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Effect of Rashba parameter on the thermal entanglement of electronic spin and induced subband states in a nanowire

F. Amiri and M. M. Golshan*

Physics Department, College of Sciences, Shiraz University, Shiraz, Iran
E-mail: golshan@susc.ac.ir

Abstract

In the present work we use the negativity to study the effect of Rashba parameter on the thermal entanglement of electronic spin and subband states inside a quasi-one-dimensional Rashba nanowire, in a perpendicular uniform magnetic field. We assume that the nanowire is held at a temperature T, so that both spin and subband states, with definite probabilities, are present. The partially transposed density matrix is shown to be block-diagonal, whose eigenvalues are readily obtained. By analyzing these eigenvalues, it is shown that, even at high temperatures there always exist negative eigenvalues, so that the system of electronic spin and subbands inside a Rashba nanowire is never separable. Moreover, we show that the negativity, at certain temperatures, exhibits maxima. The temperatures at which the entanglement is maximal strongly depend upon the Rashba parameter. We further present graphs of negativity as functions of temperature for different Rashba parameters, showing that the maximal entanglement occurs at lower temperatures for larger Rashba parameters. The novel results in the present article shows how the behavior of spin-subband thermal entanglement depends upon an externally controllable agent.

Keywords*:* Negativity; nanowire; Rashba effect

1. Introduction

It is a sound prediction that entanglement, arising from nonlocal quantum correlations, forms the basic ingredient for many applications in quantum information processing [1, 2]. In this regard, the question of how to implement and control the entanglement has been the focus of several reports [3, 4]. In most of these treatments the entanglement is investigated at absolute zero temperature, so that the quantum system is in pure states. However, due to the interaction with the environment, as a heat reservoir, temperature should be taken into account since thermal fluctuations may lead to disentanglement [5, 6].

Among the many proposals for realization of entanglement, hetrostructural systems, in particular, nanowires, have attracted much interest [7]. The simplest method of controlling the spin states of the electrons in nanostructures is to apply a uniform, time independent magnetic field and model the structural confinement as an externally applied electric field. As a consequence, both the space inversion and time reversal symmetries are broken, leading to the removal of degeneracies. Moreover, in suitably chosen hetrostructures, due to different band gaps, there appears an asymmetric potential, leading to the spin-orbit coupling; the Rashba effect [8, 9]. It has been shown and experimentally verified that the Rashba effect is modified when external gate voltages are applied [10, 11]. It is then evident that there are two externally controllable agents, magnetic field and Rashba parameter, that govern the spin-subband entanglement. In the present work, therefore, we shall concentrate on the effect of temperature on the entanglement of electronic spin states and thermally induced subband excitations (mixed states) in a quasi-one-dimensional Rashba nanowire. In particular, the behavior of such entanglements under different values of Rashba parameters is also discussed. Even though there are several measures to determine the entanglement (inseparability) of mixed states [12, 13], we use the concept of negativity [14, 15], which proves to be more suitable for the problem in hand [16]. A bipartite quantum system is disentangled (separable) if its density matrix can be written as $\rho = \sum_i p_i \, \rho_i^A \otimes \rho_i^B$ where $p_i \geq 0$, $\sum_i p_i = 1$ and $\rho_i^{A,B}$ represents the density matrices for the subsystems. The elements of ρ are given by $\langle a, b | \rho | a', b' \rangle$, where $\{|a\rangle\}$ and $\{|b\rangle\}$ form the orthonormal basis for each subsystem. It has been shown that for the composite system to be separable,

*Corresponding author
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it is necessary that the partially transposed density matrix, defined as, $\rho^{Pt} = \langle a', b|\rho|a, b'\rangle = \langle a, b'|\rho|a', b\rangle$, has no negative eigenvalues [17]. Conversely, if ρ^{Pt} possesses even a single negative eigenvalue, then the quantum system is entangled (inseparable). Quantitatively, this criterion may be expressed in terms of the negativity, defined as, $N = \sum_n Max(0, -\lambda_n)$, where λ_n's are the eigenvalues of ρ^{Pt}. It then follows that the state of the composite system is separable (disentangled) when the negativity is null, otherwise it is entangled [14]. The aforementioned criterion becomes a sufficient condition for 2×2 or 2×3 bipartite quantum systems [18]. In the following we find it expedient to explicitly use the density matrices instead of operators.

