A COMPUTATIONALLY EFFICIENT ADAPTIVE CONTROLLER FOR ROBOTIC MANIPULATORS USING THE THEORY OF PASSIVE SYSTEMS*

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Abstract – This paper develops a new adaptive control law for rigid robotic manipulators, which is computationally very fast and therefore suitable for real-time implementations. This globally convergent adaptive controller is a modified version of the original scheme presented earlier, but results in simpler design conditions in comparison with the previous work in this area. The convergence properties of the proposed scheme are established through a basic theorem, using the theory of passive dynamic systems. Other aspects of this paper, such as a new general lemma on passive systems, which is utilized in proof of the basic theorem, and a procedure for determining the design parameters, are also of interest. This algorithm is simulated on a digital computer and the results confirm theoretical studies.

Keywords - Rigid manipulators, adaptive control, passive systems, computational algorithms

1. INTRODUCTION

Robot manipulators are kinematic chains of rigid links connected through revolute or prismatic joints. They have gained growing acceptance during the last decade and are widely utilized for various industrial applications, particularly in automated manufacturing tasks. The dynamics of the system are characterized by nonlinear effects and strong couplings between the individual links of the manipulator chain. In addition, the robotic system is subjected to the uncertainty of the parameters describing the dynamic properties of the grasped load. It is widely recognized that the accuracy of conventional approaches in high speed operations is greatly affected by these parameter uncertainties.

In order to improve dynamic accuracy in the tracking of fast desired motions, efforts have been devoted to develop various control algorithms such as the application of adaptive control theory to robot manipulators. The main advantage of applying adaptive control theory to robot manipulators is the controller ability to compensate the effects of configuration changes and uncertainties associated with the system and load dynamics. The desire to ensure asymptotic trajectory tracking is central to recent research in the design of adaptive motion controllers for rigid robotic manipulators. Controllers that achieve this objective for all desired trajectories and all initial conditions are said to be globally convergent. Two relatively recent surveys [2,3] indicate the breadth of the approaches taken.

The class of direct adaptive control is one of the categories of recently developed adaptive robot controllers that make use of the full robot dynamic model. In this method, by utilizing the fact that the dynamic equations of robotic motion are linear in the inertial unknown parameters, a control law is

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proposed that guarantees the asymptotic convergence of position error. These results are common in that their schemes are based on replacing the desired trajectory by the virtual reference trajectory, which leads to error equations, where its regressor is independent of joint accelerations. The elements of the regressor matrix are known, as are the calculable functions of the manipulator structural dynamics. The parameter adaptation law is based on the trajectory tracking errors and only used to estimate the unknown inertial parameters, which includes the numerical values of the masses, the link dimensions and specifications of the payload. Due to the special structure of their control law, inversion of the inertia matrix is not required. The original contributions in this area can be found in [4,5]. Modification of the original scheme to enhance the robustness properties is presented in [6,7], while the design of a hybrid adaptive-robust approach has been outlined in [8]. Passivity-based direct adaptive controllers that exploit the passivity properties of mechanical manipulators have been considered in [9,10]. In [11,12], the exponential trajectory following of a similar scheme is proven without requiring any stringent conditions. Implementation aspects and experimental results of approach have been discussed in [13].

However, the practical applications of the above-mentioned schemes (with one exception [11]) are restricted by serious drawbacks due to computational complexities. They require on-line calculation of a large amount of nonlinear functions of the joint positions and velocities, which is more time-consuming. This problem is very critical in following fast desired trajectories that need fast sampling rates. Therefore, motivations for computationally efficient adaptive controllers are the ease and speed of implementation of the control algorithms using inexpensive hardware.

Our correspondence follows the work of Sadegh and Horowitz [1], who first proposed a modified version of the original direct scheme. The modification consists of utilizing the desired trajectories in the computation of the nonlinear terms of the regressor matrix instead of the actual quantities. Usually, the desired trajectories are known in advance, therefore the computations can be executed off-line and the results retrieved from memory as needed. This modification reduces the computational burden and moderates the scheme for practical applications. In spite of this efficiency, the design parameters of the algorithm must be extracted from a set of complex conditions to satisfy the positive definiteness of the so-called controller matrix. In addition, the convergence of the method was analyzed with difficulty through the Lyapunov approach, using stringent conditions.

