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PRODUCTION OF ENDURANCE TIME EXCITATION FUNCTIONS: THE CMA EVOLUTION STRATEGY APPROACH*

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Abstract– Endurance Time (ET) method is a recently developed response-history based analysis procedure for seismic assessment and structural design in which structures are subjected to a predesigned intensifying excitation function, and their performance is evaluated based on their response at different excitation levels. Generating efficient excitation functions, which is essential for functionality of the method, leads to a complex large-scale optimization problem. In this paper, the Covariance Matrix Adaptation Evolution Strategy (CMA-ES), which has found many applications in solving continuous optimization problems, is employed to produce the excitation functions. The results reveal the good performance of the algorithm in generating ET excitation functions (ETEF) with reasonable accuracy and time efficiency.

Keywords– Endurance time method, seismic response history analysis, intensifying excitation function, the covariance matrix adaptation evolution strategy

1. INTRODUCTION

In earthquake engineering, the need for developing advanced and more efficient structural analysis tools seems to be neverending. This need is intensified by two major factors. Firstly, the increasing demands for building more complex structures due to modern architectural or functional requirements. Secondly, the growing tendency in utilizing the progressive seismic mitigation technologies. Various limitations of conventional seismic analysis procedures, as well as remarkable advances in the field of computational technology, have motivated researchers to develop more reliable and robust methods for optimal seismic design of structures [1]. Time-history based procedures are prevalent analytical methods, since in these procedures, nearly all sorts of complicated material and geometry can be directly included in the analysis. Though the model complexity is not considered as an obstacle (at least theoretically), extensive computational demand has prohibited the widespread application of such analyses in practice [2].

A comparison of the options and limitations of available analysis procedures implies that nonlinear response-history based procedures are the future approaches, despite their complexity and considerable computational demand. A response-history based analysis is the only method that makes it possible to incorporate nearly all sources of nonlinear and time dependent effects directly in the analysis [1]. Endurance Time method is a response-history based procedure that aims to improve on complexity and computational demand of this class of analysis tools.

The concept of the endurance time is quite straightforward and is similar to the well-known exercise test in medicine; subjecting the system to an increasing demand and monitoring its response stage-by-stage. In ET method, structures are subjected to a specially designed intensifying excitation function and

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their seismic performance is evaluated based on the time duration needed to satisfy the required design objective. Maximum drift or displacement, maximum stress ratio or any other desired parameter or damage can be selected as the design objectives [1]. During recent years, this basic concept has evolved into a working procedure that offers three major benefits:

- Significant reduction of the computational time required for a realistic estimation of the seismic response at multiple intensity levels.
- Simplicity and sensibility of the concept for engineering applications.
- Applicability to nearly all types of structures and dynamic systems regardless of their complexity.

The basic concepts of the method were developed in [3]. After successful production of the second generation of ETEFs, its application considering linear behavior was extended by [4]. A preliminary study of nonlinear analysis and predicting various damage indexes were published in [5]. Nonlinear analysis of SDOF systems considering different material models was performed in [6]. This was followed by analysis of MDOFs as described in [7]. The procedure has recently been extended to multi-component analysis [8, 9].

Generating efficient dynamic excitations is essential for success of the ET method. An input function is considered to be efficient if the results estimated by an ET analysis are consistent with the response of the different structures under real earthquakes. The excitation functions currently applied in ET method have two specific properties: (I) these functions are intensifying as their amplitude increases with time, (II) these functions are optimized such that the response spectrum of any segment from t = 0 to $t = t_1$ is proportional to a template response spectrum with a scale factor that linearly increases with time [2]. Generating ETEFs with these properties leads to a complex large-scale optimization problem [10]. This is where the heuristic algorithms, which have been successfully applied to solve a wide range of engineering problems [11-14], come into the scene.

The meta-heuristic developed in this paper belongs to a class of Evolution Strategies. In the nineteenth century, Mendel was the first to state the preliminary concepts of heredity from parents to offsprings [15]. Then in 1859, Darwin presented the theory of evolution in his famous book *On the Origin of Species* [16]. In the 1980s, these theories of creation of new species and their evolution have inspired computer scientists in designing Evolutionary Algorithms (EA). Different types of EAs have evolved independently during the past 40 years: Genetic Algorithms [17], Evolution Strategies [18], Evolutionary Programming [19], and Genetic Programming [20]. Each of these constitutes a different approach; however, they are inspired by the same principles of natural evolution. EAs are the most studied population-based metaheuristics and this has promoted the field known as Evolutionary Computation [15].

The CMA-ES is a stochastic method for continuous optimization of non-linear, non-convex problems, which was first introduced by Hansen et al. [21]. In an ES, new candidate solutions are sampled according to a multivariate normal distribution. Pair-wise dependencies between the variables in this distribution are described by a covariance matrix. The CMA is a method to update the covariance matrix of this distribution. The CMA-ES is a second-order optimization approach, where only the ranking between candidate solutions is exploited for learning the sample distribution, and neither derivatives nor even the objective function values are required by the method. This makes the method feasible on ill-conditioned and non-continuous, as well as on multimodal or noisy problems [15].

After this opening section, the paper is organized as follows: Section 2 explains the concept of ET method. In section 3, characteristics of ETEFs and the objective function of the optimization problem are presented. Section 4 introduces the CMA-ES. Production of ETEFs and related discussions are provided in section 5 and finally the paper is concluded in section 6.

