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AN ELASTOPLASTIC CONSTITUTIVE MODEL FOR COARSE-GRAINED SOILS*

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Abstract– This paper developed a new elastoplastic model for coarse-grained soils. Plastic potential surface was established by solving a differential dilatancy equation, which is obtained via the triaxial test results. This model obeys a non-associated flow rule; therefore, the yield surface is not consistent with the plastic potential surface. Expression of the yield surface was established similar to the plastic potential surface, but its shape is influenced not only by the stress ratio η but the mean stress p. A unified hardening parameter that is independent of stress path is adopted, so that negative dilatancy and positive dilatancy properties of coarse-grained soils could both be described. In this research, elastoplastic formula of the proposed model is deduced. This model was also evaluated with several groups of test results under different stress paths (i.e. conventional triaxial tests, constant mean stress test and constant stress ratio test, etc.). Results showed that model predictions agree well with the test results.

Keywords– Elastoplastic model, coarse-grained soils, dilatancy equation, non-associated flow rule, unified hardening parameter, stress path

1. INTRODUCTION

With the development of roller compaction technology, coarse-grained soils are widely used in rock-fill dam, road foundation, airport, among other applications. These coarse-grained soils are large, angular, and granular rock materials blasted from the parent rock, and have a large range of particle size. Mechanical properties of coarse-grained soils are significantly different from sand or clay in compressibility, dilatancy, strength, particle breakage *etc.*

In the past years, many scholars were dedicated to the study of coarse-grained soils and achieved many useful conclusions. Several constitutive models were also proposed to describe the mechanical behaviors of the coarse-grained soils. Duncan [1] proposed a nonlinear strength criterion of the coarse-grained soils, i.e. friction angle decreases with the increase of confining pressure. Shen [2-3] introduced a two-yield-surface model for coarse-grained soils, which is widely used in rock-fill dam analysis for its simplicity and practicality. Alonso & Oldecop *et al.* [4-6] proposed the Barcelona Basic Model (BBM) and the Rock-fill Model (RM), which were also applied to analyze the Beliche Dam. Yao *et al.* [7-8] took into account the particle breakage in coarse-grained soils and developed a model within the framework of Cambridge model (Roscoe, Schofield & Wroth) [9]. Yao *et al.* [10] also modified this model to reflect the behaviors of coarse-grained soils under cyclic loadings, within the "sub-loading" concept. Liu *et al.* [11-12] studied the critical state of the coarse-grained soils and proposed a constitutive model according to the generalized plasticity theories. Time-dependent behaviors of coarse-grained soils were also investigated theoretically (Alonso & Oldecop) [13] and practically (Zhu *et al.*) [14].

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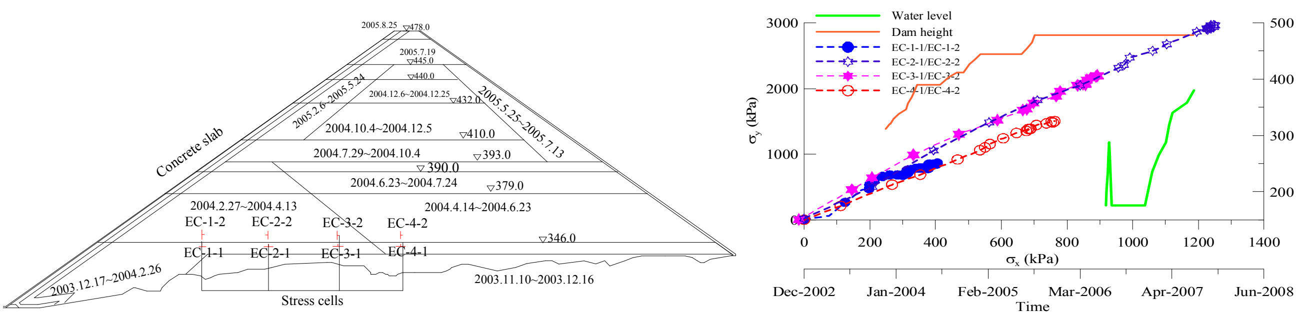
In the current stage, there are still some shortcomings that need to be solved in those models. For the Barcelona Basic Model (BBM) or Rock-fill Model (RM), model parameters are not easy to determine, so its practicality is limited. Wang *et al.* [15] pointed out that the original two-yield-surface model cannot reasonably reflect the soil behaviors under different stress paths, therefore, the two-yield surface model was revised. Test results show that plastic flow direction of coarse-grained soils cannot be described with the original dilatancy equation (proposed by [9]), therefore, the model proposed by Yao *et al.* may not predict behaviors of coarse-grained soils properly; according to the results of this paper, there is a rather large error between the model predictions and tests results when the specimen is along the constant stress ratio stress path. Constant stress ratio stress path is considered to be an important condition in rock-fill dam engineering; many in-situ measurements showed that, during the construction period of the rock-fill dam, stress ratio of soil element remains almost constant. Monitoring data of the Sanbanxi Concrete-Faced Rock-fill Dam during its construction are shown in Fig. 1. Positions of the stress cells as well as the construction schedule are presented in Fig. 1 (a). As can be seen, four groups of stress cells were installed at the elevation of 346.2m; each group has two stress cells to respectively monitor the vertical stress σ_y and horizontal stress σ_x. The relationship of σ_y and σ_x is plotted in Fig. 1 (b). If the orientations of the first principle stress σ_1 and third principle stress σ_3 are considered to be consistent with σ_y and σ_x, it can be said that, the principle stress ratio (σ_1/σ_3) remains almost constant during the period of rock-fill dam construction. Thus, models proposed for coarse-grained soils should consider the constant stress ratio stress path as an important condition.