In this work, we present a new adaptive control law for robotic manipulators, which is computationally very fast. The convergence properties of the proposed scheme are established through a basic theorem using the theory of passive dynamic systems. A new general lemma on passive systems, which was first proposed in [14], is utilized in proof of the basic theorem. This contribution results in a simpler set of design conditions in comparison with the previous works in this area. In order to cope with the parameter uncertainties, a parameter estimation law is considered which preserves the passivity property of the overall system.

This paper first presents a commonly used mathematical model of robotic motion and explains its properties. Thereafter, design of a computationally efficient adaptive controller for the tracking control of robotic manipulators is outlined to guarantee zero position tracking errors. Then, the convergence analysis of the proposed scheme is discussed, and finally the results are shown by simulation studies.

2. MATHEMATICAL MODEL OF ROBOT MANIPULATORS

The dynamics of an n-degree of freedom manipulator, in the absence of friction and other disturbances, can be obtained from the Lagrange-Euler equation of motion in the following form

$$H(q,\theta)\ddot{q} + C(q,\dot{q},\theta)\dot{q} + g(q,\theta) = \tau(t)$$
 (1)

 q, \dot{q}, \ddot{q} is the n×1 vector of joint angles, rates, and accelerations, H(q) is the n×n matrix, also called the generalized inertia matrix, $C(q, \dot{q})\ddot{q}$ is the n×n vector of centripetal and coriolis torques, g(q) is the n×1 vector of gravitational torques, $\tau(t)$ is the n×1 vector of joint torques or forces applied by accounters, and θ is the r×1 vector of unknown inertial parameters of link and payload.

The set of differential Eqs. (1) is highly nonlinear and consists of inertia loading, coupling reaction forces among the various joints, and gravity loading effects. Despite these complexities, the robotic system has several fundamental properties which can be exploited to facilitate adaptive control system design. These properties can be stated as follows

- Property 1. As noted by several authors (e.g. [3]), the two n×n matrices H and C are not independent. Specifically, given a proper definition of the matrix C, the matrix H-2C will be skew-symmetric.
- Property 2. The generalized inertia matrix is symmetric and positive definite [3].
- Property 3. The robotic system is an n-input/n-output dynamic system, which defines a passive mapping from the generalized torque vector to joint velocity vector [9].
- Property 4. The dynamic equations of motion are linear in the inertial parameters [15]. Therefore with a suitable choice of the parameter vector, Eq. (1) can be written as

$$H(q,\theta)\ddot{q} + C(q,\dot{q},\theta)\dot{q} + g(q,\theta) = Y(q,\dot{q},\ddot{q})\theta \tag{2}$$

where $Y(q, \dot{q}, \ddot{q})$ is an n×r matrix of known functions and $\theta \in R^r$ contains unknown parameters. The regressor $Y(q, \dot{q}, \ddot{q})$ is bounded for bounded q, \dot{q}, \ddot{q} . Note that the dimension of the parameter vector depends on the particular choice of parameters that is not unique.

3. ADAPTIVE CONTROLLER DESIGN FOR ROBOT MANIPULATORS

The controller design objective can be stated as follows, given the desired trajectory, and with some or all of the manipulator parameters being unknown, derives a control algorithm and an estimation law for the unknown parameters such that they force the manipulator to track the desired trajectory with desirable velocity after an initial adaptive process.

Let us denote the desired trajectories of position and velocity by $q_d(t)$ and $\dot{q}_d(t)$, respectively. In addition, assume that $\dot{q}_d(t)$ is differentiable, and denote its derivative by $\ddot{q}_d(t)$. The value of $\ddot{q}_d(t)$ is referred to as the desired acceleration. Given $q_d(t)$ and the manipulator output q(t), the trajectory tracking error is defined as

$$\widetilde{q}(t) = q(t) - q_d(t) \tag{3}$$

In the sequel, a set of time-based trajectories will be introduced and the control law is obtained considering them. The virtual reference acceleration is described by the following linear dynamics

$$\ddot{q}_r(t) = \ddot{q}_d(t) - K(p)\tilde{q}(t) \tag{4}$$

where K(p) is a differential relation and p denotes the differentiating operator. The time dependence of all signals will be removed for simplicity in the subsequent analyses. A new error that is the difference of actual and virtual velocity is defined as follows

$$e = \dot{q} - \dot{q}_r \tag{5}$$

which can be represented in the frequency domain as

$$E(s) = (sI + \frac{1}{s}K(s))\widetilde{Q}(s) = G^{-1}(s)\widetilde{Q}(s)$$
(6)

