)$ and $\bigcup B(X) \subseteq T(X)$,

(2) the pairs $\{A,T\}$ and $\{B, S\}$ are weakly commuting of type (KB) at coincidence points in X,

(3) $\delta(Ax, By) \le \max\{cd(Tx, Sy), c\delta(Tx, Ax), c\delta(Sy, By), aD(Tx, By) + bD(Sy, Ax)\}$,

for all $x, y \in X$, where $0 \le c < 1, a, b \ge 0, a + b < 1, cmax\{\frac{a}{1-a}, \frac{b}{1-b}\} < 1$.

Suppose that one of the mappings $S(X)$ and

$T(X)$ is complete subspace of X. Then A, B, S and T have a unique common fixed point.

Remark 3.1. Theorem 3.1 improves and generalizes the results of Ahmed [22], Sharma, Deshpande and Pathak [19].

If we put $R = H = I$ (:the identity mapping) and $A = B, S = T$ in Theorem 3.1, we get the following:

Theorem 3.3. Let S be a self mapping of a metric space (X, d) and $A : X \to B(X)$ a set-valued mapping satisfying the following conditions:

(1) $\bigcup A(X) \subseteq S(X)$,

(2) the pair $\{A, S\}$ is weakly commuting of type (KB) at coincidence points in X,

(3) $\delta(Ax, Ay) \le \max\{cd(Sx, Sy), c\delta(Sx, Ax), c\delta(Sy, Ay), aD(Sx, Ay) + bD(Sy, Ax)\}$,

for all $x, y \in X$, where $0 \le c < 1, a, b \ge 0, a + b < 1, c\max\{\frac{a}{1-a}, \frac{b}{1-b}\} < 1$.

Suppose that $S(X)$ is complete subspace of X. Then A and S have a unique common fixed point.

Remark 3.2. Theorem 3.3 improves and generalizes the results of Fisher [20], Sastry and Naidu [21].

Theorem 3.4. Let S be a self mapping of a metric space (X, d) and $A : X \to B(X)$ a set-valued mapping satisfying the following conditions:

(1) $\bigcup A^n(X) \subseteq S^m(X)$,

(2) the pairs $\{A^n, S^m\}$ are weakly commuting of type (KB) at coincidence points in X,

(3) $\delta(A^n x, A^n y) \le \max\{cd(S^m x, S^m y), c\delta(S^m x, A^n x), c\delta(S^m y, A^n y), aD(S^m x, A^n y) + bD(S^m y, A^n x)\}$,

for all $x, y \in X$, where $0 \le c < 1, a, b \ge 0, a + b < 1, c\max\{\frac{a}{1-a}, \frac{b}{1-b}\} < 1$.

Suppose that one of the mappings $S^m(X)$ is complete subspace of X. Then A and S have a unique common fixed point.

Proof: If we set $A = B = A^n$ and $S = T = S^m$, in Theorem 3.1 A^n and S^m have a unique common fixed point in X. That is, there exists $z \in X$ such that $A^n(z) = \{S^m(z)\} = \{z\}$. Since $A^n(Az) = A(A^n z) = Az$, it follows that Az is a fixed point of A^n and S^m and hence $Az = z$. Similarly, we have $Sz = z$.

Theorem 3.5. Let S and T be two self mappings of a metric space (X, d) and two sequences set-valued mappings $A_i, B_j : X \to B(X)$ for all $i, j \in N$ satisfying the following conditions:

(1) there exists $i_0, j_0 \in N$ such that $\bigcup A_{i_0}(X) \subseteq S(X)$ and $\bigcup B_{j_0}(X) \subseteq T(X)$,

(2) *the pairs* $\{A_{i_0}, T\}$ and $\{B_{j_0}, S\}$ are weakly commuting of type (KB) at coincidence points in X,

(3) $\delta(A_i x, B_j y) \le \max\{cd(Tx, Sy), c\delta(Tx, A_i x), c\delta(Sy, B_j y), aD(Tx, B_j y) + bD(Sy, A_i x)\}$,

for all $x, y \in X$, where $0 \le c < 1, a, b \ge 0, a + b < 1, c\max\{\frac{a}{1-a}, \frac{b}{1-b}\} < 1$ and if one of the mappings $S(X)$ and $T(X)$ is a complete subspace of X. Then A_i, B_j, S and T have a unique common fixed point for all $i, j = 1, 2, \ldots$.

Proof: By Theorem 3.1, the mappings A_{i_0}, B_{j_0}, S and T for some $i_0, j_0 \in N$ have a unique common fixed point in X. That is, there exists a unique point $z \in X$ such that $\{Sz\} = \{Tz\} = \{z\} = A_{i_0} z = B_{j_0} z$.

