IJST (2012) A3: 259-266
Iranian Journal of Science & Technology

http://www.shirazu.ac.ir/en

Non-abelian lovelock-born-infeld topological black holes

N. Farhangkhah[1]* and N. Bostani[2, 3]

[1]*Department of Physics, Shiraz Branch, Islamic Azad University, Shiraz, Iran*
[2]*Key Laboratory of Particle Astrophysics, Institute of High Energy Physics, Chinese Academy of Sciences, Beijing 100049, China*
[3]*Research Institute for Astrophysics and Astronomy of Maragha (RIAAM), Maragha, Iran*
E-mail: nfkh@iaushiraz.net

Abstract

The asymptotically AdS solutions of the Einstein gravity with hyperbolic horizons in the presence of So(n(n-1)/2-1,1) Yang-Mills fields governed by the non-Abelian Born-Infeld Lagrangian are presented. We investigate the properties of these solutions as well as their asymptotic behavior in various dimensions. The properties of these kinds of solutions are like the Einstein-Yang-Mills solutions. But the differences seem to appear in the role of the mass, charge and born-Infeld parameter β, in the solutions. For example, in Einstein-Yang-Mills theory the solutions with non-negative mass cannot present an extreme black hole while that of in Einstein-Yang-Mills-Born Infeld theory can. Also, the singularities in higher dimensional Einstein-Yang-Mills theory for non-negative mass are always spacelike, while depending on choosing the parameters, we can find timelike singularities in the similar case of Einstein-Yang-Mills-Born-Infeld theory. We also extend the solutions of Einstein to the case of Gauss-Bonnet and third order Lovelock gravities. It is shown that, these solutions in the limits of $\beta\to 0$, and $\beta\to\infty$, represent pure gravity and gravity coupled with Yang-Mills fields, respectively.

Keywords: Lovelock gravity; Einstein gravity; Gauss-Bonnet gravity; Yang Mills field; Born Infeld theory

1. Introduction

Non-linear electrodynamics was proposed in the 1930's to remove singularities associated with charged pointlike particles. Among the non-linear theories of electrodynamics Born-Infeld (BI) theory [1] is distinguished, since BI type actions arise in many different contexts in superstring theory [2, 3]. Also, in the case of a D-brane, it is known that the effective action on the brane is Born-Infeld action if all derivative terms are neglected [4]. The effective action of several D-branes is very important in the development of the understanding of the non-perturbative superstring theory, such as Matrix theory [5]. Tseytlin [6] proposed that if all derivative terms are neglected, the effective action on the branes is a non-Abelian generalization of the Born-Infeld action. There are many outstanding classical solutions to both the Abelian and non-Abelian Born-Infeld theories. Born-Infeld black holes in (A)dS spaces have been discussed in [7]. A type of a particle-like solution in the non-Abelian Born-Infeld model was obtained by Gal'tsov and Kerner (GK) [8]. A much closer relationship between these two particle-like configurations becomes clear in [9, 10, 11]. Among theories of non-Abelian gauge fields, the Yang-Mills theory may be regarded as the most fundamental one in elementary particle physics. The role of the Yang-Mills field in gravity has become an interesting topic of studies. Physical significance of the particle-like solutions of Einstein-Yang-Mills (EYM) field equations found by Bartnik and McKinnon (BK) [12], as well as their possible role in the string-inspired models remains rather obscure. The attention towards the Einstein-Yang-Mills system became even more after the discovery of the first known example of hairy black holes [13, 14], which are not uniquely characterized by their conserved charges and so violate manifestly the no-hair conjecture theory. In particular, particle-like, soliton-like and black hole solutions in the combined Einstein-Yang-Mills (EYM) models in different dimensions, shed new light on the complex features of compact object in these models [15]. (See [16] for an overview).

With the advent of string theory, the possibility to have extra-dimensions became one of the most promising possibilities to extend the standard model of particles physics. Higher dimensional theories of gravity may present some new features which are absent in four dimensions. Indeed in four dimensions the only gravitational action that can be

*Corresponding author
Received: 27 November 2011 / Accepted: 6 May 2012

built from curvature invariants leading to second order equations in the metric is the Einstein-Hilbert action. The situation changes in higher dimensions. In five dimensions one can add, for example, a Gauss-Bonnet term to the action. In higher dimensions one can add higher curvature powers to the action. Such higher curvature power theories leading to second order equations for the metric are known as Lovelock theories [17]. There are many considerable works in Lovelock gravity (for example see [18]).