2. CONCEPT OF THE ET METHOD

In order to explain the concept of endurance time, consider a hypothetical test in which the prototypes of three design alternatives for a structure are placed on a shaking table and subjected to an intensifying excitation until their complete failure. Based on the order that these structures fail, one can comparatively categorize them as the worst, average and the best performer. Fig. 1 indicates a numerical presentation of what happens in the test. If the variations of appropriate design parameters are monitored, then it can be readily seen which design works better and approximately by how far. It should be mentioned that these increasing demand curves represent the maximum absolute value of the response from the start up to a particular time [1].

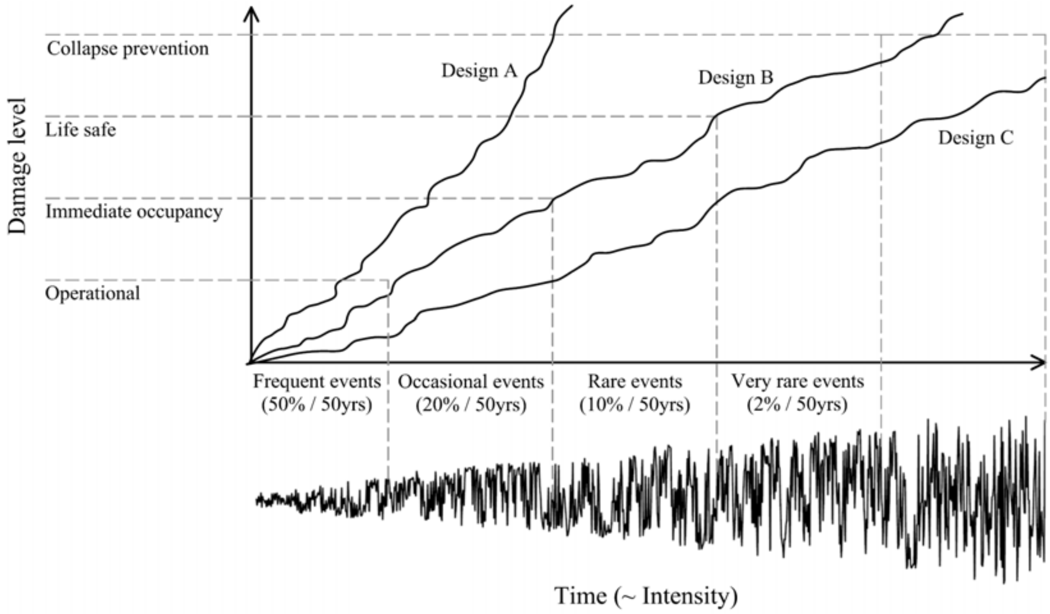

Fig. 1. Basic concept of the ET analysis

The concept seems to be simple enough. Now the question is whether it is possible to establish a meaningful correlation between the intensity of an intensifying excitation and those of ground motions. It turns out that the concept of response spectrum can be used quite effectively in producing intensifying excitation functions. The point is that the response spectrum strongly reflects two major characteristics of any ground motion, i.e. the intensity and the frequency content. Two dynamic excitations with similar response spectrum produce almost similar responses in most structures. Thus, if the response spectrum of the ETEFs at a particular time can be generated to match a particular response spectrum corresponding to, say the average response spectrum of a set of ground motions, then the produced response at that time can be considered as a good estimator of the expected average response of the structure when subjected to those ground motions [1].

3. ET EXCITATION FUNCTIONS

As stated in the previous section, the concept of response spectrum can be used effectively in producing the excitation functions. A typical code design spectrum can be considered as a good starting point to approach this problem. In this way, the problem reduces to generating an intensifying acceleration function that produces a response spectrum matching a set of up scaling design spectrums at all times. The particular time corresponding to the scale factor of unity is called the target time, t_{Target} . The response spectrum produced by the ETEF at all times before the target time should be less, and at all times after the target time should be greater than the considered design spectrum. This means that we assume the overall shape of the target spectrum remains the same and target spectrums at various times are scaled versions of the same spectrum, which is called the template spectrum hereafter [1]. This requirement is given by:

$$S_{aC}(T,t)=\frac{t}{t_{\text{Target}}}S_{aC}(T) \tag{1}$$

where $S_{aC}(T)$ is the template spectrum, $S_{aC}(T,t)$ is the target spectrum to be approached at time *t* of the ETEF, and *T* is the fundamental period of the structure. This formula simply states that the acceleration response produced by the ETEF at a particular time *t* should remain proportional to the considered template spectrum and scale up in a linear manner as a function of time [1].

Displacement spectrum is also important for characterizing a dynamic excitation. Target displacement spectrum can be defined as a function of the template acceleration spectrum considering the linear behavior and common simplifications applied in structural dynamics:

$$S_{uC}(T,t)=\frac{t}{t_{\text{Target}}}S_{aC}(T)\times\frac{T^2}{4\pi^2} \tag{2}$$

where $S_{uC}(T,t)$ is the target displacement spectrum to be approached at time *t* of the ETEF [1].

Now, the next question is whether and how such a record can actually be produced. Obviously, from an analytical viewpoint, this is a complex problem that should be tackled. One practical approach to handle this problem is through an optimization procedure, the objective function of which can be formulated as:

$$Minimize \quad F(a_g)=\int_0^{T_{\max}}\int_0^{t_{\max}}\{[S_a(T,t)-S_{aC}(T,t)]^2+\alpha[S_u(T,t)-S_{uC}(T,t)]^2\}dt\,dT \tag{3}$$

where $S_a(T,t)$ and $S_u(T,t)$ are the acceleration and displacement response spectrum, respectively. a_g is the excitation function as the optimization variable and *α* is a weight factor considered equal to unity in this research [1]. While the formulation for producing the ETEF by (3) seems to be compact and straightforward, in practice its solution has been shown to be highly complicated and computationally demanding. As will be shown in section 5, the CMA-ES turns out to be effective in providing good quality solutions to this problem.