(a) Positions of the stress cells and construction schedule (b) Relationship of σ_y and σ_x

Fig. 1. In-situ measurements of the Sanbanxi Dam

In this paper, a new elastoplastic model was developed for coarse-grained soils, dilatancy equation of the coarse-grained soils was proposed based on the conventional triaxial test results; by solving the differential equation, expression of potential surface can be obtained. Expression of the yield surface was also proposed according to the deformation characteristics of coarse-grained soils. The new model adopts a unified hardening parameter [16], so that positive dilatancy and negative dilatancy in the shearing process could both be described. In this research, elastoplastic formula of the proposed model was also presented. Then the proposed model was validated with triaxial test results under various stress paths.

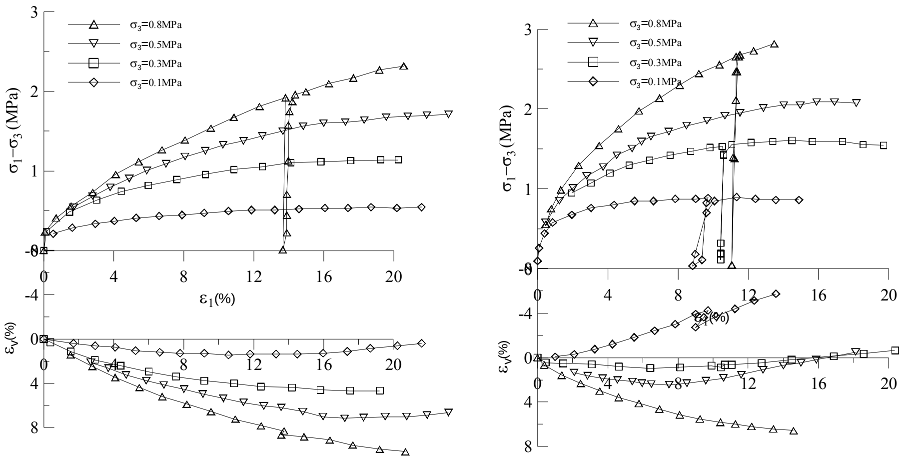
2. DILATANCY MECHANISM AND DILATANCY EQUATION OF COARSE-GRAINED SOILS

a) Dilatancy mechanism of the coarse-grained soils

Typical triaxial test results of two groups of Lechago shale rock-fills are presented in Fig. 2 [17]. In the shearing process, coarse-grained soils showed a volume expansion in low confining pressure but a volume contraction in high confining pressure. This discrepancy may be mainly caused by particle breakage.

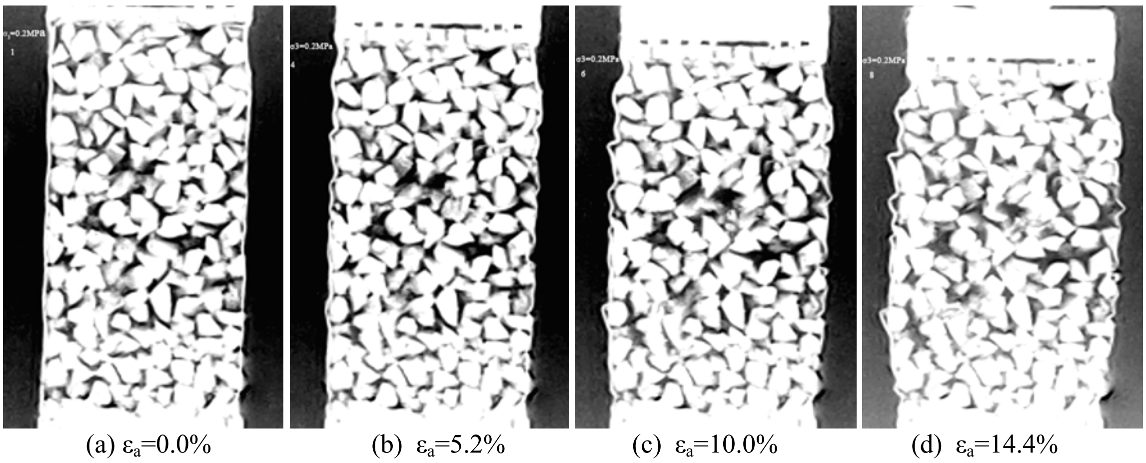
Terzaghi [18] suggested that a possible reason for large deformation of coarse-grained soils could be the breakage of rock particles and the subsequent rearrangement of the granular structure into a more stable position. After, the work of Marsal [19] obtained a similar conclusion. In low confining pressure, there is slight particle breakage; particle rearrangement caused the expansion of soil volume. Conversely, high confining pressure leads to more particle breakage and contraction of the soil volume.

Recently, CT technology [20] was used to track the traces of the soil particles in shearing test. Figure 3 presents the CT images of coarse-grained soils sourced from the Shuibuya Rock-fill Dam with a confining pressure of 0.2MPa. When the axial strain reached 0.0%, 5.2%, 10.0%, 14.4%, respectively, the same cross-section of the specimen was scanned. According to Fig. 3, at the beginning of the test (ε_a=0.0%), soil particles were tightly packed and embedded in each other. At the beginning, soil skeleton was rather stable. In the shearing process, soil particles were moving or rotating, which results in dilatancy of soil volume. Figure 3(d) presents the specimen that close to failure state, shear bands also formed during this period.