In the current writing, we will study the topological black hole of Lovelock gravity in the presence of non-abelian version of Born-Infeld theory in (n+1)-dimensions with the gauge group $So(n(n-1)/2-1,1)$. The spherically symmetric solutions in the presence of Yang-Mills field governed by Born-Infeld Lagrangian are obtained in [11]. The coupling of the two non-linear, Yang-Mills and Born-Infeld fields, is a candidate for effective action for superstrings and D-branes, besides the fundamental importance in the context of gravitation theory. To obtain an exact solution, we use a modified version of Wu-Yang ansatz [19] which was originally introduced in $N=4$ field theory and for the first time was applied by Yasskin [20]. This ansatz has been used to find the EYM and GBYM solutions with negative cosmological constant in higher dimensions [21, 22].

The outline of this paper is as follows. In Section 2. The field equations of Lovelock gravity in the presence of the energy-momentum tensor of the coupling of Yang-Mills and Born-Infeld fields is introduced. In section 3, we obtain the (n+1)-dimensional solutions of the Einstein gravity and investigate their properties in various dimensions. Section 4 is devoted to the general solutions to the Gauss-Bonnet and Lovelock gravity, and their properties respectively. We finish our paper with some concluding remarks.

2. Field Equations

As was shown by Lovelock in the early seventies [17], the possible corrections to Einstein gravity are quite limited, since the only symmetric, divergence free tensor that can be constructed out of the metric and its first two derivatives in a (n+1)- dimensional space-time is

$$G_{\mu\nu}=\sum_{k=0}^{[n/2]}\alpha'_k\,G^{(k)}_{\mu\nu}=8\pi T_{\mu\nu} \tag{1}$$

$$G^{\mu}{}_{\nu}{}^{(k)}=-\frac{1}{2^{k+1}}\delta^{\mu i_1 i_2\cdots i_{2k}}_{\nu j_1 j_2\cdots j_{2k}}\,R^{j_1 j_2}_{i_1 i_2}\cdots R^{j_{2k-1}j_{2k}}_{i_{2k-1}i_{2k}}$$

where $[d/2]$ denotes the integer part of $d/2$, $T_{\mu\nu}$ is the energy-momentum tensor and α'_k are Lovelock constants which represent the coupling of the terms in the whole Lagrangian and give the proper dimensions. Usually, in order of the Einstein gravity to be recovered in the low energy limit, the constant α'_0 should be identified as the cosmological constant up to a constant, $\alpha'_0=-2\Lambda$, and α'_1 should be positive (for simplicity one may take $\alpha'_1=1$). As In this context, we are interested in asymptotically anti-de Sitter (AdS) solutions, we set $\Lambda=-n(n-1)/2l^2$.

We want to examine the Lovelock-Yang-Mills-Born-Infeld system for a compact, semi-simple gauge group G with structure constants C^a_{bc}. The basic elements of the model are $(M, g_{\mu\nu}, A^{(a)}_\mu)$, where M is the spacetime manifold with metric $g_{\mu\nu}$, and $A^{(a)}_\mu$ are the gauge potentials. The metric tensor of the gauge group is [21]

$$\gamma_{ab}=-C^c_{ad}C^d_{bc}/\left|\det C^c_{ad}C^d_{bc}\right|^{1/N},$$

where N is the dimension of the gauge group, the Latin indices a, b.... go from 1 to N, and the repeated indices are understood to be summed over.

In order to obtain the field equations, we consider the EBI action in which, instead of the electromagnetic field we employ the non-Abelian YM field [23]. Consequently, in Eq.(1), $T_{\mu\nu}$ is the energy momentum tensor given as

$$T_{\mu\nu}=\frac{1}{4\pi}\left(\frac{\gamma_{ab}}{\Gamma}F^{(a)}_{\mu\lambda}F^{(b)\lambda}_{\nu}+\frac{1}{4}g_{\mu\nu}L_m\right) \tag{2}$$

where therein

$$\Gamma=\sqrt{1+\frac{\gamma_{ab}}{2\beta^2}F^{(a)}_{\lambda\sigma}F^{(b)\lambda\sigma}} \tag{3}$$

and

$$F^{(a)}_{\mu\nu}=\nabla_\mu A^{(a)}_\nu-\nabla_\nu A^{(a)}_\mu+\frac{1}{2e}C^a_{bc}A^{(b)}_\mu A^{(c)}_\nu \tag{4}$$

In the equations (2),(3) and (4), β is the Born-Infeld parameter which has the dimension of $length^{-2}$, $F^{(a)}_{\mu\nu}$'s are the Yang-Mills-Born-Infeld fields, e is the coupling constant and L_m, the matter term, is the F(2) nonabelian and nonlinear action density

$$L_m = 4\beta^2(1-\Gamma). \quad (5)$$

In the limit $\beta \to 0$, L_m becomes equal to zero and in the limit $\beta \to \infty$ it reduces to the standard Yang-Mills form, $L_m = -\gamma_{ab} F_{\mu\nu}^{(a)} F^{(b)\mu\nu}$, so we expect to obtain the corresponding solutions in both these limits. Also, we note that the Born-Infeld Lagrangian is obtained, if one substitutes $F_{\mu\nu}^{(a)}$'s with the electromagnetic field, $F_{\mu\nu} = \nabla_\mu A_\nu - \nabla_\nu A_\mu$, in which A is the electromagnetic potential.