Some measures other than the objective function are also required to assess the quality of the ETEFs, e.g., the total absolute error, the absolute error of acceleration response spectra and the absolute error of displacement response spectra, which are respectively as follows:

$$E^t(a_g)=\sqrt{\frac{\sum_{i=1}^{n}\sum_{j=1}^{m}\{[S_a(i,j)-S_{aC}(i,j)]^2+\alpha[S_u(i,j)-S_{uC}(i,j)]^2\}}{n\times m}} \tag{4}$$

$$E^a(a_g)=\sqrt{\frac{\sum_{i=1}^{n}\sum_{j=1}^{m}[S_a(i,j)-S_{aC}(i,j)]^2}{n\times m}} \tag{5}$$

$$E^u(a_g) = \sqrt{\frac{\sum_{i=1}^{n}\sum_{j=1}^{m}[S_u(i,j) - S_{uC}(i,j)]^2}{n \times m}} \tag{6}$$

4. THE CMA-ES

In this section, a brief description of the CMA-ES is presented. For more information about terminology and details, interested readers may refer to [22]. A summary of the algorithm together with a table of the default strategy parameters and their values are provided in Appendix A.

a) Basic equation: sampling

In the CMA-ES, a population of new search points (individuals, offspring) is generated by sampling a multivariate normal distribution. At each generation, the basic equation for sampling is:

$$x_k^{(g+1)} \sim m^{(g)} + \sigma^{(g)} N(0, \boldsymbol{C}^{(g)}) \qquad for\ \ \mathrm{k} = 1, \ldots, \lambda \tag{7}$$

where

$N(0, \boldsymbol{C}^{(g)})$ is a multivariate normal distribution with zero mean and covariance matrix $\boldsymbol{C}^{(g)}$,
$\boldsymbol{x}_k^{(g+1)} \in R^n$, k-th offspring from generation $g + 1$,
$\boldsymbol{m}^{(g)} \in R^n$, mean value of the search distribution at generation g,
$\sigma^{(g)} \in R_+$, overall standard deviation, step-size, at generation g,
$\boldsymbol{C}^{(g)} \in R^{n \times n}$, covariance matrix at generation g,
$\lambda \geq 2$, population size, number of offspring.

To define the complete iteration step, the remaining problem is how to calculate $\boldsymbol{m}^{(g+1)}$, $\boldsymbol{C}^{(g+1)}$ and $\sigma^{(g+1)}$ for the next generation $g + 1$. The next three subsections will deal with these problems.

b) Selection and recombination

The new mean $\boldsymbol{m}^{(g+1)}$ of the search distribution is a weighted average of μ selected points from the sample:

$$\boldsymbol{m}^{(g+1)} = \sum_{i=1}^{\mu} \omega_i \boldsymbol{x}_{i:\lambda}^{(g+1)} \tag{8}$$

where

$\mu \leq \lambda$ is the parent population size, i.e. the number of selected points.

$\omega_i \in R_+$, positive weight coefficients for recombination. $\sum_{i=1}^{\mu} \omega_i = 1$ and $\omega_1 \geq \omega_2 \geq \ldots \geq \omega_\mu \geq 0$.

$\boldsymbol{x}_{i:\lambda}^{(g+1)}$, i-th best individual out of $\boldsymbol{x}_1^{(g+1)}, \ldots, \boldsymbol{x}_\lambda^{(g+1)}$ from (8). The index $i : \lambda$ denotes the index of the i-th ranked individual and $f(\boldsymbol{x}_{1:\lambda}^{(g+1)}) \leq f(\boldsymbol{x}_{2:\lambda}^{(g+1)}) \leq \ldots \leq f(\boldsymbol{x}_{\lambda:\lambda}^{(g+1)})$, where f is the objective function to be minimized.

The measure $\mu_{eff} = (\sum_{i=1}^{\mu} \omega_i)^{-1}$ will be used in the following and can be paraphrased as variance effective selection mass. From the definition of ω_i in (8), $1 \leq \mu_{eff} \leq \mu$ is derived.

c) Adapting the covariance matrix

The CMA-ES is based on two adaptation principles, which make it an efficient procedure for multimodal continuous problems. Firstly, a maximum-likelihood principle, based on the idea to increase the probability of successful candidate solutions and search steps. For this purpose, the algorithm updates the covariance matrix of the distribution such that the likelihood of already applied successful steps is increased. Rank-μ-update performs this principle [15]. Secondly, an evolution path principle, based on memorizing the time evolution path of the distribution mean. These paths contain substantial information about the correlation between consecutive steps. The evolution paths are exploited in two ways. One path is used for the covariance matrix adaptation procedure and facilitates a possibly much faster variance increase of favorable directions. Rank-one-update performs this. The other path is used to conduct an additional step-size control that effectively prevents premature convergence yet allows a faster convergence (see Section 4.4) [15].

