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INFLUENCE OF THE SCALE EFFECT ON THE MECHANICAL PARAMETERS OF COARSE-GRAINED SOILS*

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Abstract– The mechanical parameters of coarse-grained soils are often obtained via indoor or field tests. In these tests, it is necessary to reduce the particle size of the original graded soils due to the size limitation of the testing apparatus. Therefore, several scale methods (e.g., the equivalent substitute method and parallel gradation method) have been proposed to reduce the size of the original graded soils to the proper testing size. However, the mechanical parameters will be different if different scale methods are adopted, a phenomenon that has been termed the "scale effect". In this research, a group of large-scale oedometer tests were conducted with specimens that were downsized using different scale methods. The results show that (1) when adopting the same scale methods, the compression modulus increases with the increase in the nominal maximum particle size. (2) For the same nominal maximum particle size, samples that are downsized using the equivalent substitute method have a higher compression modulus and degree of particle breakage than those adopting the parallel gradation method. (3) Then Duncan-Chang *E-B* model parameters were back calculated from oedometer tests using an immune genetic algorithm (IGA). These mechanical parameters were also used in a three-dimensional (3D) finite element method analysis of the Pankou Rock-fill Dam. The manner in which the scale method influences rock-fill dam deformation is also discussed.

Keywords– Oedometer test, scale effect, scale method, nominal maximum particle size, IGA

1. INTRODUCTION

In recent years, coarse-grained soils have enjoyed increasing use in dam construction, offshore structures, airports, and road foundations, among other applications. These soils are large, angular, and granular rock materials blasted from parent rock. The particle size of coarse-grained soils can reach 0.8-1.0 m, and can even exceed 1.2 m in some cases. In conventional tests, the maximum particle size allowed is only limited to 0.06 m due to the size limitation of the testing apparatus [1]. With the development of large-scale testing apparatuses, the allowed maximum particle size can reach 0.2-0.3 m; however, it is still not feasible to test the original graded coarse soils directly. Therefore, several scale methods (e.g., the equivalent substitution method, parallel gradation method, and combination method) have been proposed to reduce the size of the original graded soils to the proper testing size. However, the mechanical parameters will be different if different scale methods are adopted; this phenomenon has been termed the "scale effect".

In the 1940s, Bishop and Henkel [2] began to address the scale effect in coarse-grained soils; Marachi et al. [3] studied the influence of the scale effect on the strength parameters of coarse-grained soils,

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sourced from the Oroville Dam. Li et al. [4] studied the parallel gradation method and deduced a correction factor to extend the test results from downsized soils to the original graded soils.

In this study, the scale effect of coarse-grained soils was investigated by performing large-scale field oedometer tests. The tests were conducted during the construction of the Pankou Concrete-Faced Rock-fill Dam (CFRD). Specimens were prepared using different scale methods, and the test results obtained for these specimens were compared. An immune genetic algorithm (IGA) was adopted to back calculate the Duncan-Chang *E-B* model parameters from the oedometer tests. These parameters were also used in a three-dimensional (3D) finite element analysis of the Pankou Rock-fill Dam; within this context, the influence of the scale effect on the deformation of a rock-fill dam is discussed.

2. LARGE-SCALE FIELD OEDOMETER TEST

a) Brief introduction to basic scale methods

The grain size characteristics of coarse-grained soils are evaluated by sieve analysis. A nest of sieves is prepared by stacking test sieves on top of one another, with the largest opening at the top followed by sieves of successively smaller openings. A sample of dry soil is poured into the top sieve and shaken until each particle has dropped to a sieve with an opening too small to pass through, and the particles are retained. Then, the mass of soil retained on each sieve can be weighed; the percent retained in each sieve is calculated by dividing this mass by the total specimen weight.

Original graded coarse soils cannot be tested directly due to the limited apparatus size. Thus, several scale methods have been proposed to reduce the particle size of soils. The following are the three most commonly used scale methods [5].