The transfer function matrix K(s) specifies the performance of the outer loop of adaptive controller. Let us define this relation as

$$K(s) = s\Lambda \tag{7}$$

where Λ is a diagonal matrix with positive diagonal element λ_k . With this choice, Eq. (5) can be rewritten as

$$e = \dot{\widetilde{q}} + \lambda_K \widetilde{q} \tag{8}$$

Utilizing the desired trajectories $q_d(t)$ and Eqs. (1),(3) and (8), the adaptive control law is proposed as

$$\tau = \hat{H}(q_d)\ddot{q}_d + \hat{C}(q_d, \dot{q}_d)\dot{q}_d + \hat{g}(q_d) - K_D e - N(e, \tilde{q})$$
(9)

where (\hat{r}) denotes the *a priori* known or estimate of the same terms in the robot dynamics and K_D is a diagonal positive definite gain matrix. $N(e, \tilde{q})$ is a nonlinear compensating feedback term that will be introduced later. The role of this additional term is to compensate for the additional error caused by using the desired trajectory outputs, instead of the actual joint outputs. Exploiting the property 4 of robot arm dynamics, Eq. (9) can be written as

$$\tau = Y(q_d, \dot{q}_d, \ddot{q}_d) \hat{\theta} - K_D e - N(e, \tilde{q})$$
(10.1)

$$= \phi(q_d, \dot{q}_d, \ddot{q}_d) - K_D e - N(e, \tilde{q})$$
 (10.2)

where ϕ is an n×1 vector whose elements are nonlinear functions of desired trajectories and linear functions of inertial parameters. The closed loop error dynamic will be obtained by combining Eq. (9) and system dynamic equation

$$Y(q_{d}, \dot{q}_{d}, \ddot{q}_{d})\widetilde{\Theta} = H(q)\dot{e} + C(q, \dot{q})e + K_{D}e + N(e, \tilde{q}) + \{Y(q, \dot{q}, \dot{q}_{r}, \ddot{q}_{r}) - Y(q_{d}, \dot{q}_{d}, \ddot{q}_{d})\}\Theta$$
(11)

or

$$\widetilde{\phi}(q_d, \dot{q}_d, \ddot{q}_d) = H(q)\dot{e} + C(q, \dot{q})e + K_D e + N(e, \widetilde{q}) + \Delta Y(e, \widetilde{q})\theta$$
(12)

where

$$Y(q_d, \dot{q}_d, \ddot{q}_d)\widetilde{\Theta} = \widetilde{\phi}(q_d, \dot{q}_d, \ddot{q}_d)$$
(13.1)

$$\Delta Y(e,\tilde{q}) = Y(q,\dot{q},\dot{q}_r,\ddot{q}_r) - Y(q_d,\dot{q}_d,\ddot{q}_d)$$
(13.2)

and $\widetilde{\boldsymbol{\theta}}$ is the parameter estimation error which is defined as

$$\widetilde{\theta} = \hat{\theta} - \theta \tag{14}$$

As it can be seen from Eq. (12), an additional error is introduced. The following lemma provides explicit bounds on $\Delta Y(e, \tilde{q})$.

Lemma-1 [1]: If H(q), $C(q, \dot{q})$ and g(q) are C-infinity functions of their variables and have partial derivatives with respect to q, that of

$$\left\| \frac{\partial H}{\partial q} \right\|, \left\| \frac{\partial C}{\partial q} \right\|, \left\| \frac{\partial g}{\partial q} \right\| \in L\infty$$
 (15)

then it can be shown that there exist positive valued functions α , β and η such that

$$-e^{T}\Delta Y(e,\widetilde{q})\theta \leq e^{T}(\lambda_{k}H + \alpha I)e + e^{T}(-\lambda_{k}^{2}H + \beta I)\widetilde{q} + \eta(\|e\|^{2}\|\widetilde{q}\| + \lambda_{k}\|e\|\|\widetilde{q}\|^{2})$$

$$(16)$$

As it was mentioned in the last section, by choosing a suitable set of inputs and outputs, the robotic system defines a passive mapping. This important property has inspired a number of researchers to extend the passivity theory to robotic systems. The two following lemmas provide important results from the theory of passive systems and will be used for the convergence analysis of the proposed adaptive controller.