In what follows we study the thermal entanglement of electronic spin states and structural subbands, in a quasi-one-dimensional nanowire, under the influence of electric (position-dependent) and magnetic (uniform) fields, with due attention to the Rashba spin-orbit coupling. At the strong magnetic field limit the governing Hamiltonian is similar to the famous Janes-Cummings Hamiltonian in quantum optics for the interaction of two-level atoms and a single-mode quantized electromagnetic field [19]. Forming the partially transposed density matrix and calculating its eigenvalues, we find a simple condition under which the eigenvalues become negative. Moreover, we demonstrate that such a condition is always met, so that the system of electronic spin and subband excitations is never separable. Furthermore, it is shown that the entanglement of electronic spin-subbands becomes maximal at a certain temperature whose values (both the maxima and temperatures) strongly depend upon the Rashba parameter. Since the Rashba parameter is externally controllable (through gate potentials and suitable hetrostructures), the material presented in this article may be employed to control the behavior of electronic spin-subband thermal entanglement.

The remainder of this article is organized as follows. In Sec. 2 we present the model, the corresponding Hamiltonian along with the "bare" and "dressed" states. The thermal density matrix, the partially transposed density matrix and its eigenvalues are presented in Section 3. The condition under which these eigenvalues become negative is also discussed in this section. In Section 4 we show that the condition for negative eigenvalues is always satisfied. In this section the behavior of thermal entanglement is also discussed. Finally, some concluding remarks are made in section 5.

2. The Hamiltonian, "bare" and "dressed" states

For a quasi-one-dimensional quantum wire with Rashba spin-orbit coupling, in a perpendicular magnetic field, $\vec{B}$, laterally confined by a parabolic potential, the Hamiltonian is [11]

$$H = \frac{\left[\vec{P} + \frac{e}{c}\vec{A}\right]^2}{2m^*} + \frac{1}{2}m^*\omega_0^2 x^2 + \frac{1}{2}g\,\mu_B B\sigma_z - \frac{\alpha}{\hbar}\left[(\vec{P} + \frac{e}{c}\vec{A})\times\vec{\sigma}\right]_z , \quad (1)$$

where m^*, g and $\mu_B = \dfrac{e\hbar}{2mc}$ are, respectively, the effective mass, Landé factor and Bohr magneton of the electron, while $\vec{\sigma}$ is the vector of the Pauli matrices. The wire is assumed to lie in the $x-y$ plane, so that the magnetic field defines the z-axis. Taking the Landau gauge, $\vec{A} = -B(0, x, 0)$, the Hamiltonian of (1) becomes

$$H = \hbar\omega_c\left[a^\dagger a + \frac{m^*}{4m_0}g\sigma_Z + \frac{l_B}{\sqrt{2}\,l_{so}}\left(a\sigma_+ + a^\dagger\sigma_-\right)\right], \quad (2)$$

where $\omega_C = eB/m^*c$ is the cyclotron frequency, $l_B = \sqrt{\hbar c/eB}$ (in effect gives the radius of electronic spin precession), $l_{so} = \hbar^2/2m^*\alpha$, with α being the Rashba coupling (giving the effective radius of electronic orbital motion). In obtaining (2) the confining potential along with parts of the vector potential is presented by the harmonic oscillators creation (annihilation) operators, $a^\dagger(a)$,and the limit of strong magnetic field, i.e., $\omega_0 \ll \omega$ has been assumed. We have also disregarded the vacuum state of the oscillator, since it contributes a constant phase to the density operator. It is emphasized that the last term of (2) with the usual spin ladder operators, $\sigma_\pm$ is, in fact, a representation of Rashba spin-orbit coupling. The electronic Hilbert space is spanned by the "bare" states $\{|n,+\rangle\}$ and $\{|n,-\rangle\}$, $n = 0,1,2,...,$ where $|\pm\rangle$ denotes the spin states, while $|n\rangle$ represents the subband indices. The matrix representation of (2), with respect to the "bare" states $|1\rangle = |n,+\rangle$ and $|2\rangle = |n+1,-\rangle$ is of the form

$$H = \hbar\omega_c\begin{pmatrix} E_0 & & & \bigcirc \\ & H_1 & & \\ & & H_2 & \\ & & & \ddots \\ \bigcirc & & & & H_n \\ & & & & & \ddots \end{pmatrix} \quad (3)$$