Suppose that there exists $i \in N$ such that $i \ne i_0$. Then, we have

$$\delta(A_i z, z) = \delta(A_i z, B_{j_0} z)$$
$$\le \max\{cd(Tz, Sz), c\delta(Tz, A_i z), c\delta(Sz, B_{j_0} z), aD(Tz, B_{j_0} z) + bD(Sz, A_i z)\}$$
$$\le \max\{c, b\}\delta(A_i z, z),$$

which is a contradiction. Hence, for all $i \in N$, it follows that $A_i z = z$.

Similarly, for all $j \in N$, we have $B_j z = z$. Therefore, for all $i, j \in N$, we have
$A_i z = B_j z = z = \{Sz\} = \{Tz\}$.

References

[1] Abd EL-Monsef, M. E., Abu-Donia, H. M. & Abd-Rabou, Kh. (2007). Common fixed point theorems of single and set-valued mappings on 2-metric spaces. *Applied Math. and Information Sciences J., 1*(2), 185-194.

[2] Abd EL-Monsef, M. E., Abu-Donia, H. M. & Abd-Rabou, Kh. (2009). New types of common fixed point theorems 2-metric spaces. *Chaos, Soliton and Fractals, 41,* 1435-1441.

[3] Abu-Donia, H. M. & Abd-Rabou, Kh. (2009). Common fixed point theorems for weakly compatible mappings in fuzzy metric spaces. *Journal of fuzzy Mathematics, 17*(2), 377-388.

[4] Abu-Donia, H. M. & Abd-Rabou, Kh. (2010). Common fixed theorems for hybrid mappings in fuzzy metric spaces. *Journal of Fuzzy Mathematics, 18*(1), 95-112.

[5] Kyzyska, S. & Kubiaczyk, I. (1998). Fixed point theorems for upper semicontinuous and weakly-weakly upper semicontinuous multivalued mappings. *Math. Japonica, 47*(2), 237-240.

[6] Sessa, S. & Khan, M. S. (1994). Some remarks in best approximation theory. *Math. J. Toyoma Univ., 17,* 151-165.

[7] Sessa, S. (1982). On a weak commutativity condition of mappings in fixed point considerations. *Publ. Inst. Math. (Beograd) 32, 46,* 149-153.

[8] Jungck, G. (1986). Compatible mappings and common fixed points. *Int. J. Math. Math. Sci., 9,* 771-779.

[9] Jungck, G. & Rhoades, B. E. (1993). Some fixed point theorems for compatible maps. *Int. J. Math. Math. Sci., 16*(3), 417-428.

[10] Jungck, G. & Rhoades, B. E. (1998). Fixed points for set valued functions without continuity. *Indian J. of Pure Appl. Math., 16*(3), 227-238.

[11] Kannan, R. (1968). Some results on fixed points. *Bull. Cal. Math. Soc., 60,* 71-76.

[12] Pant, R. P. (1994). Common fixed points of noncommuting mappings. *J. Math. Anal. Appl., 188,* 436-440.

[13] Pant, R. P. (1998). Common fixed point theorems for contractive maps. *J. Math. Anal. Appl., 226,* 284-289.

[14] Pant, R. P. (1999). Common fixed points of Lipschitz type mappings pairs. *J. Math. Anal. Appl., 240,* 280-283.

[15] Pant, R. P. (1999). Discontinuity and fixed points. *J. Math. Anal. Appl., 240,* 284-289.

[16] Pathak, H. K., Cho, Y. J. & Kang. (1997). Remarks on R-weakly commuting mappings and common fixed point theorems. *Bull. Korean Math. Soc., 34,* 247-257.

[17] Kubiaczyk, I. & Deshpande, B. (2008). Noncompatibility, discontinuity in consideration of common fixed point of set and single valued maps. *SEA Bull. Math, 32,* 467-474.

[18] Chang, T. H. (1993). Fixed point theorems of contractive type set valued mappings. *Math. Japonica, 38(4),* 675-690.

[19] Sharma, S., Deshpande, B. & Pathak, R. (2008). Common fixed point theorems for hybrid pairs of mappings with some weaker conditions. *Fasciculi Mathematica, 39,* 71-86.

[20] Fisher, B. (1985). Common fixed points of mappings and set valued mappings on a metric space. *Kyungpook Math. J., 25,* 35-42.

[21] Sastry, K. P. R. & Naidu, S. V. R. (1980). Fixed point theorems for generalized contraction mappings. *Yokohama Math. J., 28,* 15-29.

[22] Ahmed, M. A. (2003). Common fixed point theorems for weakly compatible mappings. *Rocky Mountain Journal of Mathematics, 33*(4), 1189-1203.

[23] Fisher, B. & Sessa, S. (1989). Two common fixed point theorems for weakly commuting mappings. *Periodica Math. Hungarica, 20*(3), 207-218.

[24] Sharma, S. & Deshpande, B. (2007). Fixed point theorems for set and single valued maps without continuity and compatibility. *Demonstratio Mathematica, XL*(3), *649-658.*