Variation with respect to the gauge potentials A_μ yields the YM equations.

$$\nabla_\mu\left(\frac{1}{\Gamma} F^{\mu\nu(a)}\right) + \frac{1}{\Gamma e} C_{bc}^{a} A_\mu^{(b)} F^{(c)\mu\nu} = 0, \quad (6)$$

we consider the metric of a spacetime with hyperbolic horizon of the following form:

$$ds^2 = -f(r)\, dt^2 + \frac{dr^2}{f(r)} + r^2 d\,\Omega_{n-1}^2 \quad (7)$$

where
$d\Omega_{n-1}^2 = d\theta_1^2 + \sinh\theta_1^2 d\theta_2^2 + \sinh\theta_1^2 \sum_{i=3}^{n-1} \prod_{j=2}^{i-1} \sin\theta_j^2 d\theta_i^2$
is the line element of $(n-1)$- dimensional hypersurface with constant negative curvature and volume V_{n-1}.

In order to obtain the gauge potential in higher dimensional spacetimes we use the (n+1)-dimensional Wu-Yang Ansatz [19] as follows

$$A^{(a)} = \frac{e}{r^2}\left(x_i dx_n - x_n dx_i\right); \quad i = 1...n-1,$$

$$A^{(b)} = \frac{e}{r^2}\left(x_i dx_j - x_j dx_i\right); \qquad i < j,$$

where a and b run from 1 to $n-1$ and n to $n(n-1)/2$, respectively and

$$x_1 = r\sinh\theta_1 \prod_{j=2}^{n-1} \sin\theta_j,$$

$$x_i = r\sinh\theta_1 \cos\theta_{n-i+1} \prod_{j=2}^{n-i} \sin\theta_j; \; i = 2...n-1,$$

$$x_n = r\cosh\theta_1.$$

The Lie algebra of this gauge group is $So(n(n-1)/2-1,1)$ with the metric tensor of $\gamma_{ab} = \varepsilon_a \delta_{ab}$, of which there is no sum on a and

$$\varepsilon_a = \begin{cases} -1 & 1 \le a \le n-1 \\ 1 & n \le a \le \dfrac{n(n-1)}{2} \end{cases}$$

Substituting Eqs. (7) and (4) in (3), we have

$$\Gamma(r) = \sqrt{1 + \frac{(n-1)(n-2)e^2}{2\beta^2 r^4}} \quad (8)$$

3. Static solutions in Einstein gravity

In this section the goal is to find the exact solutions of Einstein gravity in the presence of YMBI fields. In this case, the field equation (1) becomes

$$G_{\mu\nu}^{(1)} - \frac{n(n-1)}{2l^2} g_{\mu\nu} = 8\pi T_{\mu\nu}. \quad (9)$$

$G_{\mu\nu}^{(1)}$ is just the Einstein tensor. To find the function $f(r)$, one may use any components of Eq. (9). The tt-component of the above equation using (2) and (8) is

$$\left[r^{n-2}(1+f)\right] - \frac{n}{l^2} r^{n-1} - \frac{4\beta^2}{n-1} r^{n-1}(1-\Gamma) = 0 \quad (10)$$

For $N = n+1 = 5$ the solution is obtained to be:

$$f(r) = -1 - \frac{3m}{r^2} + \frac{r^2}{l^2} - \frac{\beta^2 r^2}{3}(1-\Gamma_4) - \frac{2e^2 \ln(r)}{r^2} - \frac{e^2 \ln(1+\Gamma_4)}{r^2},$$

where $\Gamma_4 = \Gamma(n=4)$. For $N = n+1 > 5$ the solution of the field equation (10) is:

$$f(r) = -1 - \frac{(n-1)m}{r^{n-2}} + \frac{r^2}{l^2} + \frac{4\beta^2 r^2}{n(n-1)}\left[1 - \hat{o}(r)\right], \quad (11)$$