Equivalent substitution method. The percent mass of a specimen retained on sieve *i* is calculated using the following formula:

$$P_i = \frac{P_{oi}}{P_5 - P_{d\max}} P_5 \tag{1}$$

where P_{i} is the percent mass of a specimen retained on sieve *i*; P_{oi} is the percent mass of the original graded soil retained on sieve *i*; P_5 is the cumulative percent mass of the original graded soil retained on 5-mm sieve; and P_{dmax} is the percentage of particles coarser than the maximum permissible particle size in the original graded soil.

This method keeps the percentage of particles finer than 5 mm unchanged, although the shape of the particle-size distribution curve changes.

Parallel gradation method. The percentage of particles finer than *d/n* in a specimen can be obtained using the following formula:

$$p_{d/n} = p_{do} \tag{2}$$

where P_{do} is the percentage of particles finer than *d* in the original graded soils and *n* is the scale coefficient, defined by Eq. (3).

$$n = d_{\max o} / d_{\max e} \tag{3}$$

where d_{maxo} is the maximum particle size of the original graded soils and d_{maxe} is the maximum permissible particle size. Through this method, the particle-size distribution curves of the samples and original graded soils are exactly parallel, and the coefficient of uniformity C_{u} and coefficient of curvature C_{c} are constant.

Combination method. This method combines the equivalent substitution method and parallel gradation method. First, the parallel gradation method is used to reduce the percentage of oversized particles to below 40%, and then, the equivalent substitution method is used to reduce to the maximum permissible size.

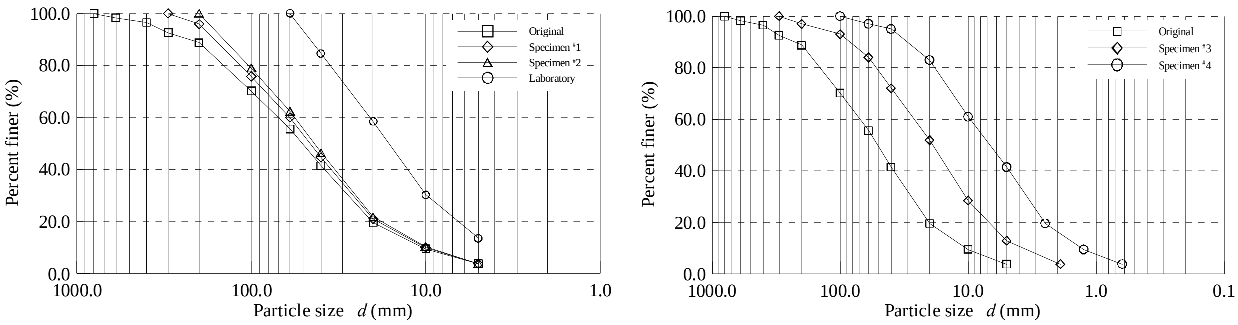
b) Physical characteristics of the testing soils

The coarse-grained soils used in the oedometer tests were sourced from the primary rock-fill of the Pankou CFRD. First, the coarse soils were roller compacted in a testing field, as shown in Fig. 1 (a) and (b); then, these coarse-grained soils were sieved into several fractions, as shown in Fig. 1 (c). Preliminary tests were carried out to identify the basic properties of the intact rock. The parent rock of the test rock-fill was silicalite, containing 75% quartz, 20% graphite, 2% muscovite, 2% iron objects, and 1% trace substances. The dry density of the parent rock was 2.60g/cm^3, and its uniaxial compressive strength was 117.0 MPa in the dry state.

Testing field (b) Coarse soils after roller compaction (c) Sieving into several fractions

Fig. 1. Testing soils used in the oedometer tests

Four specimens downsized using different scale methods were tested. The particle-size distribution curve of each sample is plotted in Fig. 2. The properties of each sample are shown in Table 1. The mechanical behaviors of the coarse soils are closely related to their dry densities [6]; in all tests, the initial dry density of each soil was 2.06 g/cm^3.