Where $E_0 = -\frac{m^*}{4m_0} g$ (coming from $|0,-\rangle$) is a 1×1 matrix, while each $H_n (n=1,2,...)$ is a 2×2 one with elements,

$$(H_n)_{11} = n + \frac{m*}{4m_0} g\,, \quad (H_n)_{22} = (n+1) - \frac{m*}{4m_0} g\,, \qquad (H_n)_{12} = (H_n)_{21}{}^* = \frac{l_B}{\sqrt{2}\, l_{so}}\sqrt{n+1}. \tag{4}$$

From (3) it is seen that the ground state corresponds to $|0,-\rangle$ which is separable, so that at absolute zero the negativity should vanish. The eigenstates of each block, the "dressed" states, $|\Psi_{1n}\rangle$ and $|\Psi_{2n}\rangle$, along with the eigenvalues, E_{1n} and E_{2n}, may be straight forwardly calculated. One finds

$$\begin{aligned} |\Psi_{1n}\rangle &= \alpha_{1n}|n,+\rangle + \alpha_{2n}|n+1,-\rangle, \\ |\Psi_{2n}\rangle &= \alpha_{2n}|n,+\rangle - \alpha_{1n}|n+1,-\rangle, \end{aligned} \tag{5}$$

where,

$$\alpha_{1n} = \frac{2\gamma_n}{\sqrt{2\eta^2 + 8\gamma_n{}^2 - 2\sqrt{\eta^2 + 4\gamma_n{}^2}}}, \tag{6}$$

with $|\alpha_{2n}| = \sqrt{1-|\alpha_{1n}|^2}$ and

$$\eta = \omega_c(\frac{m^* g}{2m} - 1), \quad \gamma_n = \frac{\omega_c\, l_B}{\sqrt{2}\, l_{so}}\sqrt{n+1}, \tag{7}$$

while,

$$E_{1n,2n} = \hbar\left[\omega_c(n+\frac{1}{2}) \pm \frac{1}{2}\sqrt{\eta^2 + 4\gamma_n{}^2}\right]. \tag{8}$$

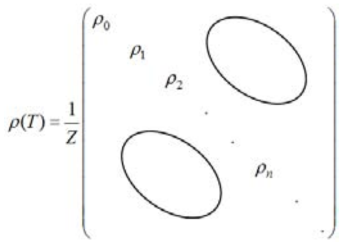
Since $\sqrt{\eta^2 + 4\gamma_n{}^2}$ cannot vanish for any n, the absence of degeneracy is noted. It is evident from (7) that neither eigenvalues can assume negative values. In the next section (5) and (8) are used to form the thermal density matrix.

3. The thermal density matrix and negativity

The thermal density operator for the present system is

$$\rho(T) = \frac{1}{Z}\sum_{n=0}^{\infty}(e^{-\beta E_{1n}}|\Psi_{1n}\rangle\langle\Psi_{1n}| + e^{-\beta E_{2n}}|\Psi_{2n}\rangle\langle\Psi_{2n}|) + \frac{1}{Z} e^{\beta(\frac{\hbar\omega_c}{4}\frac{m^*}{m_0}g)}|0,-\rangle\langle 0,-|, \tag{9}$$

where $\beta = (KT)^{-1}$, Z is the partition function and $|\Psi_{in}\rangle, i=1,2,$ are the "dressed" states with eigenvalues E_{in} given in (5) and (8), respectively. When $|\Psi_{in}\rangle$'s and E_{in} are substituted into (9) the thermal density matrix, describing the system of electronic spin and subband excitations at a temperature T, is obtained as



(10)

where, again, $\rho_0 = \exp\left(\beta(\frac{\hbar\omega_c}{4}\frac{m^*}{m_0}g)\right)$ is a 1×1 matrix while each ρ_n is a 2×2 one with elements (in the "bare" representation),

$$\begin{aligned} &(\rho_n)_{11} = A_{n+1}(T), \quad (\rho_n)_{22} = B_{n+1}(T), \\ &(\rho_n)_{12} = (\rho_n)_{21}^* = C_{n+1}(T). \end{aligned} \tag{11}$$

With

$$\begin{aligned} A_{n+1}(T) &= \exp(-\beta E_{1n})|\alpha_{1n}|^2 + \exp(-\beta E_{2n})|\alpha_{2n}|^2, \\ B_{n+1}(T) &= \exp(-\beta E_{1n})|\alpha_{2n}|^2 + \exp(-\beta E_{2n})|\alpha_{1n}|^2, \end{aligned} \tag{13}$$

and

$$C_{n+1}(T) = \alpha_{1n}\alpha_{2n}(\exp(-\beta E_{1n}) - \exp(-\beta E_{2n}))\,. \tag{14}$$