Lemma-2 [3]: Consider the differential equation

$$H(q)\dot{e} + C(q,\dot{q})e + K_D e = Y(q,\dot{q},\dot{q}_r,\ddot{q}_r)\widetilde{\Theta} = \Psi$$
(17)

if the mapping from '-e' to ' ψ ' is passive and the transfer function G(s) defined in (6) has stable poles, then \tilde{q} and $\dot{\tilde{q}}$ will be globally asymptotically stable.

In the following a new general lemma on passive systems will be presented.

Lemma-3: If the mapping, -e' to ' ϕ ' is passive, then for any arbitrary vector x and any positive definite matrix A, the following result can be stated

$$\int_{0}^{T} (-\phi^{T} e + x^{T} A \dot{x}) dt \ge -\gamma_{0}^{2}$$

$$\tag{18}$$

Proof: Individual integration of the two terms at the left side of Eq. (18) yields the result.

In the sequel, the convergence properties of Eq. (12) will be examined. The basic results and convergence proof of the method are introduced in the following theorem.

Theorem-1: Suppose that the nonlinear compensating vector is defined as

$$N(e, \widetilde{q}) = \lambda_n \|\widetilde{q}\|^2 e \tag{19}$$

If the mapping from '-e' to $\tilde{\phi}$ is passive, the transfer function G(s) has stable poles and the designing parameter λ_k is chosen such that $\lambda_k^2 H - \beta I$ be positive definite, then other designing parameters λ_n and λ_d can be selected such that \tilde{q} and \tilde{q} are globally asymptotically convergent, i.e., both \tilde{q} (t) and \tilde{q} (t) will tend to zero from any initial condition.

Proof: Choose a Lyaponuv function candidate, due to lemma 3, as follows

$$V(e, \widetilde{q}, \widetilde{\phi}) = \frac{1}{2} \int_{e}^{T} H(q)e + \gamma_0^{2+} \int_{0}^{T} \left[-\phi^{T} e + \widetilde{q}^{T} (\lambda_k^{2} H - \beta_I) \dot{\widetilde{q}} \right] dt$$
 (20)

differentiating Eq. (20) along the trajectories of closed loop system yields

$$\dot{V}(e,\tilde{q},\tilde{\phi}) = -e^T K_D e - e^T N(e,\tilde{q}) - e^T \Delta Y(e,\tilde{q}) \tilde{\theta} + \tilde{q}^T (\lambda_k^2 H - \beta I) \dot{\tilde{q}}$$
(21)

it can be easily shown that

$$e^{T}N(e,\widetilde{q}) \ge \lambda_{n} \|e\|^{2} \|\widetilde{q}\|^{2}$$
(22)

Utilizing this fact and Eq. (16) results

$$\dot{V}(e,\widetilde{q},\widetilde{\phi}) \leq -e^{T} k_{D} e - \lambda_{n} \|e\|^{2} \|\widetilde{q}\|^{2} + e^{T} (\lambda_{K} H + \alpha I) e - e^{T} (\lambda_{k}^{2} H - \beta I) \widetilde{q} +$$

$$\eta(\|e\|^{2} \|\widetilde{q}\| + \lambda_{k} \|e\| \|\widetilde{q}\|^{2}) + \widetilde{q}^{T} (\lambda_{k}^{2} H - \beta I) e - \widetilde{q}^{T} (\lambda_{k}^{3} H - \lambda_{k} \beta I) \widetilde{q}$$
(23)

since the inertia matrix is symmetric, two similiar terms in Eq. (23) vanish and it can be written

$$\dot{V}(e,\widetilde{q},\widetilde{\phi}) \leq -e^{T} \left(K_{D} - \lambda_{k}H - \alpha I\right)e - \widetilde{q}^{T} \left(\lambda_{k}^{3}H - \lambda_{k}\beta I\right)\widetilde{q} - \lambda_{n} \|e\|^{2} \|\widetilde{q}\|^{2} + \eta \left(\|e\|^{2} \|\widetilde{q}\| + \lambda_{k} \|e\| \|\widetilde{q}\|^{2}\right)$$
(24)

let us define the maximum and minimum eigenvalues of H(q) to be λ_M and λ_m , respectively. Therefore the result follows