1. Rank-μ-update: Choosing $\boldsymbol{C}^{(0)}$ to be the unity matrix, the new covariance matrix $\boldsymbol{C}^{(g+1)}$ is given by:

$$\begin{aligned}\boldsymbol{C}^{(g+1)} &= (1-c_\mu)\boldsymbol{C}^{(g)} + c_\mu \frac{1}{\sigma^{(g)^2}} \boldsymbol{C}_\mu^{(g+1)} \\ &= (1-c_\mu)\boldsymbol{C}^{(g)} + c_\mu \sum_{i=1}^{\mu} \omega_i\, \boldsymbol{y}_{i:\lambda}^{(g+1)} \boldsymbol{y}_{i:\lambda}^{(g+1)^{\mathrm{T}}}\end{aligned} \tag{9}$$

where

$0 \le c_\mu \le 1$ is learning rate for updating the covariance matrix. For $c_\mu = 1$, no prior information is retained and for $c_\mu = 1$, no learning takes place.

$\boldsymbol{y}_{i:\lambda}^{(g+1)} = (\boldsymbol{x}_{i:\lambda}^{(g+1)} - \boldsymbol{m}^{(g)}) / \sigma^{(g)}$, the selected steps.

This covariance matrix update is called rank-μ-update, because the sum of outer products in (9) is of rank μ. The number $1/c_\mu$ is the backward time horizon, which says approximately 37% of the information in $\boldsymbol{C}^{(g+1)}$ is older than the last $1/c_\mu$ generations.

2. Rank-one-update: A sequence of successive steps, in which the strategy takes over a number of generations is called an evolution path. An evolution path can be expressed by a sum of consecutive steps. This summation is referred to as cumulation. The evolution path of the distribution mean is expressed by:

$$\boldsymbol{p}_c^{(g+1)} = (1-c_c)\boldsymbol{p}_c^{(g)} + \sqrt{\boldsymbol{c_c}(2-c_c)\mu_{eff}}\, \frac{\boldsymbol{m}^{(g+1)} - \boldsymbol{m}^{(g)}}{\sigma^{(g)}} \tag{10}$$

where

$\boldsymbol{p}_c^{(g))} \in R^n$ is the evolution path at generation g and $\boldsymbol{p}_c^{(0)} = 0$.

c_c, learning rate for updating the evolution path.

In the final algorithm the relationship (10) is slightly modified, see Appendix A.

The rank-one-update of the covariance matrix $\boldsymbol{C}^{(g)}$ via the evolution path $\boldsymbol{p}_c^{(g+1)}$ is defined as:

$$\boldsymbol{C}^{(g+1)} = (1-c_1)\boldsymbol{C}^{(g)} + c_1 \boldsymbol{p}_c^{(g+1)} \boldsymbol{p}_c^{(g+1)^{\mathrm{T}}} \tag{11}$$

where

c_1 is the learning rate for the rank one update.

Figure 2 demonstrates the concept behind the covariance matrix adaptation in the CMA-ES algorithm. As the generations progress, the algorithm approaches the global optimum while simultaneously the distribution shape adapts to an ellipsoidal landscape and the search is directed along an evolution path.

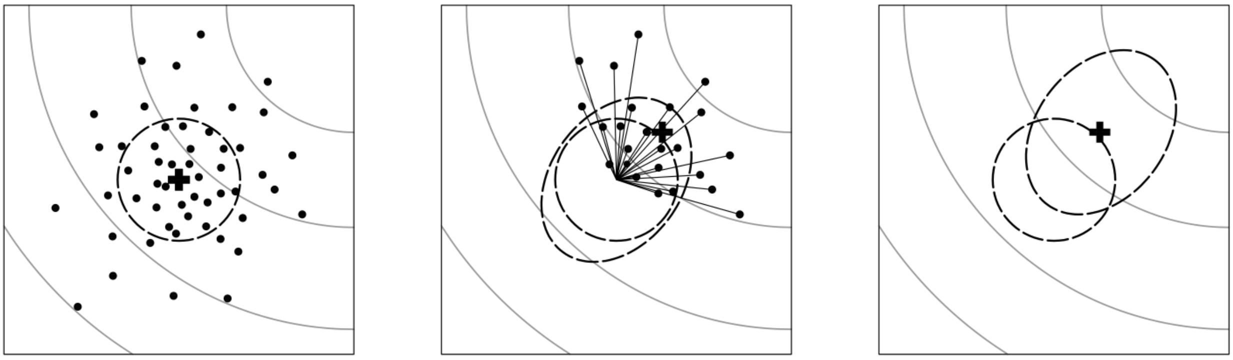

Fig. 2. Estimation of the search distribution for the second generation. Left: producing the first population by sampling of N (0, I). Middle: selection of the new parents and updating the covariance matrix; solid lines determine the selected steps. Right: search distribution of the next generation (dashed ellipsoid). Contour lines (grayed) indicate that the strategy should move toward the upper right corner

d) Step-size control

The covariance matrix adaptation, introduced in the last section, does not explicitly control the "overall scale" of the distribution. Step-size control defines how much the distribution ellipsoid should be elongated or shortened, to achieve an optimal scale. The evolution path is utilized to control the step-size.

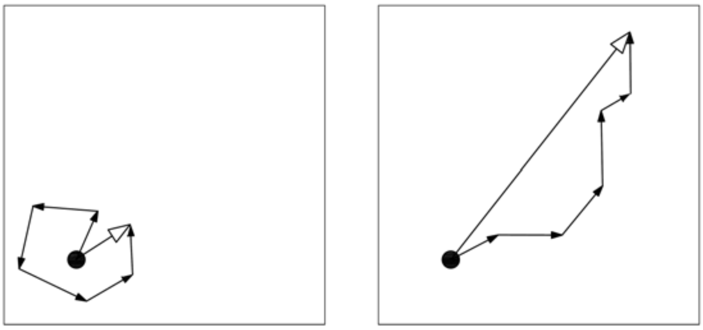

Fig. 3. Two evolution paths of respectively six steps from various situations. The length of the evolution paths is remarkably different and is exploited for step-size control

The length of an evolution path is exploited, based on the following reasoning. Whenever the evolution path is short, single steps cancel each other out as is shown in Fig. 3 (left). Hence, they are called anti-correlated. If steps annihilate each other, the step-size should be decreased. Whenever the evolution path is long, the single steps are pointing to similar directions and they are called correlated, Fig. 3 (right). Because the steps are similar, the same distance can be covered by fewer but longer steps into the same directions. Consequently, the step-size should be increased.