(a) Equivalent substitution method (b) Parallel gradation method

Fig. 2. Particle-size distribution curves of original graded soils and specimens

Table1. Properties of the specimens used in oedometer tests

	ρ_d (g/cm^3)	d_{max} (mm)	e_0	C_u	C_c	Scale method	Remarks
Specimen 1	2.06	300.0	0.267	5.94	2.04	*Equivalent substitution*	*ρ_d= dry density d_{max} = maximum particle size e_0 = initial void ratio C_u = coefficient of uniformity C_c = coefficient of curvature
Specimen 2	2.06	200.0	0.267	5.82	1.31	*Equivalent substitution*	
Specimen 3	2.06	300.0	0.267	6.19	1.01	*Parallel gradation*	
Specimen 4	2.06	100.0	0.267	6.19	1.01	*Parallel gradation*	
Laboratory	2.06	60.0	0.267	5.17	1.20	*Combination method*	

c) Apparatus and procedures of the oedometer tests

The oedometer tests were conducted in a testing tunnel. The testing tunnel measured 12 m in length with a cross section of 2 m × 2 m (width × height). A cylindrical container measuring 0.65 m in height and 1.33 m in diameter was placed on the floor of the testing tunnel. The surroundings of the container were strengthened with reinforced concrete to prevent lateral deformation of the sample. Four hydraulic jacks were used to exert a vertical force. Displacement meters were installed to record the vertical displacement of the samples (Figs. 3 and 4). Coarse aggregates were compacted into the container so that the samples reached the desired dry density of 2.06 g/cm^3.

(a) Filling of the container (b) Testing specimen

Fig. 3. Oedometer test specimen

(a) Compression of soils (b) Soils after testing

Fig. 4. Compression of the coarse-grained soils

<u>*Compression modulus*</u>

Curves of vertical stress versus settlement are shown in Fig. 5. The figure clearly shows that the settlement of specimen 4 is the largest, while that of specimen 1 is the smallest. By comparing specimen 1 with specimen 2 or specimen 3 with specimen 4, it can be seen that when the same scale method is adopted, settlement increases with the decrease in d_{max}. Even when adopting the other scale methods, the settlement also tends to increase with the decrease in d_{max}. As shown by the results, d_{max} plays an important role in the behavior of the coarse soils. By comparing specimen 3 with specimen 1, it can be seen that the settlement of the equivalent substitution method is smaller than that obtained using the parallel gradation method, but the difference is relatively small when d_{max} values of the specimen are both 0.3 m. Specimen #4 experiences relatively greater deformation than the other three groups, which implies that the soil properties are closely related to d_{max}, particularly when the scale coefficient n is large.

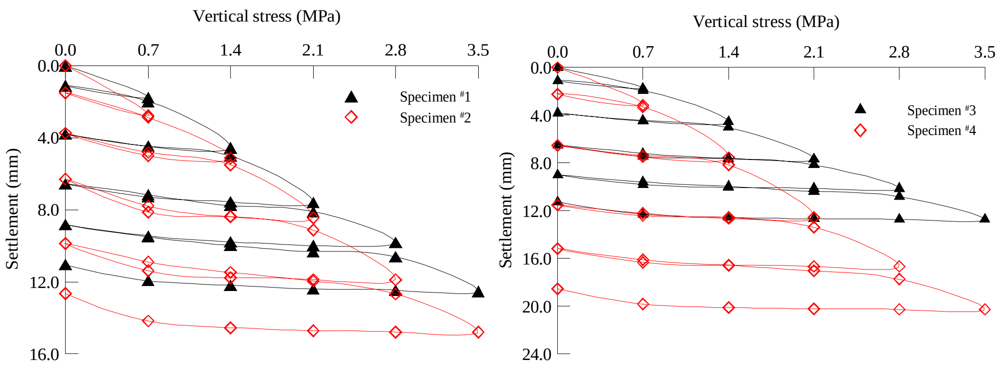


Fig. 5. Relationship between vertical stress and settlement during the oedometer testing

Table 2. Compression modulus in field tests

	Compression modulus (MPa)					Reloading modulus (MPa)
	0 - 0.7	0.7 - 1.4	1.4 - 2.1	2.1 - 2.8	2.8 - 3.5	
Specimen 1	211.2	140.8	126.7	181.0	140.8	833.8
Specimen 2	140.8	158.4	115.2	115.2	126.7	586.0
Specimen 3	211.2	140.8	126.7	158.4	140.8	785.4
Specimen 4	115.2	90.5	79.2	97.5	105.6	609.5