$\hat{o}(r)$ is defined as:

$$\hat{o}(r) = \frac{n}{r^n}\int r^{n-1}\Gamma(r)\,dr = {}_2F_1\left(\left[-\frac{1}{2}, -\frac{n}{4}\right], \left[1-\frac{n}{4}\right], \Gamma^2 - 1\right) \quad (12)$$

In which ${}_2F_1([a,b],[c],z)$ is hypergeometric function. It is apparent that in the limit $\beta \to 0$ Eq. (11) will be

$$f(r) = -1 - \frac{(n-1)m}{r^2} + \frac{r^2}{l^2}$$

which is the AdS solution to the Einstein equation. Using the fact that ${}_2F_1([a,b],[c],z)$ has a convergent series expansion for $|z|<1$, we can find the limit of the $f(r)$ for large r and β as

$$f(r) \to -1 - \frac{(n-1)m}{r^{n-2}} + \frac{r^2}{l^2} - \frac{(n-2)e^2}{(n-4)r^2} \qquad (13)$$

As expected, this is the solution to the Einstein-Yang-Mills equations introduced in [21]. Also, the Born-Infeld solution is obtained when e is the electromagnetic charge parameter in the Eq (8).

The global structure of the spacetime is characterized by properties of the singularities and horizons. It is easy to show that the Kretschmann scalar $R_{\mu\nu\lambda\kappa}R^{\mu\nu\lambda\kappa}$ diverges at $r=0$ and is finite everywhere else, so $r=0$ is an essential singularity. The behavior of the solution $f(r)$ at infinity is dominated by the term $r^2\left(1/l^2 + 4\beta^2/n(n-1)\right)$, and so one can see that $f(r) \to +\infty$ for large r's. Also, one can see that the function $f(r)$ has a negative or positive value near $r=0$ depending on the value of $-\frac{(n-1)m}{r^{n-2}} - \frac{4\beta^2 r^2}{n(n-1)}\hat{o}(r)$, as this is the dominating term in $f(r)$ near $r=0$. The hypergeometric function ${}_2F_1\left(\left[-\frac{1}{2},-\frac{n}{4}\right],\left[1-\frac{n}{4}\right],\Gamma^2-1\right)$ has different forms in different dimensions. By considering an integer number a, we obtain the explicit form of the hypergeometric function as follows.

3.1. n = 8a,a = 1,2,..

For these values of n, as in ${}_2F_1([a,b],[c],z)$, c is always an integer, the integral (12) is expected to include a logarithmic term. As a case for $n=8$ the metric function $f(r)$ is

$$\begin{aligned} f &= -1 - \frac{7m}{r^6} + \frac{r^2}{l^2} + \frac{\beta^2 r^2}{14}(1-\Gamma_8) \\ &- \frac{3e^2}{4r^2}\Gamma_8\left[1 - ln\left(r^2 + r^2\Gamma_8\right)^{\frac{\Gamma_8^2-1}{\Gamma_8}}\right] \end{aligned} \qquad (14)$$

where $\Gamma_8 = \Gamma(n=8)$. Of course one should note that this solution in $\beta \to +\infty$ reduces to give the relation (13) as expected, and the logarithmic term vanishes in this case. But other than this case, this logarithmic case plays a significant term in the properties of the Yang-Mills-Born-Infeld solutions. To see these properties we obtain the limit of $f(r)$ near $r=0$ for these values of $n=8a$. In this case we obtain:

$$f \to -\frac{(8a-1)}{r^{8a-2}}\left[m - \lambda e^{4a}\ln(\varsigma)\right],$$

where

$$\lambda = \frac{(8a-1)^{2a-2}(4a-1)^{2a-1}}{2^{4a-1}\beta^{4a-2}}\binom{4a-1}{2a} \qquad (15)$$

$$\varsigma = \frac{(8a-1)(4a-1)e^2}{\beta^2} \qquad (16)$$

When $m > \lambda e^{4a}\ln(\varsigma)$, $f(r)$ is negative and as $f(r) \to +\infty$, when $r \to +\infty$, so $f(r)$ has certainly one real root and a black hole with one horizon exists. For non-negative mass, if $\varsigma > 1$ and $m < \lambda e^{4a}\ln(\varsigma)$, $f(r)$ is positive near $r=0$ and the solution may present an extreme black hole, a black hole with two horizons or a spacetime without a horizon. This is a property that does not happen in the Einstein-Yang-Mills theory as the spacetime always presents naked singularity for non-negative mass.

Also, for non-negative mass of this case the singularity at $r=0$ is timelike, but in the Einstein-Yang-Mills theory the singularity for the dimensions higher than five is always spacelike.