To decide whether the evolution path is long or short, the length of the path is compared with its expected length under random selection, which is equal to the expectation of the Euclidean norm of a N (0, **I**) distributed random vector. If selection biases the evolution path to be longer than expected, σ is increased, and vice versa.

To calculate the step-size, a conjugate evolution path is constructed because the expected length of the evolution path p_c from (10) depends on its direction. Initialized with $\boldsymbol{p}_\sigma^{(0)} = 0$, the conjugate evolution path is given by:

$$\boldsymbol{p}_\sigma^{(g+1)} = (1-c_\sigma)\boldsymbol{p}_\sigma^{(g)} + \sqrt{\boldsymbol{c}_\sigma(2-c_\sigma)\mu_{eff}}\, \boldsymbol{C}^{(g)^{-\frac{1}{2}}} \frac{\boldsymbol{m}^{(g+1)} - \boldsymbol{m}^{(g)}}{\sigma^{(g)}} \tag{12}$$

where

$\boldsymbol{p}_\sigma^{(g)} \in R^n$ is the conjugate evolution path at generation g.

c_σ, the learning rate.

$\boldsymbol{C}^{(g)^{-\frac{1}{2}}} = \boldsymbol{B}^{(g)} \boldsymbol{D}^{(g)^{-1}} \boldsymbol{B}^{(g)^{\mathrm{T}}}$, this transformation produces the expected length of $\boldsymbol{p}_\sigma^{(g+1)}$ independent of its direction.

The step-size update is formulated as:

$$\sigma^{(g+1)} = \sigma^{(g)} \, exp\,(\frac{c_\sigma}{d_\sigma}(\frac{\| p_\sigma^{(g+1)} \|}{\mathrm{E}\| \; N(0,\mathbf{I}) \|} - 1)) \tag{13}$$

where

d_σ is damping parameter that scales the change magnitude of $\sigma^{(g)}$.

E ‖ N (0, **I**) ‖ = $\sqrt{n}(1 - \frac{1}{4n} + \frac{1}{21n^2})$, expected length under random selection where n is the search space dimension.

5. GENERATING ET EXCITATION FUNCTIONS VIA THE CMA-ES

The design spectrum of BHRC (Standard No. 2800) for stiff soil (type II) is employed as the template spectrum in this study [23]. Time duration of the excitation functions is 20 seconds, which is composed of 2000 acceleration points in 0.01-second time steps. The target time is selected to be 10th second and that is when the response spectrum of a SDOF system with a damping ratio of 5% should match the template spectrum having a scale factor of unity and remain proportional to it at all other times. Maximum fundamental period of the system is considered to be 5 seconds. The periodic range from 0.0 to 5.0 seconds is divided into 0.005-second time steps. According to Standard No. 2800, the template spectrum for soil type II is expressed by:

$$\begin{cases} B = 1 + 1.5(\dfrac{T}{0.1}) & T < 0.1 \\ B = 2.5 & 0.1 \le T < 0.5 \\ B = 2.5(\dfrac{0.5}{T})^{\frac{2}{3}} & 0.5 \le T \end{cases} \tag{14}$$

$$S_{aC}(T) = \frac{0.35BI}{R}$$

where I is the importance factor of the structure taken as 1.0 and R is the response reduction factor that is not applied (i.e. R = 1.0) [22].

A three-member series of ETEFs, labeled as ET-ES series, was produced by exploiting the CMA-ES. One record of the series is depicted in Fig. 4 and the acceleration and displacement response spectra of this record at 5th, 10th, 15th and 20th seconds are demonstrated in Fig. 5. This is the result of optimizing the satisfaction of 2,000,000 equations (response in 1000 periods at 2000 time spots) by 2000 variables.