For soils, the compression modulus E_s determined by the oedometer test is defined as follows:

$$E_s = \frac{(1+e_1)(p_2 - p_1)}{e_1 - e_2} \tag{4}$$

where e_1 is the initial void ratio corresponding to vertical stress p_1 and e_2 is the void ratio corresponding to vertical stress p_2 after an stress increment $\Delta p = p_2 - p_1$. Therefore, the compression modulus is stress level dependent. The compression modulus and reloading modulus of each sample are listed in Table 2.

As shown, the equivalent substitute method yields a higher compression modulus than the parallel gradation method for equal d_{max} values. Thus, the equivalent substitute method may compensate for the decrease in particle size. From this perspective, samples downsized by the equivalent substitute method may exhibit mechanical behavior similar to that of the original graded soils. According to Table 2, the reloading modulus is also related to d_{max}; when using the parallel gradation method, the reloading modulus increases by 28.9% when d_{max} increases from 0.1 m to 0.3 m. By comparing specimen 2 with specimen 3, it can be seen that although d_{max} of specimen 2 is greater than that of specimen 4, its reloading modulus is smaller; therefore, the scale method may have a greater impact on the reloading modulus than d_{max}.

Particle breakage

Terzaghi [7] suggested that the deformation of rock-fill may be caused by the breakage of rock particles. Particle breakage typically occurs in the vicinity of high contact stress. This breakage could lead to the rearrangement of the granular structure of specimens. Particle breakage is determined by many factors, including particle shape, size, density, and strength of the parent rock. Marsal [8] and Hardin [9] proposed different methods to evaluate the degree of particle breakage. In the present study, Marsal's method was used, and the particle breakage index B_g was defined as the sum of the positive values of W_k (Eq.5).

$$B_g = \sum \Delta W_k, \quad \Delta W_k = \begin{cases} W_{ki} - W_{kf} & if \quad W_{ki} - W_{kf} \geq 0 \\ 0 & if \quad W_{ki} - W_{kf} < 0 \end{cases} \tag{5}$$

where W_{ki} is the percent retained on sieve k before testing and W_{kf} is the percent retained on sieve k after testing. The particle-size distribution curves are plotted in Fig. 6.