(a) Specimen A (b) Specimen B

Fig. 2. Typical curves of shale rock-fills in triaxial test [17]



(a) ε_a=0.0% (b) ε_a=5.2% (c) ε_a=10.0% (d) ε_a=14.4%

Fig. 3. Particles motion in the triaxial tests scanned with CT [20]

b) Dilatancy equation of the coarse-grained soils

Dilatancy equation determines the relationship between the incremental plastic strain and the current stress state (or internal variables), which is significantly important in establishing a constitutive model. The following are several classical dilatancy equations.

Equation (1) is the dilatancy equation of the original Cambridge model (Schofield & Wroth) [21]

$$d_g = \frac{d\varepsilon_v^p}{d\varepsilon_s^p} = M - \eta \tag{1a}$$

In which, M is a material parameter; $\eta = q/p$ is the stress ratio; ε_v^p is the plastic volumetric strain; ε_s^p is the generalized plastic shear strain.

$$p = (\sigma_1 + \sigma_2 + \sigma_3)/3 \tag{1b}$$

$$q = \frac{1}{\sqrt{2}}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right]^{1/2} \tag{1c}$$

$$\varepsilon_s^p = \frac{\sqrt{2}}{3}\left[(\varepsilon_1^p - \varepsilon_2^p)^2 + (\varepsilon_2^p - \varepsilon_3^p)^2 + (\varepsilon_3^p - \varepsilon_1^p)^2\right]^{1/2} \tag{1d}$$

$$\varepsilon_v^p = \varepsilon_1^p + \varepsilon_2^p + \varepsilon_3^p \tag{1e}$$

This original Cambridge model is proposed for "normal consolidated" or slightly "over consolidated" clay, but not suitable for coarse-grained soils.

Li and Dafalias [22] proposed a dilatancy equation considering influence of the density and stress level on the soils, shown in Eq. (2)

$$d_g = \frac{d\varepsilon_v^p}{d\varepsilon_s^p} = d_1\left(e^{m\psi} - \frac{\eta}{M}\right) \tag{2}$$

In which, d_1, m and M are material parameters, and the state variable ψ is defined as

$$\psi = e - e_{cr} \tag{3}$$

In which e is the current void ratio; e_{cr} is the critical state void ratio. Note that this equation takes into account factors that are not included in classic dilatancy theories.

In order to find a proper dilatancy equation for coarse-grained soils, several groups of test results are chosen, including Lechago Dam shale rock-fills [17], Shuibuya Dam Limestone rock-fills, Oroville Dam sandy-gravel fills [1], Diorite rock-fills [11-12]. According to Eq. (1) and Eq. (2), $d\varepsilon_v^p / d\varepsilon_s^p$ is related to stress ratio $\eta = q/p$, therefore, we plot relationship of $d\varepsilon_v^p / d\varepsilon_s^p$ and $\eta = q/p$ in Figs. 4~8. According to the unloading stress path in Fig. 2, elastic strain is very small in coarse-grained soils; therefore, we obtain the following formula:

$$\frac{d\varepsilon_v^p}{d\varepsilon_s^p} \approx \frac{d\varepsilon_v}{d\varepsilon_s} \tag{4}$$

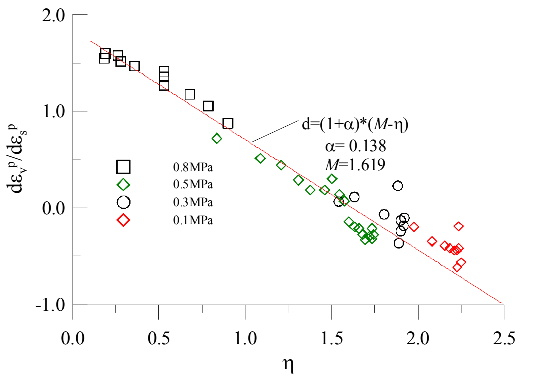


Fig. 4. Lechago Dam Shale rock-fills (sample A)

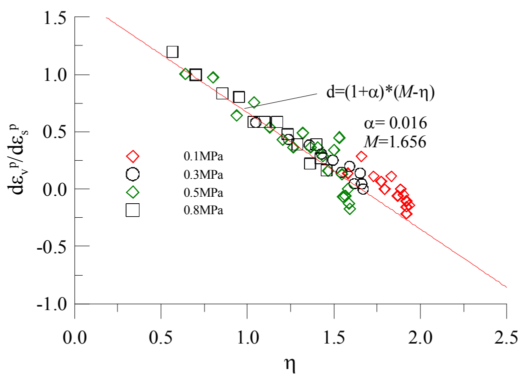


Fig. 5. Lechago Dam Shale rock-fills (sample B)