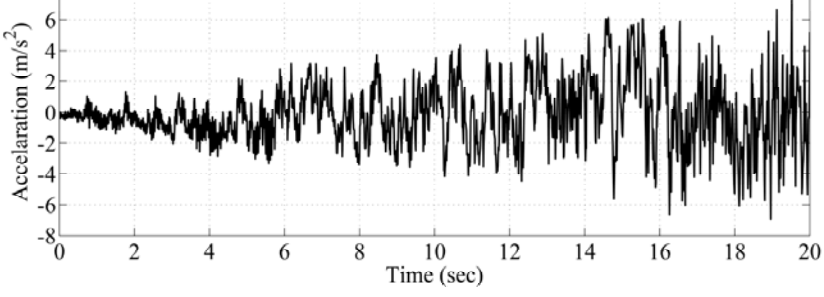

Fig. 4. ET-ES01 excitation function

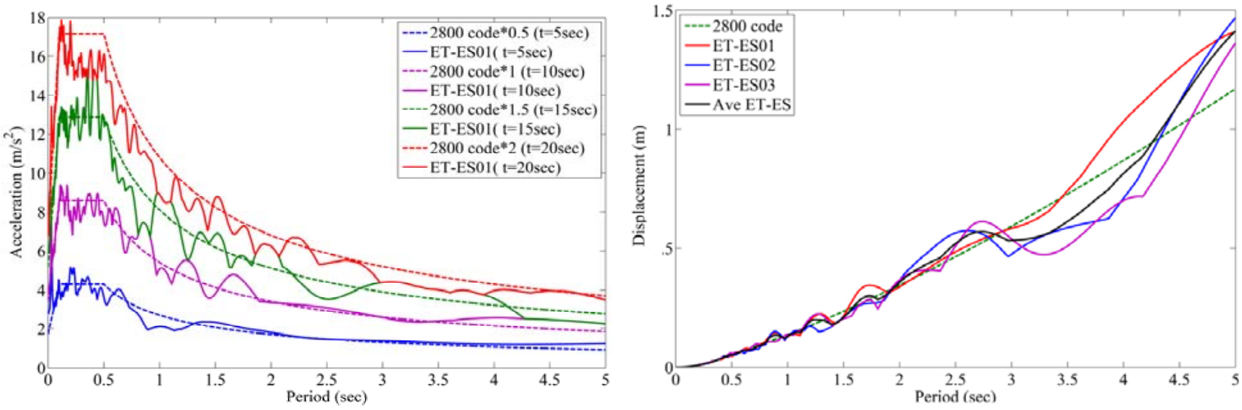

Fig. 5. Acceleration and displacement response spectra of ET-ES01 at 5th, 10th, 15th and 20th seconds

In order to reduce the level of scatter, various alternative schemes can be adopted. One of the simplest procedures in this regard is to average the result from several ETEFs in order to reduce the level of a scatter around the target [1]. Preliminary studies show that by averaging the results from multiple ETEFs, a reasonably accurate estimate of the response can be made [4]. The average response from three records of the ET-ES series at the target time is shown in Fig. 6. As can be seen, the average response has a better fit with the target spectrum.

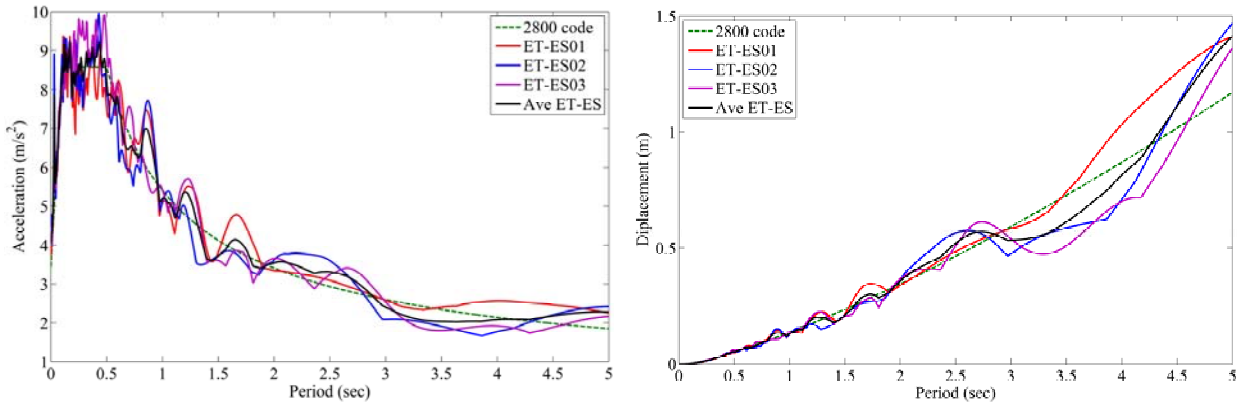

Fig. 6. Acceleration and displacement response spectra of ET-ES series at the target time (the 10th second).

Table 1. Calculated errors of ET-ES series

	Absolute acceleration response spectra error	Absolute displacement response spectra error	Total absolute error
ET-ES01	0.4629	0.0762	0.4691
ET-ES02	0.4703	0.0710	0.4766
ET-ES03	0.4787	0.0831	0.4858
Average	0.4706	0.0768	0.4772
Ave ET-ES	0.3172	0.0516	0.3214

Measure of the absolute acceleration and displacement response spectra errors and the total absolute error of the ET-ES series are provided in Table. 1. As can be seen, the average of total absolute error of the series is 0.4772, whereas the error of average response of the series is 0.3214. Thus, by averaging the results of the three records, the amount of deviation is reduced about 33 percent.

For a more precise performance assessment, the calculated errors of the best available ETEF, generated on the same template spectrum with the ET-ES series, are provided in Table 2. ETA20a01, ETA20a02 and ETA20a03 form a series of ETEFs that are the result of a mathematical programming approach, applies a classical unconstrained optimization procedure [2]. It is interesting to note that the error of the average response of the ETA20a series is still about 19 percent less than that of the ET-ES series. Nevertheless, quality is not the only concern in production of ETEFs, but the time efficiency is also important. The optimization time of ET-ES series is about one third of the time spent for the ETA20a series. This reduction is noticeable due to heavy computational time cost in the generation of ETEFs. The CPU-time consumption of the program, coded in MATLAB®, for generating each record of the ET-ES series was roughly 45 hours, which is obtained using an Intel® Core™ i7 @ 2.0 GHz processor equipped with 8 GBs of RAM. This means, by employing the CMA-ES for production of ETEFs rather than the classical unconstrained optimization procedure, a significant improvement in computational effort can be achieved, accompanied by a slight loss of accuracy. Data of both series are available online [20].