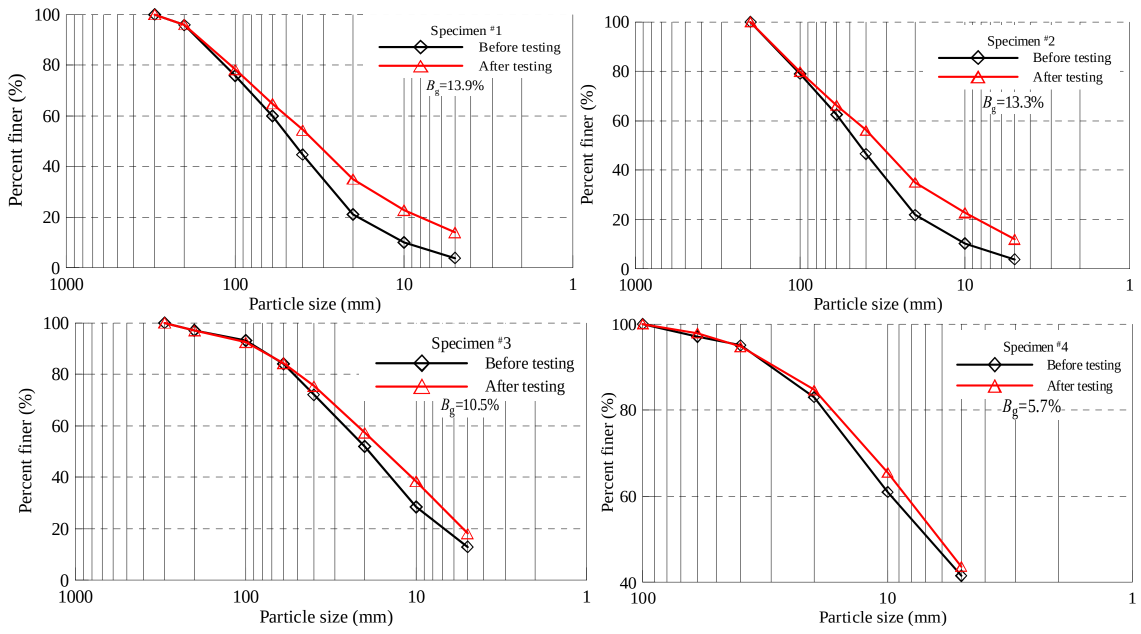


Fig. 6. Particle-size distribution curves before and after testing

By comparing specimen 3 with specimen 4, we find that B_g is strongly affected by the maximum particle size d_{max} in the parallel gradation method. B_g increases with the increase in d_{max}. When d_{max} is 0.1 m, particle breakage is negligible. By comparing specimen 1 with specimen 2, it can be observed that B_g is not strongly affected by d_{max}; B_g increases by only 0.6% when d_{max} increases from 0.2 m to 0.3 m.

As is known, particle breakage typically occurs in the vicinity of high stress contacts. Thus, the greater the number of contact points, the lower the degree of particle breakage. According to the particle-size distribution curves, the specimen downsized using the parallel gradation method exhibited a greater number of contact points than the specimen downsized using the equivalent substitution method; this phenomenon is more pronounced for small d_{max} values, which explains why the equivalent substitution method induces a higher degree of particle breakage than the parallel gradation method.

3. FINITE ELEMENT ANALYSIS OF THE PANKOU DAM WITH BACK-ANALYZED PARAMETERS

a) Back analyzed Duncan-Chang E-B model parameters from oedometer tests

Duncan-Chang E-B model

In the Duncan-Chang *E-B* model [10], tangent modulus E_t, tangent bulk modulus B_t, reloading modulus E_{ur} and internal friction angle φ are respectively expressed as follows:

$$E_t = Kp_a\left(\frac{\sigma_3}{p_a}\right)^n\left[1-\frac{R_f\left(1-\sin\phi\right)\left(\sigma_1-\sigma_3\right)}{2C\cdot\cos\phi+2\sigma_3\sin\phi}\right]^2 \tag{6a}$$

$$B_t = K_b p_a\left(\frac{\sigma_3}{p_a}\right)^m \tag{6b}$$

$$E_{ur} = k_{ur} p_a(\frac{\sigma_3}{p_a})^{n_{ur}} \tag{6c}$$

$$\varphi = \varphi_0 - \Delta\varphi\log(\frac{\sigma_3}{p_a}) \tag{6c}$$

where K, n, R_f, K_b, m, C, φ, k_u and n_{ur} are material parameters; σ_1 and σ_3 are the first and third principal stresses, respectively, p_a is the atmospheric pressure; and φ_0 and $\Delta\varphi$ are strength parameters.

Brief introduction to IGA

The IGA is a stochastic optimization method [11-12] that is suitable for a variety of nonlinear problems [13-14]. In the present study, the IGA method was used together with the finite element method to back calculate the *E-B* model parameters from the oedometer tests [15].

The following was selected as the objective function:

$$F(X) = \sum_{i=1}^{m}\sum_{j=1}^{n}\omega_{ij}(u_{ij}^c / u_{ij}^m - 1.0)^2 \tag{7}$$

The objective function was used to measure the error between calculated displacement and displacement measured by the oedometer test. The IGA was used to search for a group of parameters to make the corresponding calculated and measured settlements best approximate to one another. In Eq. (7), m is the number of load steps; n is the number of monitoring points; u_{ij}^c is the calculated displacement of monitoring point j at the ith load step; u_{ij}^m is the measured displacement of monitoring point j at the ith load step; ω_{ij} is the weight of monitoring point j at the ith load step. The parameters were back analyzed in two steps. First, five loading parameters were back analyzed. Second, two reloading parameters were back analyzed. In each step, these unknown variables could be written as a row vector. The strength of the coarse soils was assumed to have not been affected by the scale method used, based on the results of Marachi *et al* [3].