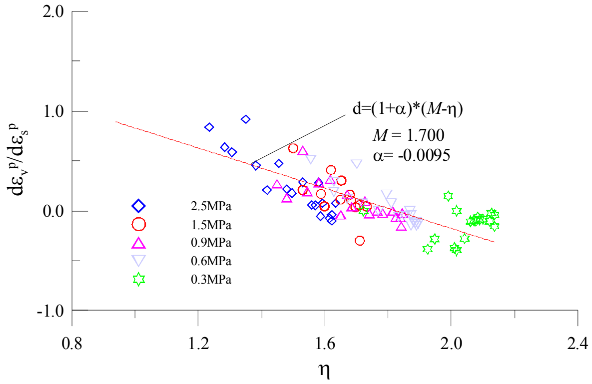


Fig. 6. Shuibuya Dam Limestone rock-fills

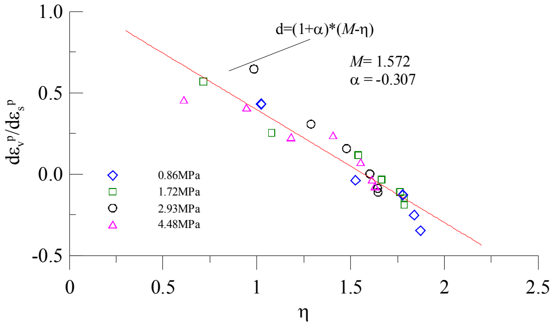


Fig. 7. Oroville Dam sandy-gravel

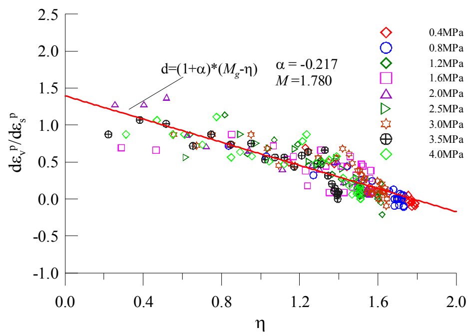


Fig. 8. Diorite rock-fills

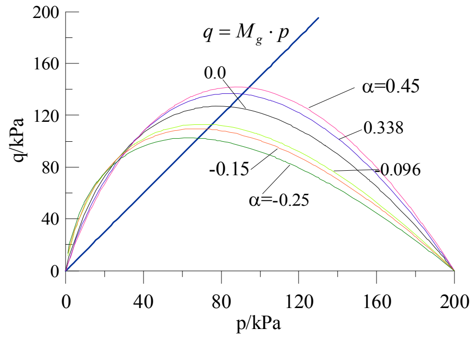


Fig. 9. Plastic potential surfaces with different values of α

According to Figs. 4~ 8, the relationship between $d\varepsilon_v^p / d\varepsilon_s^p$ and stress ratio $\eta = q / p$ can be described linearly. In Fig. 8, Diorite rock-fills were sheared from a rather low confining pressure 0.4MPa to a high confining pressure 4.0MPa; it can be seen that, the straight line agrees well with the test results. Obviously, $d\varepsilon_v^p / d\varepsilon_s^p < 0$ indicates the volume expansion, on the contrary, $d\varepsilon_v^p / d\varepsilon_s^p > 0$ indicates the volume contraction. Thus, the following expression is used to describe dilatancy properties of the coarse-grained soils

$$d_g = \frac{d\varepsilon_v^p}{d\varepsilon_s^p} = (1+\alpha)(M_g - \eta) \tag{5}$$

In which, α and M_g are material parameters.

3. EXPRESSIONS OF THE POTENTIAL SURFACE AND YIELD SURFACE

a) Plastic potential surface

In the proposed model, stress dilatancy equation is expressed as Eq. (5). The orthogonality condition is

$$dp \cdot d\varepsilon_v^p + dq \cdot d\varepsilon_s^p = 0 \tag{6}$$

According to Eq. (5) and Eq. (6), the following differential equation is obtained:

$$\frac{dq}{dp} = -\frac{d\varepsilon_v^p}{d\varepsilon_s^p} = -(1+\alpha)\left(M_g - \frac{q}{p}\right) \tag{7}$$

Solving the Eq. (7), expression of the plastic potential surface is written as

$$g = q - M_g p\left[1 + \frac{1}{\alpha}\right]\left[1 - \left[\frac{p}{p_g}\right]^{\alpha}\right] \tag{8}$$

In which, p_g is the intersection of the plastic potential surface and the p-axis. Plastic potential surfaces are depicted in Fig. 9 for different values of parameter α .

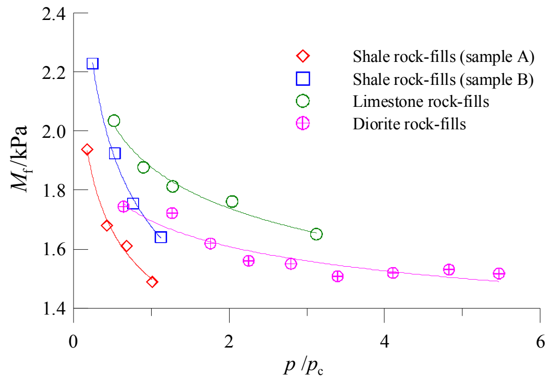


Fig. 10. Relationship between M_f and p/p_c

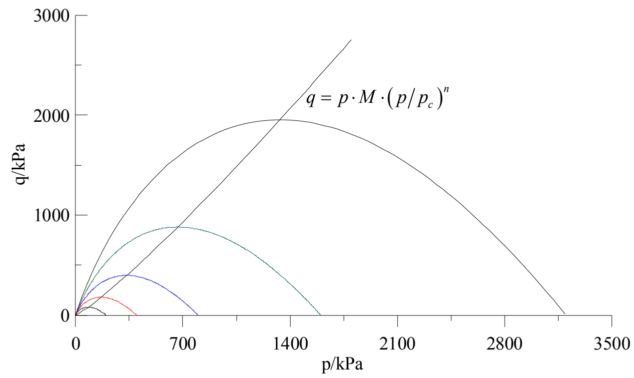


Fig. 11. Yield surfaces on p-q plane

b) Yield surface of the proposed model

It is generally accepted that friction angle of coarse-grained soils decreases with the increase of confining pressure. In other words, stress ratio at the failure is not constant on the p-q plane; therefore, stress ratio at the failure is expressed as

$$M_f = M \cdot \left(\frac{p}{p_c} \right)^{-n} \tag{9}$$

In which, p_c is the reference crushing stress, M and n are material parameters that can be obtained by fitting the test data. Relationships between M_f and p/p_c are plotted in Fig. 10. Reference crushing stress of each group is listed in Table.1.