$$\dot{V}(e, \widetilde{q}, \widetilde{\phi}) \leq -\left(\lambda_{d} - \lambda_{k} \lambda_{M} - \alpha - \frac{1}{4} \eta\right) \|e\|^{2} - \left(\lambda_{k}^{2} \lambda_{m} - \beta - \frac{1}{4} \eta\right) \lambda_{K} \|\widetilde{q}\|^{2} - \left(\lambda_{k}^{2} \lambda_{m} - \beta - \frac{1}{4} \eta\right) \lambda_{K} \|\widetilde{q}\|^{2} - \left(\lambda_{k}^{2} \lambda_{m} - \beta - \frac{1}{4} \eta\right) \lambda_{K} \|\widetilde{q}\|^{2} - \left(\lambda_{k}^{2} \lambda_{m} - \beta - \frac{1}{4} \eta\right) \lambda_{K} \|\widetilde{q}\|^{2} - \left(\lambda_{k}^{2} \lambda_{m} - \beta - \frac{1}{4} \eta\right) \lambda_{K} \|\widetilde{q}\|^{2} - \left(\lambda_{k}^{2} \lambda_{m} - \beta - \frac{1}{4} \eta\right) \lambda_{K} \|\widetilde{q}\|^{2} - \left(\lambda_{k}^{2} \lambda_{m} - \beta - \frac{1}{4} \eta\right) \lambda_{K} \|\widetilde{q}\|^{2} - \left(\lambda_{k}^{2} \lambda_{m} - \beta - \frac{1}{4} \eta\right) \lambda_{K} \|\widetilde{q}\|^{2} - \left(\lambda_{k}^{2} \lambda_{m} - \beta - \frac{1}{4} \eta\right) \lambda_{K} \|\widetilde{q}\|^{2} - \left(\lambda_{k}^{2} \lambda_{m} - \beta - \frac{1}{4} \eta\right) \lambda_{K} \|\widetilde{q}\|^{2} - \left(\lambda_{k}^{2} \lambda_{m} - \beta - \frac{1}{4} \eta\right) \lambda_{K} \|\widetilde{q}\|^{2} - \left(\lambda_{k}^{2} \lambda_{m} - \beta - \frac{1}{4} \eta\right) \lambda_{K} \|\widetilde{q}\|^{2} - \left(\lambda_{k}^{2} \lambda_{m} - \beta - \frac{1}{4} \eta\right) \lambda_{K} \|\widetilde{q}\|^{2} - \left(\lambda_{k}^{2} \lambda_{m} - \beta - \frac{1}{4} \eta\right) \lambda_{K} \|\widetilde{q}\|^{2} - \left(\lambda_{k}^{2} \lambda_{m} - \beta - \frac{1}{4} \eta\right) \lambda_{K} \|\widetilde{q}\|^{2} - \left(\lambda_{k}^{2} \lambda_{m} - \beta - \frac{1}{4} \eta\right) \lambda_{K} \|\widetilde{q}\|^{2} - \left(\lambda_{k}^{2} \lambda_{m} - \beta - \frac{1}{4} \eta\right) \lambda_{K} \|\widetilde{q}\|^{2} - \left(\lambda_{k}^{2} \lambda_{m} - \beta - \frac{1}{4} \eta\right) \lambda_{K} \|\widetilde{q}\|^{2} - \left(\lambda_{k}^{2} \lambda_{m} - \beta - \frac{1}{4} \eta\right) \lambda_{K} \|\widetilde{q}\|^{2} - \left(\lambda_{k}^{2} \lambda_{m} - \beta - \frac{1}{4} \eta\right) \lambda_{K} \|\widetilde{q}\|^{2} - \left(\lambda_{k}^{2} \lambda_{m} - \beta - \frac{1}{4} \eta\right) \lambda_{K} \|\widetilde{q}\|^{2} - \left(\lambda_{k}^{2} \lambda_{m} - \beta - \frac{1}{4} \eta\right) \lambda_{K} \|\widetilde{q}\|^{2} + \left(\lambda_{k}^{2} \lambda_{m} - \beta - \frac{1}{4} \eta\right) \lambda_{K} \|\widetilde{q}\|^{2} + \left(\lambda_{k}^{2} \lambda_{m} - \beta - \frac{1}{4} \eta\right) \lambda_{K} \|\widetilde{q}\|^{2} + \left(\lambda_{k}^{2} \lambda_{m} - \beta - \frac{1}{4} \eta\right) \lambda_{K} \|\widetilde{q}\|^{2} + \left(\lambda_{k}^{2} \lambda_{m} - \beta - \frac{1}{4} \eta\right) \lambda_{K} \|\widetilde{q}\|^{2} + \left(\lambda_{k}^{2} \lambda_{m} - \beta - \frac{1}{4} \eta\right) \lambda_{K} \|\widetilde{q}\|^{2} + \left(\lambda_{k}^{2} \lambda_{m} - \beta - \frac{1}{4} \eta\right) \lambda_{K} \|\widetilde{q}\|^{2} + \left(\lambda_{k}^{2} \lambda_{m} - \beta - \frac{1}{4} \eta\right) \lambda_{K} \|\widetilde{q}\|^{2} + \left(\lambda_{k}^{2} \lambda_{m} - \beta - \frac{1}{4} \eta\right) \lambda_{K} \|\widetilde{q}\|^{2} + \left(\lambda_{k}^{2} \lambda_{m} - \beta - \frac{1}{4} \eta\right) \lambda_{K} \|\widetilde{q}\|^{2} + \left(\lambda_{k}^{2} \lambda_{m} - \beta - \frac{1}{4} \eta\right) \lambda_{K} \|\widetilde{q}\|^{2} + \left(\lambda_{k}^{2} \lambda_{m} - \beta - \frac{1}{4} \eta\right) \lambda_{K} \|\widetilde{q}\|^{2} + \left(\lambda_{k}^{2} \lambda_{m} - \beta - \frac{1}{4} \eta\right) \lambda_{K} \|\widetilde{q}\|^{2} + \left(\lambda_{k}^{2} \lambda_{m} - \beta - \frac{1}{4} \eta\right) \lambda_{K} \|\widetilde{q}\|^{2} + \left(\lambda_{k}^{2} \lambda_{m} - \beta - \frac{1}{4} \eta\right) \lambda_{K} \|\widetilde{$$