$$\begin{cases} \{X\} = \{K, n, R_f, K_b, m\} & \text{Variables in the first step} \\ \{X\} = \{k_{ur}, n_{ur}\} & \text{Variables in the second step} \end{cases} \tag{8}$$

Back analyzed results and discussions

The change in the parameters of specimen #1 during the optimization process is shown in Fig. 7. The measured and back-analyzed curves of specimen #1 are plotted in Fig. 8, the results show that the calculated and measured curves are rather consistent. Table 3 presents the *E-B* model parameters that were back analyzed from the four specimens.

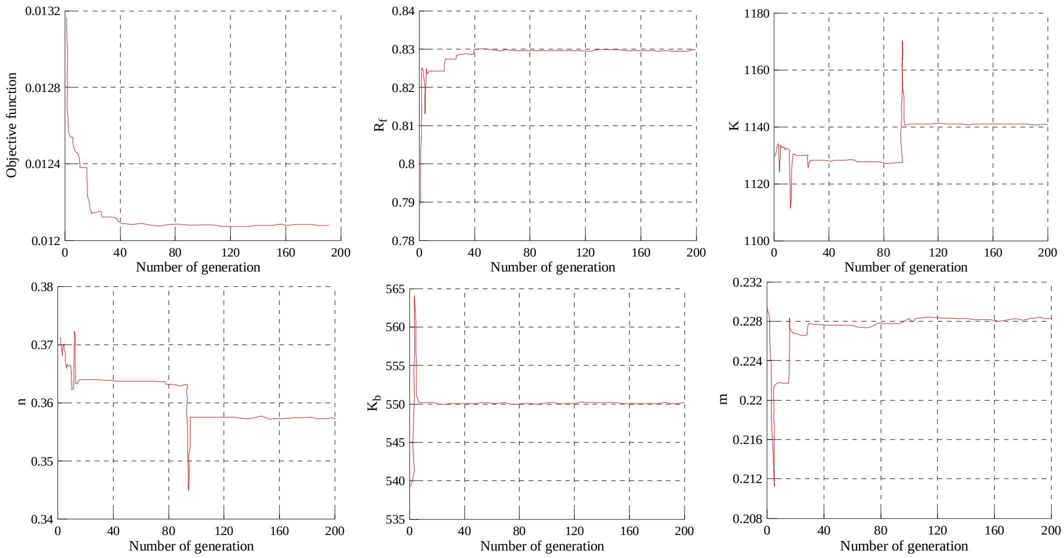


Fig. 7. Change in parameters during optimization (specimen 1)

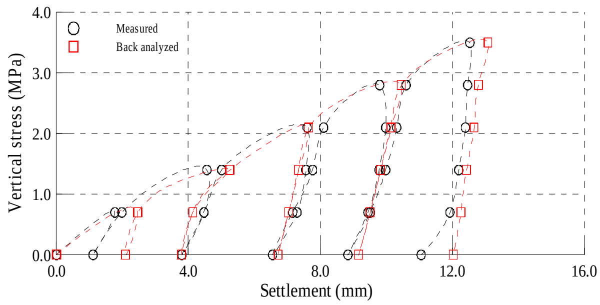


Fig. 8. Curves of the measured and back-analyzed displacement (specimen 1)

Table 3. Duncan-Chang *E-B* model parameters back analyzed from oedometer tests results

	$\varphi_0(°)$	$\Delta\varphi(°)$	R_f	K	n	K_b	m	k_{ur}	n_{ur}
Specimen 1	51.0	8.1	0.830	1141	0.35	550	0.23	2736	0.69
Specimen 2	51.0	8.1	0.770	951	0.25	419	0.13	2353	0.53
Specimen 3	51.0	8.1	0.790	1079	0.39	486	0.13	2369	0.69
Specimen 4	51.0	8.1	0.760	902	0.33	356	0.17	2964	0.69
Indoor test	51.0	8.1	0.820	930	0.26	442	0.11	/	/

b) Three-dimensional finite element analysis of the Pankou CFRD

<u>Brief introduction to Pankou CFRD</u>

The Pankou Dam, located in Hubei Province in China, stands 109 m tall. The typical zoning of the Pankou CFRD is shown in Fig. 9. Finite element meshes of the dam are shown in Fig. 10. In this study, Duncan-Chang *E-B* model parameters of primary rock-fill were obtained from oedometer tests; the parameters of other materials are listed in Table 4.