For negative mass, the possibility to have spacelike singularity exists if $\varsigma < 1$ and $|m| < |\lambda e^{4a}\ln(\varsigma)|$, while in EYM gravity the singularity for negative mass is timelike.

3.2. n = 4 (2a +1) , a= 1,2, ...

In this case, the hypergeometric function again includes a logarithmic term. For the behavior of the metric function near $r=0$ we have

$$f \to -\frac{(8a+3)}{r^{8a+2}}\left[m+\lambda e^{4a+2}\ln(\varsigma)\right],$$

Where, in this case

$$\lambda = \frac{(8a+3)^{2a-1}(4a+1)^{2a}}{2^{4a+1}\beta^{4a}}\binom{4a+1}{2a} \tag{17}$$

$$\varsigma = \frac{(8a+3)(4a+1)e^2}{\beta^2} \tag{18}$$

By choosing proper parameters, this term can be negative or positive. For negative mass this term can be positive if $\varsigma<1$, or if $\varsigma>1$ and also $|m|>|\lambda e^{4a+2}\ln(\varsigma)|$. In this case extreme black hole exists for the proper choice of the parameters. For non-negative mass also, if β be sufficiently small and $\varsigma<1$, $f(r)$ will be positive, the solution may present a black hole with one or two horizons or a spacetime without a horizon. This is also the case that does not happen in Yang-Mills theory without Born-Infeld or in Maxwell-Born-Infeld theory.

3.3. $n = 2a+1, a = 1,2,...$

In this case

$$f \to -2a\frac{m}{r^{2a-1}}$$

This is similar to the case that happens in Maxwell-Born-Infeld theory and just when $m<0$, extreme black hole may exist, and for non-negative mass spacetime always presents naked singularity.

3.4. $n = 2(4a\pm1), a = 1,2,...$

In these two cases, the integral in the relation (12) can be solved easily. For example, for $n=6$ and $n=10$ respectively, solutions are obtained as

$$f_{n=6} = -1-\frac{5m}{r^4}+\frac{r^2}{l^2}+\frac{2\beta^2r^2}{15}(1-\Gamma_6)-\frac{4e^2}{3r^2}\Gamma_6, \tag{19}$$

$$f_{n=10} = -1-\frac{9m}{r^8}+\frac{r^2}{l^2}+\frac{2\beta^2r^2}{45}(1-\Gamma_{10})-\frac{8e^2}{15r^2}\Gamma_{10}(3-2\Gamma_{10}^2), \tag{20}$$

where Γ_6 and Γ_{10} are the amount of Γ for $n=6$ and $n=10$ respectively. The behavior of $f(r)$ near $r=0$ for the case $n=2(4a-1)$ is

$$f \to -\frac{(8a-3)}{r^{8a-4}}\left[m+\frac{\lambda}{\beta^{4a-3}}e^{4a-1}\right],$$

$$\lambda = \frac{2^{6a-\frac{7}{2}}(2a-1)^{2a-\frac{3}{2}}(8a-3)^{2a-\frac{5}{2}}}{(4a-1)\binom{4a-3}{2a-1}} \tag{21}$$

While that of for $n=2(4a+1)$ is as follows

$$f \to -\frac{(8a+1)}{r^{8a}}\left[m-\frac{\lambda}{\beta^{4a-1}}e^{4a+1}\right],$$

$$\lambda = \frac{2^{8a-1}a^{2a-\frac{1}{2}}(8a+1)^{2a-\frac{3}{2}}}{(4a+1)\binom{4a-1}{2a}} \tag{22}$$

So depending on proper choosing of the parameters m and e, black hole with one or two horizons, an extreme black hole and a naked singularity may exist. Also, in these two cases spacelike or timelike singularities may exist depending on the values of m and e and β.

We know a horizon is a null hypersurface defined by $r=r_h$ such that $f(r_h)=0$ with finite curvatures, where r_h is a constant horizon radius. For all cases that $f(r)$ is positive near $r=0$, the extreme black hole may exist in which therein, both $f(r)$ and $f'(r)$ are zero on the horizon radius $r=r_{\rm ext}$ and can be calculated from (10) to be

$$r_{\rm ext} = \frac{\sqrt{\frac{n-2}{n}}\,l}{1+2\zeta}\left\{1+\zeta\left(1+\sqrt{1+\frac{2ne^2}{(n-2)l^2}\left(2+\frac{1}{\zeta}\right)}\right)\right\} \tag{23}$$

$$\zeta = \frac{4\beta^2l^2}{n(n-1)} \tag{24}$$

For these cases the spacetime of Eqs. (7) and (11) presents a naked singularity if $m<m_{\rm ext}$, an extreme black hole for $m=m_{\rm ext}$ and a black hole with two horizons provided $m>m_{\rm ext}$, where $m_{\rm ext}$ is

$$m_{\mathrm{ext}} = -r_{\mathrm{ext}}^{n-2}\left(\frac{1}{(n-1)} + \frac{r_{\mathrm{ext}}^2}{(n-1)l^2} + \frac{4\beta^2 r_{\mathrm{ext}}^2}{n(n-1)^2}\left[1-\hat{o}(r_{\mathrm{ext}})\right]\right),$$