Table 2. Calculated errors of ET-ES and ETA20a series

Total absolute error			
ET-ES01	0.4691	ETA20a01	0.4343
ET-ES02	0.4766	ETA20a02	0.4539
ET-ES03	0.4858	ETA20a03	0.4305
Average	0.4772	Average	0.4396
Ave ET-ES	0.3214	Ave ETA20a	0.2618

6. CONCLUDING REMARKS

In this paper, the CMA-ES is implemented to produce the intensifying excitation functions of the Endurance Time method. A series of ETEFs, labeled as ET-ES series, is produced for a design spectrum of BHRC code. The calculated errors of the ET-ES series confirm that the proposed procedure can be applied efficiently in the production of ETEFs with reasonable accuracy.

It is not clear how much better ETEFs could be generated using advanced optimization procedures. However, it seems that even with the current level of fitness, the produced ETEFs can be successfully put into practical use. Production of improved ETEFs that produce more precise estimates of seismic response remains an open topic.

Besides quality considerations in production of ETEFs, reducing the optimization time is also essential. Production of ETEFs, considering the high volume of computations, is an extremely time-consuming process. Therefore, developing more efficient optimization procedures is necessary to make the process practically appealing.

The CMA-ES does not require a tedious parameter tuning for its application. In fact, the choice of strategy parameters is not left to the user. Finding good strategy parameters is considered as a part of the algorithm design, and not part of its application. For the application of the CMA-ES, just an initial solution, an initial standard deviation (step-size) and, possibly, the termination criteria need to be set by the user. This makes the CMA-ES a user-friendly optimization method.

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REFERENCES

1. Estekanchi, H. E., Vafai, A., Valamanesh, V., Mirzaee, A., Nozari, A. & Bazmuneh, A. (2011). Recent advances in seismic assessment of structures by endurance time method. Seismic Risk Management in Urban Areas, A Turkish-Iranian-American Workshop, December 2010, Istanbul Turkey, PEER report.

2. Valamanesh, V., Estekanchi, H. E. & Vafai, A. (2010). Characteristics of second generation endurance time acceleration functions. *Sci. Iran.*, Vol. 17, No. 1, pp. 53-61.
3. Estekanchi, H. E., Vafai, A. & Sadeghazar, M. (2004). Endurance time method for seismic analysis and design of structures. *Scientia Iranica*, Vol. 11, No. 4, pp. 361-370.
4. Estekanchi, H. E., Valamanesh, V. & Vafai, A. (2007). Application of endurance time method in linear seismic analysis. *Eng. Struct*., Vol. 29, No. 10, pp. 2551-2562.
5. Estekanchi, H. E., Arjomandi, K. & Vafai, A. (2008). Estimating structural damage of steel moment frames by endurance time method. *J. Constr. Steel. Res.,* Vol. 64, No. 2, pp. 145-155.
6. Riahi, H. T., Estekanchi, H. E. & Vafai, A. (2009). Endurance time method - application in nonlinear seismic analysis of SDOF systems*. J. App. Sci.*, Vol. 9, No. 10, pp. 1817-1832.
7. Riahi, H. T. & Estekanchi, H. E. (2010). Seismic assessment of steel frames with endurance time method. *J. Constr. Steel. Res.,* Vol. 66, No. 6, pp. 780-792.
8. Valamanesh, V. & Estekanchi, H. E. (2010). Compatibility of the endurance time method with codified seismic analysis approaches on three-dimensional analysis of steel frames. *Struct. Des. Tall. Spec. Build.*, doi:10.1002/tal.666.
9. Valamanesh, V. & Estekanchi, H. E. (2010). A study of endurance time method in the analysis of elastic moment frames under three-directional seismic loading. *Asian J. Civil Eng.*, Vol. 11, No. 5, pp. 543-562.
10. Nozari, A. & Estekanchi, H. E. (2011). Optimization of endurance time acceleration functions for seismic assessment of structures. *Int. J. Opt. Civil Eng.*, Vol. 1, No. 2, pp. 257-277.
11. Kaveh, A. & Laknejadi, K. (2011). A hybrid multi-objective particle swarm optimization and decision making procedure for optimal design of truss structures. *Iranian Journal of Science and Technology, Transactions of Civil Engineering*, Vol. 35, No. C2, pp. 137-154.
12. Kaveh, A. & Sharafi, P. (2012). Ordering for bandwidth and profile minimization problems via charged system search method. *Iranian Journal of Science and Technology, Transactions of Civil Engineering*, Vol. 36, No. C1, pp. 39-52.
13. Kaveh, A., Talatahari, S. & Farhmand Azar, B. (2012). Optimum design of composite open channels using charged system search algorithm. *Iranian Journal of Science and Technology, Transactions of Civil Engineering*, Vol. 36, No. C1, pp. 67-77.
14. Kaveh, A. & Massoudi, M.S. (2012). Cost optimization of a composite floor system using ant colony system. *Journal of Science and Technology, Transactions of Civil Engineering*, Vol. 36, No. C2, pp. 139-148.
15. Talbi, E. G. (2009). *Metaheuristics: from design to implementation*. Wiley, New Jersey.
16. Darwin, C. (1859). *On the origin of species by means of natural selection*. John Murray, London.
17. Holland, J. H. (1975). *Adaptation in natural and artificial systems*. University of Michigan Press, Ann Arbor, MI.
18. Rechenberg, I. (1973). *Evolutionstrategie: Optimierung technischer systeme nach prinzipien derbiologischen evolution*. Frommann-Holzboog.
19. Fogel, L. J., Owens, A. J. & Walsh, M. J. (1966). *Artificial intelligence through simulated evolution.* Wiley.
20. Koza, J. R. (1992). *Genetic Programming*. MIT Press, Cambridge, MA.
21. Hansen, N., Ostermeier, A. & Gawelczyk, A. (1995). On the adaptation of arbitrary normal mutation distributions in evolution strategies: The generating set adaptation. In L. Eshelman (Ed.), *Proceedings of the Sixth International Conference on Genetic Algorithms*, Pittsburgh, Morgan Kaufmann.
22. Hansen, N. The CMA evolution strategy: a tutorial. http://www.lri.fr/~hansen/cmaesintro.html.
23. BHRC. (2005). *Iranian code of practice for seismic resistant design of buildings, Standard No. 2800-05.* 3rd edition, Building and Housing Research Center, Tehran.
24. Estekanchi, H. E. Website of the Endurance Time method, [Online]. http://sharif.edu/~stkanchi/ET.