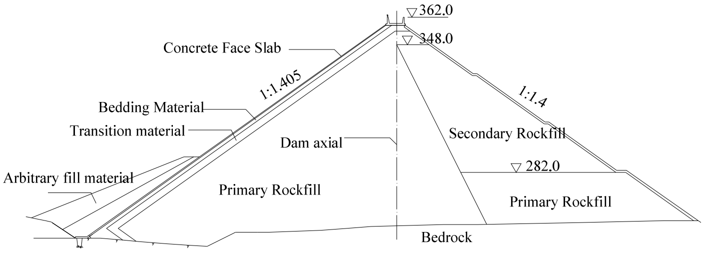


Fig. 9. Typical zoning of the Pankou CFRD

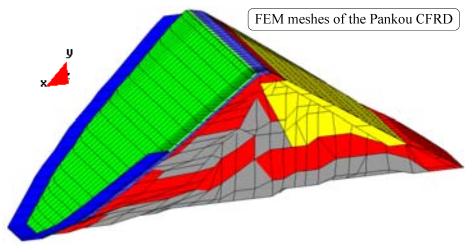


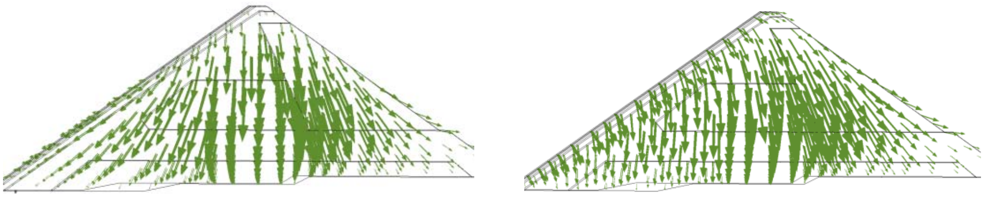
Fig. 10. Finite element meshes of the Pankou CFRD

Table 4. Duncan-Chang *E-B* model parameters of the Pankou CFRD

	φ_0(°)	$\Delta\varphi$(°)	R_f	K	n	K_b	m	k_{ur}	n_{ur}
Bedding material	51.0	8.1	0.830	1141	0.35	550	0.22	2736	0.69
Transition material	51.0	8.1	0.770	951	0.25	419	0.13	2353	0.53
Secondary rock-fill	51.0	8.1	0.790	1079	0.39	486	0.13	2369	0.69
Primary rock-fill	Primary rock-fill parameters were back analyzed from the oedometer test results in Table 3								

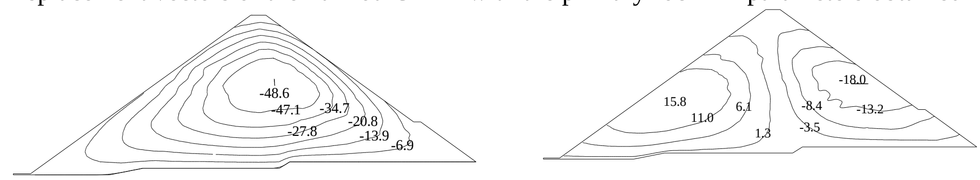
<u>Displacement of Pankou dam using parameters back analyzed from oedometer test and indoor test</u>

The displacement vectors and contours of the Pankou CFRD are shown in Figs. 11 and 12, respectively (primary rock-fill parameters obtained from specimen #1). Figs. 12a and b show the vertical and horizontal displacements upon dam completion, respectively. Figs. 12c, d and e show the vertical displacement, horizontal displacement, and deflection of the concrete slab during the full storage period, respectively. The displacement extrema of each group are listed in Table 5.