The Hawking temperature is given by

$$T = \frac{f'(r_+)}{4\pi} = \frac{(n-2)}{r_+}\Upsilon(r_+) \tag{25}$$

where r_+ is the largest real root of $f(r)$ and

$$\Upsilon(r_+) = -1 + \frac{nr_+^2}{(n-2)l^2} + \frac{4\beta^2 r_+^2}{(n-1)(n-2)}\left[1-\hat{o}(r_+) - \frac{r_+}{n}\hat{o}'(r_+)\right], \tag{26}$$

$\hat{o}'(r_+)$ is the value of the first derivative of $\hat{o}(r_+)$ at $r = r_+$. It's notable that T vanishes for $m = m_{\mathrm{ext}}$.

4. Static solutions in Lovelock gravity

The Gauss-Bonnet-Yang-Mills field equation in the presence of YMBI fields may be written as

$$G_{\mu\nu}^{(1)} + \alpha_2' G_{\mu\nu}^{(2)} - \frac{n(n-1)}{2l^2} g_{\mu\nu} = 8\pi T_{\mu\nu}, \tag{27}$$

where $G_{\mu\nu}^{(2)}$ is the second order Lovelock tensor. The tt-component of the above field equation for the metric (7) is:

$$\left[r^3 - 2\alpha_2 r(f+1)\right]f' - (n-4)\alpha_2(f+1)^2 + (n-2)r^2(f+1)$$

$$-\frac{4\beta^2}{(n-1)}r^4(1-\Gamma) - \frac{n}{l^2}r^4 = 0 \tag{28}$$

where $\alpha_2 = (n-2)(n-3)\alpha_2'$. It is a matter of calculation to show that the solution of the field equation (28), for $N = n+1 = 5$ may be written as

$$f(r) = -1 + \frac{r^2}{2\alpha_2}\left(1 \mp \sqrt{1 - \frac{4\alpha_2}{l^2} + \frac{12\alpha_2 m}{r^4} + \frac{4\alpha_2\beta^2}{3}(1-\Gamma_4) + \frac{4\alpha_2 e^2 \ln\left(r^2 + r^2\Gamma_4\right)}{r^4}}\right)$$

where $\Gamma_4 = \Gamma(n=4)$. Solving the equation (28) for $N \geq 5$, we get

$$f(r) = -1 + \frac{r^2}{2\alpha_2}\left(1 \mp \sqrt{1 - \frac{4\alpha_2}{l^2} + \frac{4(n-1)\alpha_2 m}{r^n} - \frac{16\alpha_2\beta^2}{n(n-1)}\left[1-\hat{o}(r)\right]}\right) \tag{29}$$

where $\hat{o}(r)$ was introduced in (12). There are two families of solutions which correspond to the sign in front of the square root in Eq. (29). We call the family with minus (plus) sign the minus (plus)-branch solution. The minus-branch solution reduces to the solution in the Einstein-Yang-Mills-Born-Infeld solution in the limit of $\alpha \to 0.$ On the other hand, $f(r)$ diverges for the plus-branch solution in this limit, and there is no counterpart in the Einstein theory.

We notice that $f(r)$ takes the form

$$f(r) = -1 + \frac{r^2}{2\alpha_2}\left[1 \pm \sqrt{1 - \frac{4\alpha_2}{l^2} + \frac{4(n-1)\alpha_2 m}{r^n}}\right]$$

as $\beta \to 0$. Also, in the limit $\beta \to \infty$ it takes the form

$$f(r) = -1 + \frac{r^2}{2\alpha_2}\left[1 \pm \sqrt{1 - \frac{4\alpha_2}{l^2} + \frac{4(n-1)\alpha_2 m}{r^n} + \frac{4(n-2)\alpha_2 e^2}{(n-4)r^4}}\right]$$

This is the solution to the Gauss-Bonnet-Yang-Mills equation obtained in [22] as expected.