APPENDIX A. SUMMARY OF THE CMA-ES ALGORITHM

Set parameters

Set parameters λ, μ, $\omega_{i=1,\ldots,\mu}$, c_μ, c_1, c_c, c_σ and d_σ to their default values according to Table 7.

Set evolution paths $\boldsymbol{p}_c^{(0)} = 0$, $\boldsymbol{p}_\sigma^{(0)} = 0$, and covariance matrix $\boldsymbol{C}^{(0)} = \mathbf{I}$.

Choose distribution mean $\boldsymbol{m}^{(0)} \in R^n$ and step-size $\sigma^{(g)} \in R_+$, problem dependent.

Until termination criterion met

Sample new population of search points

$$\boldsymbol{x}_k^{(g+1)} \sim \boldsymbol{m}^{(g)} + \sigma^{(g)} N(0, \boldsymbol{C}^{(g)}) \qquad \text{for } k = 1, \ldots, \lambda$$

Selection and recombination

$$\boldsymbol{m}^{(g+1)} = \sum_{i=1}^{\mu} \omega_i \boldsymbol{x}_{i:\lambda}^{(g+1)} \qquad \text{where} \quad \sum_{i=1}^{\mu} \omega_i = 1, \ \omega_i \geq 0$$

Step-size control

$$\boldsymbol{p}_\sigma^{(g+1)} = (1-c_\sigma)\boldsymbol{p}_\sigma^{(g)} + \sqrt{\boldsymbol{c}_\sigma(2-c_\sigma)\mu_{eff}}\, \boldsymbol{C}^{(g)^{-\frac{1}{2}}} \frac{\boldsymbol{m}^{(g+1)} - \boldsymbol{m}^{(g)}}{\sigma^{(g)}}$$

$$\sigma^{(g+1)} = \sigma^{(g)} \, exp\left(\frac{c_\sigma}{d_\sigma}\left(\frac{\| p_\sigma^{(g+1)} \|}{\mathrm{E}\| N(0,\boldsymbol{I}) \|} - 1\right)\right)$$

Covariance matrix adaptation

$$\boldsymbol{p}_c^{(g+1)} = (1-c_c)\boldsymbol{p}_c^{(g)} + h_\sigma^{(g+1)} \sqrt{\boldsymbol{c_c}(2-c_c)\mu_{eff}}\, \frac{\boldsymbol{m}^{(g+1)} - \boldsymbol{m}^{(g)}}{\sigma^{(g)}}$$

$$\text{where} \quad h_\sigma^{(g+1)} = \begin{cases} 1 & \text{if} \quad \dfrac{\| \mathrm{p}_\sigma^{(g+1)} \|}{\sqrt{1-(1-c_\sigma)^{2(g+1)}}} < (1.4 + \dfrac{2}{n+1})\mathrm{E}\| N(0,\mathbf{I}) \| \\ 0 & \text{Otherwise} \end{cases}$$

$$\boldsymbol{C}^{(g+1)} = (1-c_1-c_\mu)\boldsymbol{C}^{(g)} + c_1 \boldsymbol{p}_c^{(g+1)} \boldsymbol{p}_c^{(g+1)^{\mathrm{T}}} + c_\mu \sum_{i=1}^{\mu} \omega_i \, \boldsymbol{y}_{i:\lambda}^{(g+1)} \boldsymbol{y}_{i:\lambda}^{(g+1)^{\mathrm{T}}}$$

Table 3. Default strategy parameters

Selection and recombination

$$\lambda = 4 + \lfloor 3 \ln n \rfloor, \qquad \mu = \lfloor \mu' \rfloor, \qquad \mu' = \frac{\lambda}{2}$$

$$\omega_i = \frac{\omega_i'}{\sum_{j=1}^{\mu} \omega_j'}, \qquad \omega_i' = \ln(\mu'+5) - \ln i \qquad \text{for } i = 1, \ldots, \mu$$

Step-size control

$$c_\sigma = \frac{\mu_{eff}+2}{n+\mu_{eff}+5}, \qquad d_\sigma = 1 + c_\sigma + 2\max\left(0, \sqrt{\frac{\mu_{eff}-1}{n+1}} - 1\right)$$

Covariance matrix adaptation

$$c_c = \frac{4+\mu_{eff}/n}{n+4+2\mu_{eff}/n}$$

$$c_1 = \frac{2}{(n+1.3)^2 + \mu_{eff}}$$

$$c_\mu = \min\left(1-c_1, \frac{2(\mu_{eff}-2+1/\mu_{eff})}{(n+2)^2+\mu_{eff}}\right)$$