where λ_d is the minimum eigenvalue of diagonal matrix K_D . The negative definiteness of $\dot{V}(e, \widetilde{q}, \widetilde{\phi})$ will be guaranteed, if the designing parameters are chosen as follows

$$\lambda_k \ge \frac{1}{2} \sqrt{\frac{4\beta + \eta}{\lambda_m}} \tag{26.1}$$

$$\lambda_n \ge \eta(\lambda_k + 1) \tag{26.2}$$

$$\lambda_d \ge \lambda_k \lambda_m + \alpha + \frac{1}{4} \eta \tag{26.3}$$

therefore the closed loop system is globally stable and e converges to zero. Due to Eq. (6) and since G(s) has stable poles, it will be inferred from the principal theorem of linear systems, that \tilde{q} and $\dot{\tilde{q}}$ converge to zero asymptotically. Q.E.D

In order to apply the above theorem on a system with unknown parameters, we have to consider a parameter estimation law such that the passivity condition is satisfied. Now, we propose the following parameter update law that is driven by the error corresponding to the virtual reference and actual velocities \dot{q} and \dot{q}_r as defined in Eq. (5)

$$\dot{\hat{\theta}} = -PY^T (q_d, \dot{q}_d, \ddot{q}_d)e \tag{27}$$

where P is a constant positive definite weighting matrix that is used for scaling and will adjust the rate of convergence. The following theorem expresses the basic property of the gradient estimator defined in Eq. (27).

Theorem-2: The passivity condition introduced in Theorem-1 is satisfied for the estimator defined by Eq. (29). Proof - The result can be proved as follows

$$\int_{0}^{T} -\phi^{T} e \, dt = \int_{0}^{T} -\widetilde{\theta}^{T} Y^{T} \left(q_{d}, \dot{q}_{d}, \ddot{q}_{d} \right) e \, dt = \int_{0}^{T} -\widetilde{\theta}^{T} P^{-1} \dot{\widetilde{\theta}} dt$$

$$= \frac{1}{2} \widetilde{\theta}^{T} P^{-1} \widetilde{\theta} \Big|_{0}^{T} = \widetilde{\theta}^{T} (T) P^{-1} (T) \widetilde{\theta} (T) - \frac{1}{2} \widetilde{\theta}^{T} (0) P^{-1} (0) \widetilde{\theta} (0)$$
(28)

The first term in Eq. (29) is positive, since by choosing $\gamma^2 = \frac{1}{2} \tilde{\theta}^T (0) P^{-1}(0) \tilde{\theta}(0)$, it is clear that the passivity condition will be established and therefore the estimator (27) can be used to guarantee the global convergence with a suitable rate.