In the proposed model, yield surface is similar to the plastic potential surface, but influence of the mean stress p on the shape of yield surface is also included, similar to Eq. (7), a new differential equation is presented as Eq. (10).

$$\frac{dq}{dp} = -(1+\alpha)\left(M\left(\frac{p}{p_c}\right)^n - \frac{q}{p} \right) \tag{10}$$

Solving the Eq. (10) (*Appendix I*), expression of the yield surface is

$$f = q - Mp\frac{(1+\alpha)}{\alpha - n}\left(\frac{1}{p_c}\right)^n \left[p^n - \left(\frac{p}{p_x}\right)^\alpha p_x^{\ n} \right] = 0 \tag{11}$$

In which, p_x is the intersection of the plastic potential surface and the p-axis. Yield surfaces on p-q plane are depicted in Fig. 11.

c) Hardening rule of the proposed model

Referencing the previous work (Nakai) [23], the relationship between the plastic volumetric strain and the mean stress p under isotropic compression condition could be assumed as

$$\varepsilon_v^p = (c_t - c_e)\left[\left(\frac{p}{p_a}\right)^m - \left(\frac{p_0}{p_a}\right)^m \right] \tag{12}$$

In which, c_t is the compression index, c_e is the swelling index, m is a coefficient for coarse-grained soils, p_a is the atmospheric pressure.

Eq. (12) could be converted into the following form

$$p_x = p_a \cdot \left(\frac{\varepsilon_v^p}{c_t - c_e} + \left(\frac{p_0}{p_a} \right)^m \right)^{\frac{1}{m}} \tag{13}$$

Combining Eq. (11) with Eq. (13), the expression of the yield surface could be written as the following form with the plastic volumetric strain as its hardening parameter.

$$f = q - M(1+\alpha)\left(\frac{1}{p_c}\right)^n \frac{p}{\alpha - n}\left[p^n - p^\alpha K\right] = 0 \tag{14}$$

In which

$$K = \left\{ p_a \cdot \left(\frac{\varepsilon_v^p}{c_t - c_e} + \left(\frac{p_0}{p_a} \right)^m \right)^{\frac{1}{m}} \right\}^{n-\alpha} \tag{15}$$

Note that, if we adopt the plastic volumetric strain ε_v^p as the hardening parameter, the yield surface will shrink once soils dilate, which is not reasonable. In this study, a unified hardening parameter *H* developed by Yao *et al*. [16] is used. The unified hardening parameter used is written as

$$H = \int \frac{M_f^4 - \eta^4}{M_g^4 - \eta^4} d\varepsilon_v^p = \int \frac{1}{\Omega} d\varepsilon_v^p \tag{16a}$$

$$\Omega = \frac{M_g^4 - \eta^4}{M_f^4 - \eta^4} \tag{16b}$$

It can be seen that, when $\eta < M_g < M_f$, $dH > 0$, volume of the soil element contracts; when $M_g < \eta < M_f$, $dH > 0$, volume of the soil element expands; when $M_g < \eta = M_f$, the soil element reaches its failure state. Therefore, the unified hardening parameter always remains positive in the process of hardening, and can describe both the positive and negative dilatancy of soils. Eq. (17) is then used instead of Eq. (15).

$$K = \left\{ p_a \cdot \left(\frac{H}{c_t - c_e} + \left(\frac{p_0}{p_a} \right)^m \right)^{\frac{1}{m}} \right\}^{n-\alpha} \tag{17}$$

4. ELASTOPLASTIC FORMULA OF THE PROPOSED MODEL

For elastoplastic model, the total strain can be written as the sum of elastic strain and plastic strain. Elastic strain can be achieved using the Generalized Hooke's law and two elastic parameters (i.e. elastic modulus and Poisson's ratio) need to be defined. For coarse-grained soils, the elastic modulus is expressed in Eq. (18), and the Poisson's ratio is assumed to be 0.3.

$$E = \frac{3(1-2\upsilon)p_a^m}{mC_e p^{m-1}} \tag{18}$$

According to the elastoplastic theories, stress - strain relationship can be written as

$$\{d\sigma\} = \left[C^{ep}\right]\{d\varepsilon\} \tag{19}$$

In which, elastoplastic matrix $[C^{ep}]$ is written as

$$\left[C^{ep}\right]=\left[\left[C^{e}\right]-\frac{\left[C^{e}\right]\left\{\frac{\partial g}{\partial\sigma}\right\}\left\{\frac{\partial f}{\partial\sigma}\right\}^{T}\left[C^{e}\right]}{-\frac{\partial f}{\partial H}\left\{\frac{\partial H}{\partial\varepsilon^{P}}\right\}^{T}\left\{\frac{\partial g}{\partial\sigma}\right\}+\left\{\frac{\partial f}{\partial\sigma}\right\}\left[C^{e}\right]\left\{\frac{\partial g}{\partial\sigma}\right\}}\right] \tag{20}$$