The partially transposed density matrix representation of ρ is then calculated using the partially transposed (with respect to spin states) "bare" states, $|1'\rangle = |n,-\rangle$ and $|2'\rangle = |n+1,+\rangle$. The partial transposed density matrix, as the density matrix itself, (11), is block diagonal. The first block (coming from $|n,+\rangle$), being 1×1, is

$$\rho_0{}^{Pt} = A_1 \tag{15}$$

all the other blocks are 2×2, with elements,

$$\begin{aligned} &(\rho_n{}^{Pt}(T))_{1'1'} = B_n(T),\; (\rho_n{}^{Pt}(T))_{2'2'} = A_{n+2}(T), \\ &(\rho_n{}^{Pt}(T))_{1'2'} = (\rho_n{}^{Pt})^*_{2'1'} = C_{n+1}(T). \end{aligned} \tag{16}$$

Since the negativity is defined as,

$$N(T) = \sum_0^{\infty} Max\,(0, -\lambda_n(T)) \tag{17}$$

where, λ_n's are the negative eigenvalues of $\rho_n{}^{Pt}(T)$, $\rho_0{}^{Pt}(T) > 0$ does not contribute to the negativity. Diagonalization of each block of $\rho_n{}^{Pt}(T)$ gives

$$\lambda_n^{(\pm)}(T)=\tfrac{1}{2}[(B_n(T)+A_{n+2}(T)) \pm\sqrt{(B_n(T)+A_{n+2}(T))^2-4(B_n(T)A_{n+2}(T)-C_{n+1}^2(T))}\,] \quad (18)$$

as the eigenvalues. It is clear from (18) that $\lambda_n^{(-)}(T)$ becomes negative if and only if

$$A_{n+2}(T)B_n(T)-C_{n+1}^2(T)<0 \quad (19)$$

Upon substituting (12), (13) and (14) into (19), the condition for nonvanishg negativity becomes

$$\begin{aligned}&[\cosh(\frac{\Omega_n}{\tau})\cosh(\frac{\Omega_{n+2}}{\tau})+\frac{\chi}{2\Omega_n}\sinh(\frac{\Omega_n}{\tau})\cosh(\frac{\Omega_{n+2}}{\tau})\\ &-\frac{\chi}{2\Omega_{n+2}}\sinh(\frac{\Omega_{n+2}}{\tau})\cosh(\frac{\Omega_n}{\tau})-\frac{\chi^2}{4\Omega_n\Omega_{n+2}}\sinh(\frac{\Omega_{n+2}}{\tau})\sinh(\frac{\Omega_n}{\tau})\\ &-\frac{(n+1)}{\Omega_{n+1}^2}\sinh^2(\frac{\Omega_{n+1}}{\tau})]<0,\end{aligned} \quad (20)$$

where $\tau=\frac{\sqrt{2}l_{so}kT}{\hbar\omega_c l_B}$ defines the scaled, dimensionless temperature, $\Omega_n=\frac{1}{2}\sqrt{\chi^2+4n}$ and $\chi=\frac{\sqrt{2}l_{so}}{l_B}(\frac{m^*g-2m}{2m})$. In the next section we examine the condition of (20) and show that the system of electronic spin-subbands is *never* separable.

4. Results and discussions

Since all possible negative eigenvalues of the blocks in $\rho^{Pt}(T)$ participate in the negativity, we examine these eigenvalues for large excitations. Under the assumption that $n\gg\chi^2$ (≈ 225 for InAs with $\alpha=1.6\times10^{-11}\,eVm$), systematic expansion of the condition (20) in negative powers of n (at a fixed temperature) gives

$$1-\frac{e^{\frac{2\sqrt{n}}{\tau}}}{16\tau n^{\frac{3}{2}}}<0 \quad (21)$$

From (21) it is clear that for a large enough excitation, and thereafter, even at high temperatures, blocks of $\rho^{Pt}(T)$ produce negative eigenvalues. Moreover, a similar expansion of (20) for high temperatures and fixed n yields