Q.E.D

Note that the above theorems establish that the adaptive controller defined by (9) and (27) is globally asymptotically stable and guarantees zero tracking errors. In particular, any consistent parameter estimator may be used, provided it preserves the passivity performance of the overall system.

4. SIMULATION RESULTS

Simulation studies were conducted for a two-degree-of-freedom planar robot arm to test the adaptine algorithm. This manipulator can be modeled as two rigid links of lengths l_1 and l_2 with point masses at the distal ends of links, m_1 and m_2 (Fig.1). It moves in a vertical plane with gravity acting. Such a manipulator, although quite simple, is subject to joint torques due to inertial, centrifugal, coriolis and gravity effects.

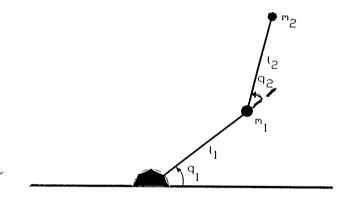


Fig. 1. Manipulator model considered for simulation study

The dynamical equations of this manipulator are as follows [14]

$$\tau_{1} = (m_{1} + m_{2})l_{1}^{2}\ddot{q}_{1} + m_{2}l_{2}^{2}\ddot{q}_{1} + 2m_{2}l_{1}l_{2}^{2}\cos(q_{2})\ddot{q}_{1} + m_{2}l_{1}l_{2}^{2}\ddot{q}_{2} + m_{2}l_{1}l_{2}\cos(q_{2})\ddot{q}_{2} -$$
(29.1)

$$m_2 l_1 l_2 \sin(q_2)(2\dot{q}_1 \dot{q}_2) + \dot{q}^2 2 + m_2 l_2 g \cos(q_1 + q_2) + (m_1 + m_2) l_1 g \cos(q_1)$$

$$\tau_2 = m_2 l_2^2 \ddot{q}_1 + m_2 l_2 \cos(q_2) \ddot{q}_1 + m_2 l_2^2 \ddot{q}_2 + m_2 l_1 l_2 \sin(q_2) \dot{q}_1^2 + m_2 l_2 g \cos(q_1 + q_2)$$
 (29.2)

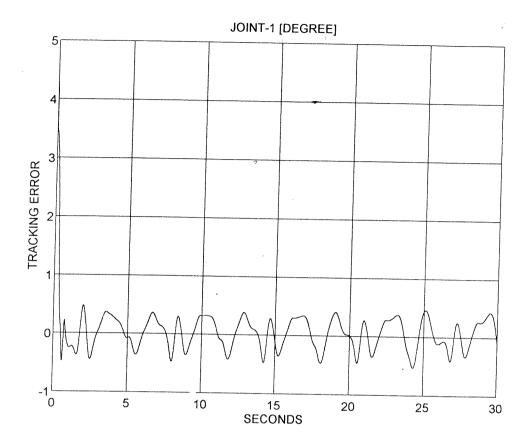
We assumed that the values of the link lengths and masses are unknown. Therefore writing the system equation in the form given in (2) results in a total of four independent parameters, which are m_1 , m_2 and l_1 . l_2 . Parameters used in the simulation were chosen to be realistic, and the maximum output torque of the actuators was limited.

The desired trajectory used in this simulation had the form

$$q_{1d} = 1.5 + \cos(2t) - \cos(4t) \tag{30.1}$$

$$q_{2d} = 1.5 + \sin(t) + \sin(2t) \tag{30.2}$$

Figures 2 through 4 show some results from a typical simulation. Figure 2 shows the tracking errors on joints 1 and 2 between the desired and actual trajectories. Plotting the actual trajectories versus the desired trajectories does not give any information and is not useful for comparison, because these two trajectories are very close. Both of the depicted variables on Fig. 2 initially had substantial errors, which were corrected by the adaptive controller over a few seconds of operation. In tuned steady state, the tracking errors are restricted to a limitation of which the bounds are less than one 0.5 percent of the desired trajectories' maximum amplitude. These remaining errors are due to the noise added to the simulation to test robustness. The magnitude of these tracking errors is consistent with the value of noise amplitude divided by the closed loop position gain. Figure 3 shows the torque command signals, which were applied to the joints of manipulator during the simulation. Figure 4 shows the unknown parameter estimations, starting from an initial value and adapting to their values. For the purpose of comparison, all the values were normalized to unity.