Elastic matrix $[C^e]$ can be obtained using Generalized Hooke's law. For the current model, it is easy to get the following relations

$$\frac{\partial g}{\partial\sigma_{ij}}=\frac{\partial g}{\partial p}\frac{\partial p}{\partial\sigma_{ij}}+\frac{\partial g}{\partial q}\frac{\partial q}{\partial\sigma_{ij}}=\frac{(1+\alpha)}{3}\left[M_g-\frac{q}{p}\right]\delta_{ij}+\frac{3\left(\sigma_x-p\delta_{ij}\right)}{2q} \tag{21}$$

$$\frac{\partial f}{\partial\sigma_{ij}}=\frac{\partial f}{\partial p}\frac{\partial p}{\partial\sigma_{ij}}+\frac{\partial f}{\partial q}\frac{\partial q}{\partial\sigma_{ij}}=\frac{(1+\alpha)}{3}\left[M\left(\frac{p}{p_c}\right)^{n}-\frac{q}{p}\right]\delta_{ij}+\frac{3\left(\sigma_x-p\delta_{ij}\right)}{2q} \tag{22}$$

$$\frac{\partial f}{\partial H}=\frac{\partial f}{\partial K}\frac{\partial K}{\partial H}=M\left(1+\alpha\right)\left(\frac{1}{p_c}\right)^{n}p_a^{\,n-\alpha}\frac{p^{\alpha+1}}{m\left(c_e-c_t\right)}\left(\frac{H}{c_t-c_e}+\left(\frac{p_0}{p_a}\right)^{m}\right)^{\frac{n-\alpha}{m}-1} \tag{23}$$

$$\left\{\frac{\partial H}{\partial\varepsilon_{ij}^{p}}\right\}^{T}=\frac{1}{\Omega}\left[1,1,1,0,0,0\right] \tag{24}$$

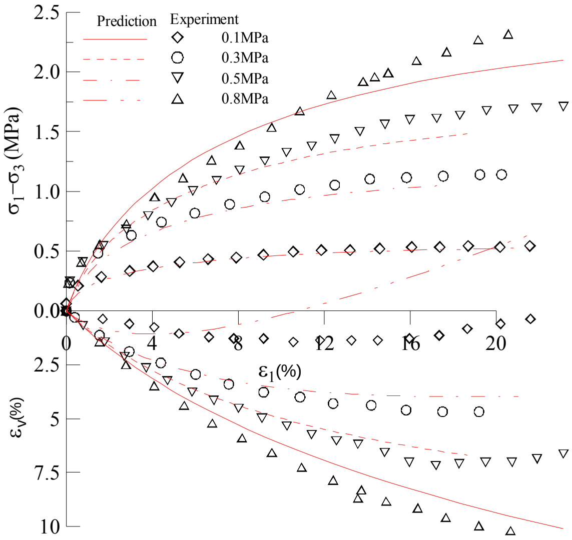
5. TEST RESULTS AND MODEL PREDICTIONS

In this model, 9 parameters are to be determined, i.e. c_t, c_e, m, M, p_c, n, α, p_0, M_g. Parameters c_t, c_e, m and p_0 can be determined in an isotropic compression test; M, p_c, n can be determined as in Fig. 10; M_g and α can be determined following the steps in Figs. 4-8. However, for coarse-grained soils, effects of "membrane penetration" may lead to a rather large error in measuring the volumetric strain; therefore, c_t, c_e, m and p_0 can also be determined via conventional triaxial tests by optimization method [24-25]. Model parameters of coarse-grained soils in this study are listed in Table. 1.

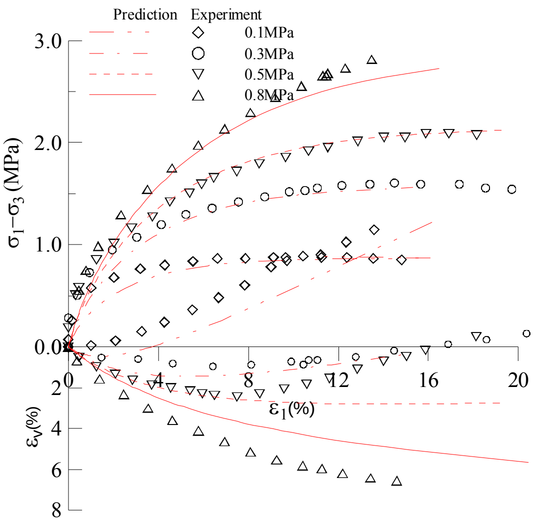
Test results and model predictions are compared in Figs. 12~14. As can be seen, the proposed model could predict the test results with good accuracy. Fig. 13 presents the comparisons between model predictions and test results in drained condition and undrained condition, as can be seen, the proposed model predicts the drained condition better than undrained condition, which may be caused by different degree of particle breakage. Fig. 14 shows the validations of the proposed model under different stress paths (i.e. conventional triaxial tests, constant mean stress tests and constant stress ratio tests) [26]. It is seen from Figs. 14(a) ~(c) that the present model gives relatively good predictions of the measured stress–strain behaviors. In particular, constant stress ratio tests results can be reasonably predicted. According to discussions in section 1, we know that constant stress ratio path is an important case in rock-fill dam engineering. Comparing the model in this study with the model proposed by Yao *et al.* [7], we find that, the model in the present study gives better predictions when specimens are along the constant stress ratio stress path. Fig. 15 shows the stress-strain curves predicted using crushing model (proposed by Yao *et al.* [7]); obviously, results in Fig. 14 are better.