$$\tau^2<\frac{\chi^2}{4}+n+1 \quad (22)$$

Again, it is observed that at any fixed (even high) temperature (except zero and infinity) the condition of (20) may be satisfied for a specific n (and thereafter). The conclusion that $\lambda_n^{(-)}$'s eventually become (and remain) negative may be verified from Figs. (1) to (5), in which $\lambda_n^{(-)}(T)$ (unnormalized) versus n, at different temperatures corresponding to $\tau=1,2,4,10$ and 100, for three values of Rashba parameters, $\alpha=1.6\times10^{-11}$ eVm (long-dashed line), $\alpha=2.4\times10^{-11}\,eVm$ (short-dashed line) and $\alpha=3.2\times10^{-11}\,eVm$ (solid line), are illustrated.The figures (and the following one) are drawn for the parametric values in InAs: $\alpha=1.6\times10^{-11}$ - $3.2\times10^{-11}\,eVm$, $g=-8$, $m^*=0.04m_o$ [11] under the influence of a magnetic field of $6.59\,Tesla$. Parethetically, we mention that for the width of the wire in the order of $100\,nm$ a confinement frequency of the order of $\omega_0\simeq10^{11}Hz$ is requred. For the limit of strong magnetic field, e. g. $\omega_0\ll\omega$, we have taken ω_c in the order of $10^{13}\,Hz$. As a result, $\omega_c\simeq10^{13}\,Hz$ yeilds a magnetic field of $6.59\,Tesla$. From these figures it is confirmed that, even at high temperatures, for sufficiently high subband indices, n_0, $\lambda_{n_0}^{(-)}(T)$ assumes a *negative* value and remains *negative* for $n>n_0$

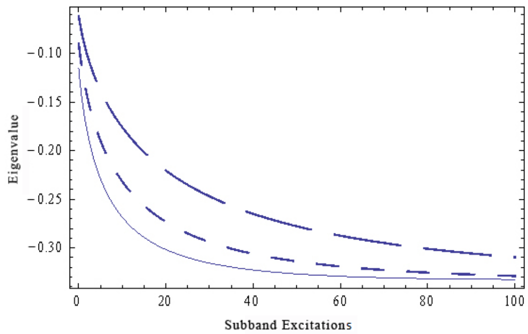


Fig. 1. Eigenvalue versus subband excitations at $\tau=1$, for different Rashba parameters: $\alpha=1.6\times10^{-11}\,eVm$ (long-dashed line), $\alpha=2.4\times10^{-11}\,eVm$ (short-dashed line) and $\alpha=3.2\times10^{-11}\,eVm$ (solid line)

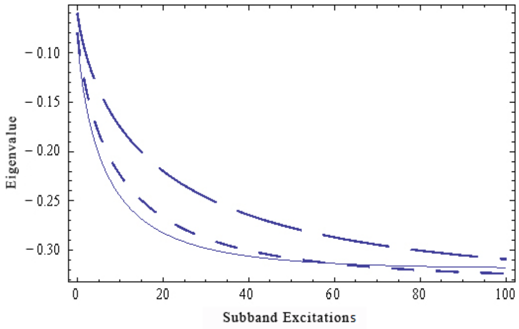


Fig. 2. Eigenvalue versus subband excitations at $\tau = 2$ for different Rashba parameters: $\alpha = 1.6\times10^{-11}\,eVm$ (long-dashed line), $\alpha = 2.4\times10^{-11}\,eVm$ (short-dashed line) and $\alpha = 3.2\times10^{-11}\,eVm$ (solid line)

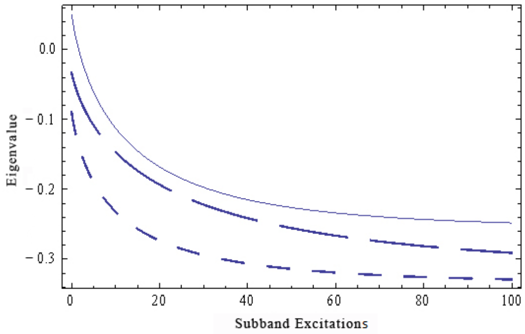


Fig. 3. Eigenvalue versus subband excitations at $\tau = 4$, for different Rashba parameters: $\alpha = 1.6\times10^{-11}\,eVm$ (long-dashed line), $\alpha = 2.4\times10^{-11}\,eVm$ (short-dashed line) and $\alpha = 3.2\times10^{-11}\,eVm$ (solid line)