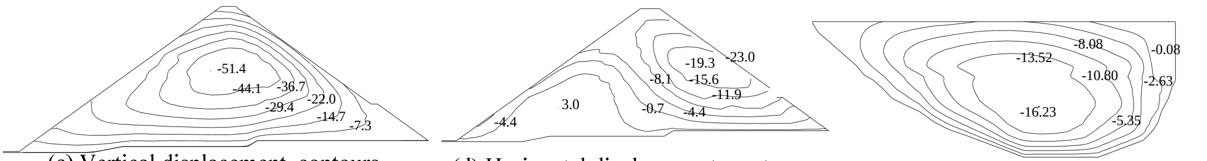


(a) At completion (b) During the full storage period

Fig. 11. Displacement vectors of the Pankou CFRD with the primary rock-fill parameters obtained from specimen 1



(a) Vertical displacement contours at completion (b) Horizontal displacement contours at completion



(c) Vertical displacement contours during the storage period (d) Horizontal displacement contours during the storage period (d) Deflection contours of Concrete Face Slab during the storage period

Fig. 12. Displacement contours of the Pankou CFRD with the primary rock-fill parameters obtained from specimen 1 (unit: cm)

Table 5. Displacement extrema of the Pankou CFRD with different groups of parameters

	Completion period			Storage period				Remarks
	V	*LU*	*LD*	*V*	*LU*	*LD*	*D*	
Specimen 1	-48.6	15.8	-18.0	-51.42	3.36	-23.06	-23.00	*V= vertical displacement*
Specimen 2	-62.5	18.7	-18.2	-68.96	2.76	-24.38	-22.11	*LU= upstream lateral displacement*
Specimen 3	-53.5	14.7	-17.1	-57.96	1.99	-21.99	-18.34	*LD= downstream lateral displacement*
Specimen 4	-69.4	18.5	-18.3	-75.76	2.57	-24.30	-23.34	*D=deflection of the concrete slab*
Laboratory	-63.0	19.8	-19.2	-68.16	3.57	-25.40	-21.71	(unit: cm)

Discussion

According to Table 5, we conclude that (1) The fourth group of parameters shows the largest vertical displacement, while the first group shows the smallest, which is consistent with the fact that specimen $^{\#}4$ exhibits the greatest compression, while specimen $^{\#}1$ exhibits the least; (2) The vertical displacement of the second group of parameters is greater than that of the first group. This finding implies that higher d_{max} values induce less settlement. The same tendency can be observed by comparing the results of the third and fourth groups of parameters; (3) The first group of parameters shows less settlement than the third group, which implies that the *equivalent substitution method* leads to lower compression than the *parallel gradation method* in the rock-fill dam; (4) By comparing the field parameters with indoor parameters, it can be seen that the dam settlement associated with the field parameters is generally lower than that associated with the indoor parameters. However, the settlement indicated by the fourth group of parameters is even lower than that indicated by the indoor parameters. This finding suggests that when d_{max} is small, the *parallel gradation method* may overestimate dam settlement; (5) The lateral displacement of the indoor test parameters is the largest.

4. CONCLUSION

In this study, original graded coarse-grained soils were downsized using different scale methods. A series of large-scale oedometer tests were conducted to study the influence of the scale effect on coarse-soil behavior.

The oedometer test results show that (1) For the same scale method, the compression modulus increases with d_{max}; (2) When the maximum particle sizes are equal, the compression modulus generated by the *equivalent substitute method* is larger than that generated by the *parallel gradation method;* (3) The particle breakage induced by the *equivalent substitute method* is greater than that induced by the *parallel gradation method;* (4) The reloading modulus is not directly related to the particle size or scale method used, which may be due to the effects of dry density and breakage; (5) The Duncan-Chang *E-B* model parameters were back analyzed from the results of the oedometer tests, which were also adopted in the 3D finite element analysis of the Pankou CFRD. The displacements calculated by these different groups of model parameters were compared. The results show that the vertical displacements calculated using the oedometer test parameters tend to be smaller than those calculated using the indoor test parameters. However, for small d_{max}, the *parallel gradation method* may lead to rather large deformations.

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