Of course, one may note that $f(r)$ is imaginary for $r < r_0$ and real for $r > r_0$ where r_0 is the largest real root of the following equation:

$$m + \frac{r^n}{(n-1)}\left(\frac{1}{4\alpha_2} - \frac{1}{l^2} - \frac{4\beta^2}{n(n-1)}\left[1-\hat{o}(r)\right]\right) = 0$$

Thus one cannot extend the spacetime to the region $r < r_0$. To get rid of this incorrect extension we introduce the new radial coordinate ρ as

$$\rho^2 = r^2 - r_0^2 \Rightarrow dr^2 = \frac{\rho^2}{\rho^2 + r_0^2}d\rho^2.$$

With this new coordinate the metric (7) becomes

$$ds^2 = -f(\rho)dt^2 + \frac{\rho^2}{\rho^2 + r_0^2}\frac{d\rho^2}{f(\rho)} + (\rho^2 + r_0^2)d\Omega^2 \tag{30}$$

where now one should substitute $r = \sqrt{\rho^2 + r_0^2}$ in Eq. (29). The new metric function has a singularity at $\rho = 0$ $(r = r_0)$.

Like in Einstein gravity, Gauss-Bonnet gravity, black hole with one or two horizons, an extreme black hole and a naked singularity may exist. Here, we just consider the condition of having extreme black holes, for which the temperature vanishes. The Hawking temperature can be obtained as

$$T = \frac{(n-2)r_+}{(r_+^2 - 2\alpha)}\left(\Upsilon(r_+) + \frac{(n-4)\alpha}{(n-2)r_+^2}\right) \tag{31}$$

where $\Upsilon(r_+)$ was given in Eq. (26) and r_+ is the radius of outer horizon. It is a matter of calculation to show that $m = m_{\rm ext}$,

$$m_{\rm ext} = \frac{2r_+^{n-2}}{n(n-1)}\left(-1 + \frac{2\alpha_2}{r_+^2} + \frac{2r_+^3\beta^2}{n(n-1)}\hat{o}'(r_+)\right)$$

is the solution of $T = 0$.

To explicitly find the solutions to the Yang-Mills-Born-Infeld equation in third order Lovelock gravity in the presence of cosmological constant, the third order Lovelock-Yang-mills field equation must be considered as

$$G_{\mu\nu}^{(1)} + \alpha_2' G_{\mu\nu}^{(2)} + \alpha_3' G_{\mu\nu}^{(3)} - \frac{n(n-1)}{2l^2} g_{\mu\nu} = 8\pi T_{\mu\nu}, \qquad (32)$$

where now $G_{\mu\nu}^{(3)}$ is the third order Lovelock tensor introduced in (24). The tt-component of the field equation (32) for the metric ansatz (7) is derived to be:

$$\begin{aligned}&\left[\alpha_3 r(f+1)^2 - 2\alpha_2 r^3(f+1) + r^5\right]f' \\ &+\frac{n-6}{3}\alpha_3(f+1)^3 - (n-4)\alpha_2 r^2(f+1)^2, \\ &+(n-2)r^4(f+1) - \frac{n}{l^2}r^6 - \frac{4\beta^2 r^6}{(n-1)}(1-\Gamma) = 0, \qquad (33)\end{aligned}$$

where prime denotes the derivative with respect to r and we define $\alpha_2' = \alpha_2/(n-2)(n-3)$ and $\alpha_3' = \alpha_3/3(n-2)(n-3)(n-4)(n-5)$ for simplicity.

First, we solve the equation (33) for the case $N = n+1 = 7$. The equation in this case admits the solution:

$$f(r) = -1 + \frac{\alpha_2 r^2}{\alpha_3}\left\{1 + \left(\sqrt{\gamma + j^2(r)} + j(r)\right)^{1/3} - \gamma^{1/3}\left(\sqrt{\gamma + j^2(r)} + j(r)\right)^{-1/3}\right\}, \qquad (34)$$

where

$$\gamma = \left(\frac{\alpha_3}{\alpha_2^2} - 1\right)^3,$$

$$j(r) = 1 - \frac{3\alpha_3}{2\alpha_2^2} + \frac{3\alpha_3^2}{2\alpha_2^3}\left(\frac{1}{l^2} - \frac{5m}{r^6} + \frac{2\beta^2}{15}(1-\Gamma_6) - \frac{4e^2}{3r^4}\Gamma_6\right).$$

Where Γ_6, as was mentioned before, is the amount of Γ, for $n = 6$. Also, for $N = n+1 > 7$ the generalized solution is the same as the relation (34) with $j(r)$ being:

$$j(r) = 1 - \frac{3\alpha_3}{2\alpha_2^2} + \frac{3\alpha_3^2}{2\alpha_2^3}\left(\frac{1}{l^2} - \frac{(n-1)m}{r^n} + \frac{4\beta^2}{n(n-1)}[1 - \hat{o}(r)]\right) \qquad (35)$$