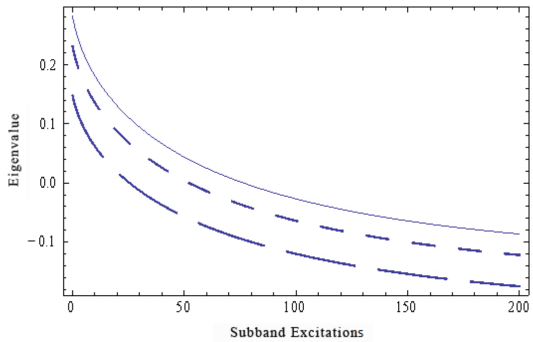


Fig. 4. Eigenvalue versus subband excitations at $\tau = 10$, for different Rashba parameters: $\alpha = 1.6\times10^{-11}\,eVm$ (long-dashed line), $\alpha = 2.4\times10^{-11}\,eVm$ (short-dashed line) and $\alpha = 3.2\times10^{-11}\,eVm$ (solid line)

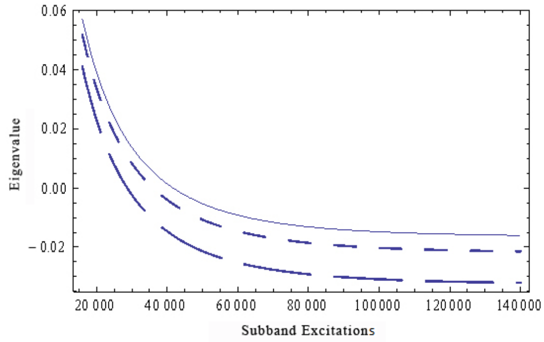


Fig. 5. Eigenvalue versus subband excitations at $\tau = 100$, for different Rashba parameters: $\alpha = 1.6\times10^{-11}\,eVm$ (long-dashed line), $\alpha = 2.4\times10^{-11}\,eVm$ (short-dashed line) and $\alpha = 3.2\times10^{-11}\,eVm$ (solid line)

From these figures, moreover, it is also observed that at relatively low temperatures, $\lambda_n^{(-)}(T)$ turns negative at smaller n_0, with larger absolute values for larger Rashba parameters. This behavior of $\lambda_n^{(-)}$'s is reversed at higher temperatures.These observations of the behavior of $\lambda_n^{(-)}$'s indicate that the negativity should rise at lower temperatures, passing through a maximum, and then reduce. The negetivity against the scaled temperature, τ, for different Rashba parameters, is depicted in Fig. (6), showing its behavior in accordance with the foregoing anticipations. The fact that electronic spin-subband states are never separable (always entangled) is also seen from Fig. (6).

6. Conclusion

In the present work we have considered the effect of Rashba spin-orbit coupling on the thermal entanglement of spin-subband states in a Rashba nanowire.

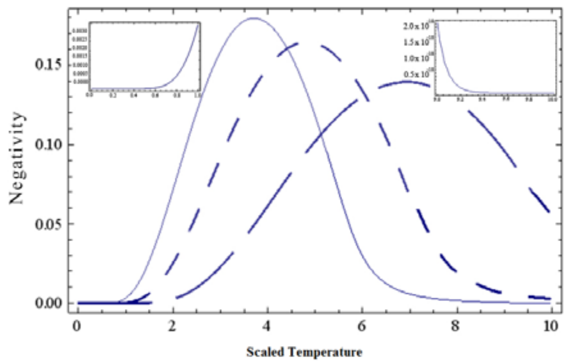


Fig. 6. Graphs of negativity versus the scaled temprature, τ, for a magnetic field of $5.69\,Tesla$ and different Rashba parameters. Left (right) inset depicts the negativity for small (large) τ 's

To this end, we have calculated the eigenvalues of partially transposed thermal density matrix and, consequently, the negativity. An analysis of the eigenvalues and the negativity indicates that (a complete discussion is presented in Section (4)),

i) The system of electronic spin state and subband excitations in a Rashba nanowire is never separable.

ii) As the temperature rises the negativity, in general, starts from zero (due to separablity of the ground state) at absolute zero, goes through a maximum and decreases to diminishing values.

iii) The maximal entanglement occurs at lower temperatures for larger Rashba spin- orbit coupling.

In conclusion, we have demonstrated that the behavior of thermal entanglement of electronic spin states and subband excitations in a Rashba nanowire may be controlled through the Rashba spin-orbit couplings. To be specific, the maximal value of entanglement, the temperature at which the maximal entanglement and the decoherence occur, one indeed controlled by the Rashba spin-orbit coupling.

Acknowledgments

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