Table.1 Model parameters of the proposed model

	c_t	c_e	m	M	p_c (kPa)	n	P_0	α	M_g
Shale rock-fills A	0.0044	0.0016	0.65	1.499	1.57e3	0.148	100	0.138	1.619
Shale rock-fills B	0.0046	0.0012	0.50	1.677	1.57e3	0.205	100	0.016	1.656
Diorite rock-fills	0.0038	0.0021	0.40	1.695	2.58e3	0.076	100	-0.217	1.780
Slightly Weathered Granite rock-fills	0.0047	0.001	0.70	1.547	3.80e3	0.11	100	0.400	1.652

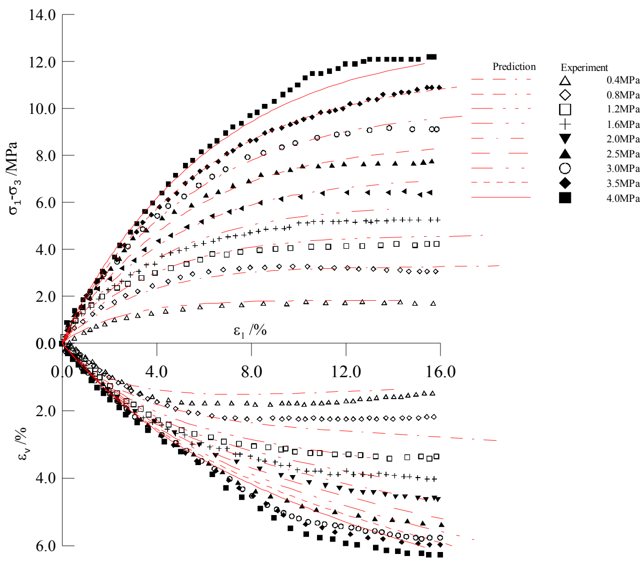


(a) Shale rock-fills (sample A)

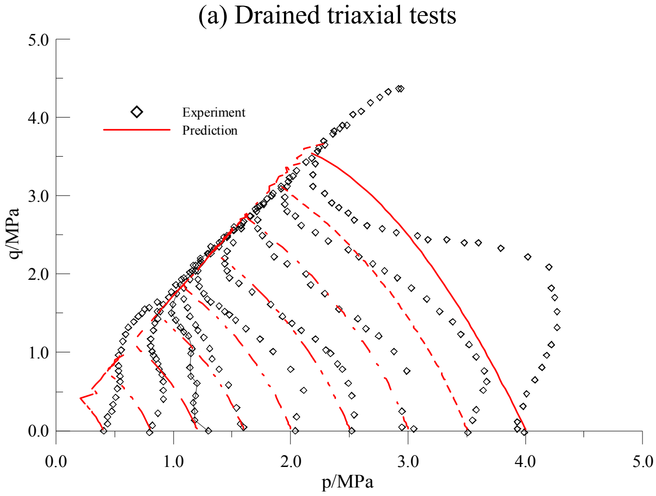


(b) Shale rock-fills (sample B)

Fig. 12. Comparison of experimental data with model predictions [17]