Now for this solution as $\beta \to 0$, $k(r)$ reduces to

$$j(r) = 1 - \frac{3\alpha_3}{2\alpha_2^2} + \frac{3\alpha_3^2}{2l^2\alpha_2^3} - \frac{3(n-1)\alpha_3^2 m}{2\alpha_2^3 r^n}, \qquad (36)$$

which gives the solution to the Third order Lovelock gravity as expected and for $\beta \to \infty$, $j(r)$ will be

$$j(r) = 1 - \frac{3\alpha_3}{2\alpha_2^2} + \frac{3\alpha_3^2}{2\alpha_2^3}\left(\frac{1}{l^2} - \frac{(n-1)m}{r^n} - \frac{(n-2)e^2}{(n-4)r^4}\right). \qquad (37)$$

The solution given by Eqs. (34) and (37) represents the AdS solution to the Third order Lovelock-Yang-Mills equation.

To see the asymptotic behavior of the solution to the Third order Lovelock gravity, we write it for a special case that $\alpha_3 = \alpha_2^2$. The solution then will be

$$f(r) = -1 + \frac{r^2}{\alpha_2} + \frac{2^{1/3}r^2}{\alpha_2}j(r)^{1/3} \qquad (38)$$

$$j(r) = -\frac{1}{2} + \frac{3}{2}\alpha_2\left\{\frac{1}{l^2} - \frac{(n-1)m}{r^n} + \frac{4\beta^2}{n(n-1)}[1 - \hat{o}(r)]\right\}. \qquad (39)$$

This solution has a singularity at $r = 0$ as the Kretschmann scalar diverges at $r = 0$.

The Hawking temperature for this solution is given by:

$$T = \frac{(n-2)r_+^3}{(r_+^2 - \alpha_2)^2}\left(\Upsilon(r_+) - \frac{(n-6)\alpha_2^2}{3(n-2)r_+^4} + \frac{(n-4)\alpha_2}{(n-2)r_+^2}\right) \qquad (40)$$

Where r_+ is the radius of event horizon. Also, in this case, we see that the black hole solutions may present an extreme black hole with horizon radius $r_{\rm ext}$, where $r_{\rm ext}$ is one of the real roots of $T = 0$.

Now by solving the equations in second and third order Lovelock theories, we deduce that the tt-component of the field equation in Lovelock gravity is

$$\sum_{i=0}^{p}\alpha_i' \frac{(n-2)!}{(n-2i)!}\frac{1}{nr^{n-3}}\left[r^n\left(\frac{-1-f(r)}{r^2}\right)^i\right]' + \frac{4\beta^2 r^2}{n(n-1)}(1-\Gamma) = 0 \qquad (41)$$

We can solve this equation to obtain the general solution in Lovelock gravity as

$$\sum_{i=0}^{p}\alpha_i' \frac{(n-2)!}{(n-2i)!}\left(\frac{-1-f(r)}{r^2}\right)^i = \frac{(n-1)m}{r^n} - \frac{4\beta^2}{n(n-1)}\left[1-\hat{\mathbf{o}}(r)\right] \quad (42)$$

5. Concluding Remarks

The solutions to Einstein and Lovelock theories were obtained considering the coupling of two non-linear fields, Yang Mills and Born-Infeld fields and the properties of the solutions were investigated. The properties of these kinds of solutions are like the Yang-Mills solutions. But the difference seems to appear in the role of the mass in the solutions, as for small r's in Yang-Mills gravity the dominant term is the term containing m , but in the the Yang-Mills-Born-Infeld, the dominant term indicates both m and $\hat{\mathbf{o}}(r)$ which includes β, the Born-Infeld parameter and e. As the function $\hat{\mathbf{o}}(r)$ takes different forms in different dimensions, we saw that this term modifies the properties of the solutions. For example, in Einstein-Yang-Mills theory the solutions with nonnegative mass cannot present an extreme black hole but we found conditions for some of the solutions in Einstein-Yang-Mills-Born Infeld theory in which an extreme black hole or naked singularity can exist for non-negative mass. The singularities in Einstein-Yang-Mills theory are always spacelike and therefore unavoidable, but the singularities in Einstein-Yang-Mills-Born-Infeld theory can be spacelike or timelike depending on the choice of parameters. We also obtained the solutions for the second and third order Lovelock theories and from that the solution for the n-order Lovelock-Yang-mills-Born-Infeld theory was introduced.

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