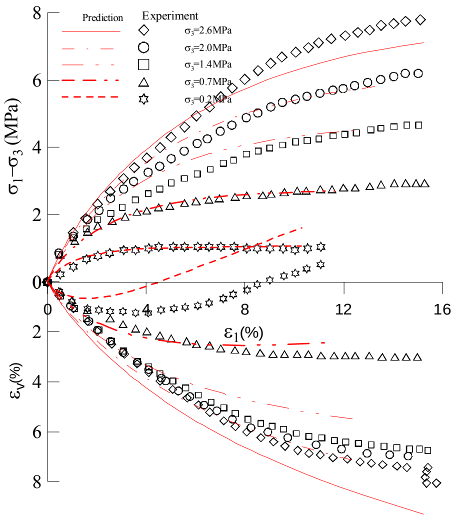


(a) Drained triaxial tests

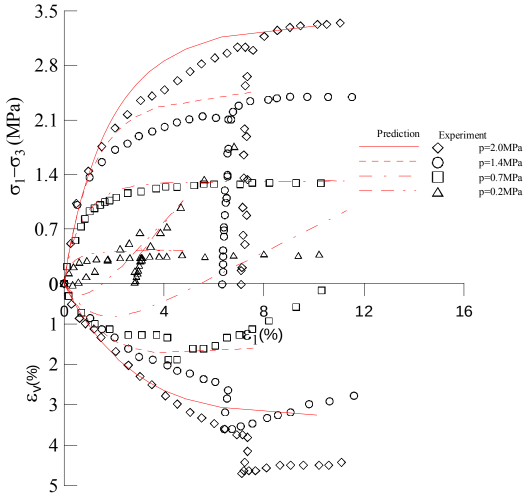


(b) Undrained triaxial tests

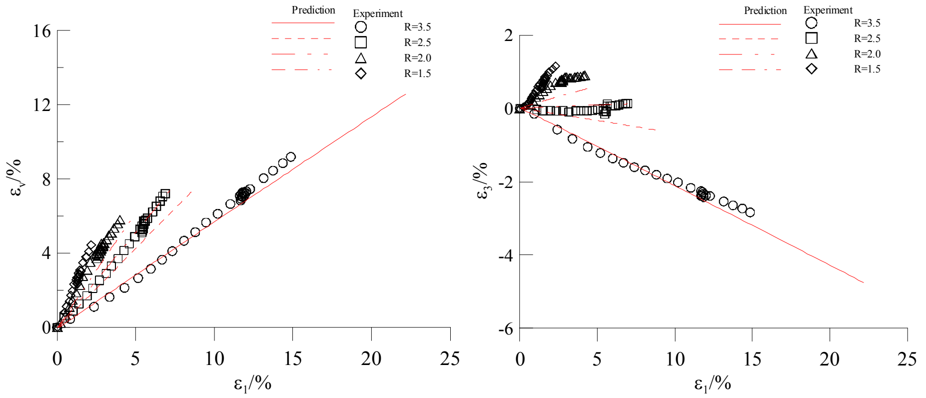
Fig. 13. Comparison of experimental data with model predictions for Diorite rock-fills [12]



(a) Conventional triaxial tests



(b) Constant p tests



(c) Constant stress ratio tests ($R = \sigma_1/\sigma_3$)

Fig. 14. Comparison of experimental data with model predictions for Slightly Weathered Granite rock-fills [26]

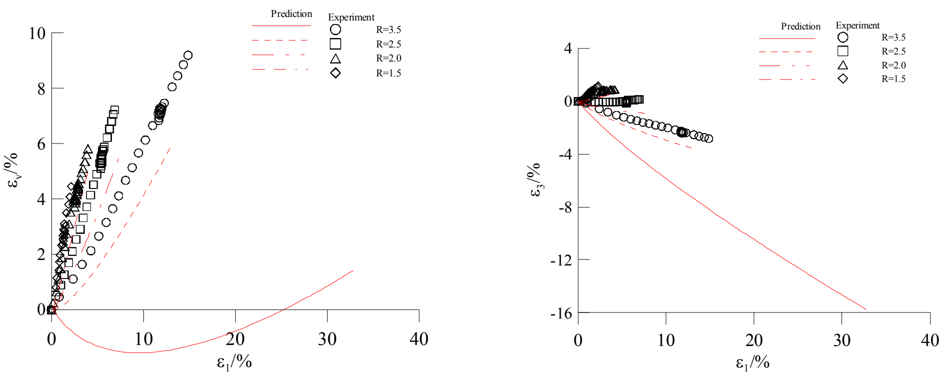


Fig. 15. Predictions of the crushing model (proposed by Yao *et al.*, 2008) under constant stress ratio stress path

Therefore, it can be seen from the above comparisons that the proposed model can reasonably describe the stress–strain characteristics of coarse-grained soils along various stress paths.

6. CONCLUSION

This paper introduced a new elastoplastic model for coarse-grained soils; the dilatancy equation was established based on triaxial test results. Plastic potential surface was obtained by solving the dilatancy equation. Expression of the yield surface was developed considering the influence of the mean stress and stress ratio. A unified hardening parameter was adopted so that dilatancy properties of coarse-grained soils can be described in the whole process of deformation. Elastoplastic formula of the proposed model was presented in detail. Nine model parameters could all be determined via conventional triaxial tests. The advantage of this model is that soil behaviors under different stress paths can be predicted. In particular, constant stress ratio path can also be reasonably predicted.

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APPENDIX I : DERIVATION OF EXPRESSION OF YIELD SURFACE

According to Eq. (10), assume the yield surface has the following expression

$$F = q - f(p) \tag{25}$$

Then

$$\frac{\partial F}{\partial q} = 1 \tag{26a}$$

$$\frac{\partial F}{\partial p} = -\frac{\partial}{\partial p} f(p) = (1+\alpha)\left(M\left(\frac{p}{p_c}\right)^n - \eta \right) \tag{27b}$$

From Eq. (27b), it is easy to get the following form.

$$\frac{df(p)}{dp} = -(1+\alpha)M\left(\frac{p}{p_c}\right)^n + (1+\alpha)\frac{f(p)}{p} \tag{28}$$

As can be seen Eq. (28) is an ordinary differential equation, and its solution is written as

$$f(p) = e^{\int \frac{(1+\alpha)}{p}dp}\left[\int -(1+\alpha)M\left(\frac{p}{p_c}\right)^n e^{\int -\frac{(1+\alpha)}{p}dp} dp + C\right] \tag{29}$$

In which C is a constant. Further, we can get the following expression.

$$f(p) = M(1+\alpha)\left(\frac{1}{p_c}\right)^n \frac{p^{n+1}}{\alpha - n} + Cp^{(1+\alpha)} \tag{30}$$

And

$$F = q - M(1+\alpha)\left(\frac{1}{p_c}\right)^n \frac{p^{n+1}}{\alpha - n} - Cp^{(1+\alpha)} \tag{31}$$

In an isotropic compression test; $p = p_x$ and $q = 0$, then constant C can be determined as follows.

$$C = -M(1+\alpha)\left(\frac{1}{p_c}\right)^n \frac{p_x^{\,n-\alpha}}{\alpha - n} \tag{32}$$

By substituting Eq. (32) into Eq. (31), we get the expression of the yield surface.

$$F = q - M(1+\alpha)\left(\frac{1}{p_c}\right)^n \frac{p}{\alpha - n}\left[p^n - \left(\frac{p}{p_x}\right)^\alpha p_x^{\,n}\right] \tag{33}$$