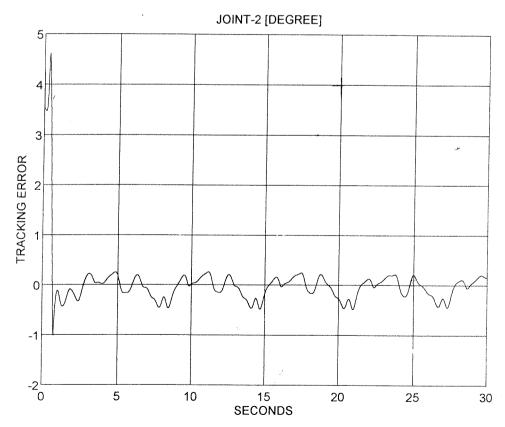
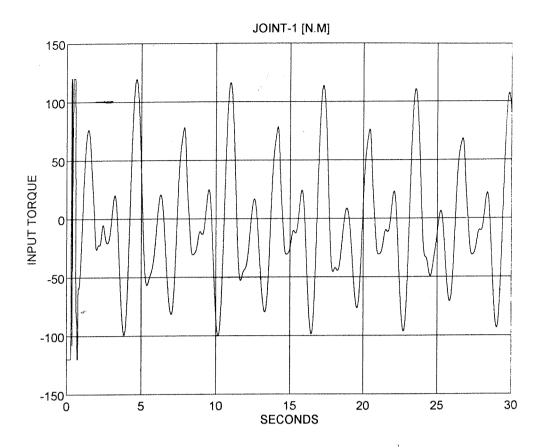


Fig.2 Tracking errors of joints



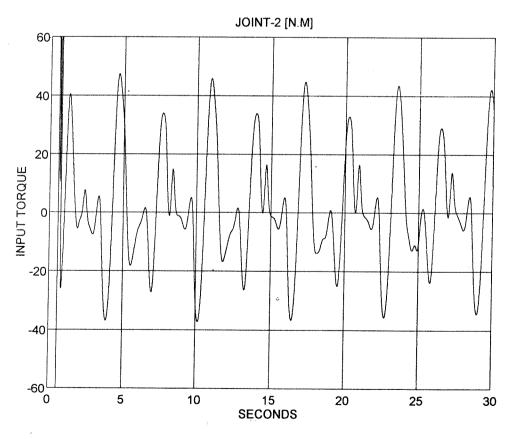


Fig.3 Input torque commands of joints

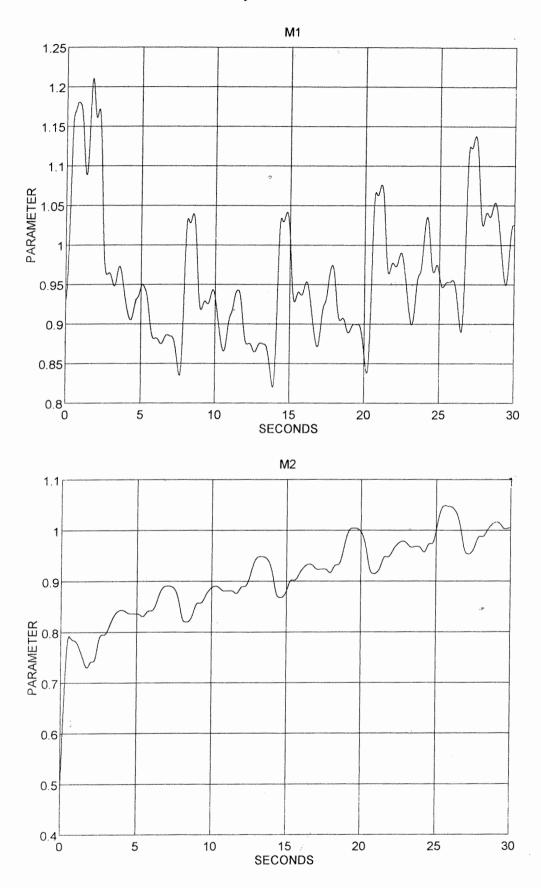


Fig. 4 a. (M1,M2) Convergence of the parameter estimation

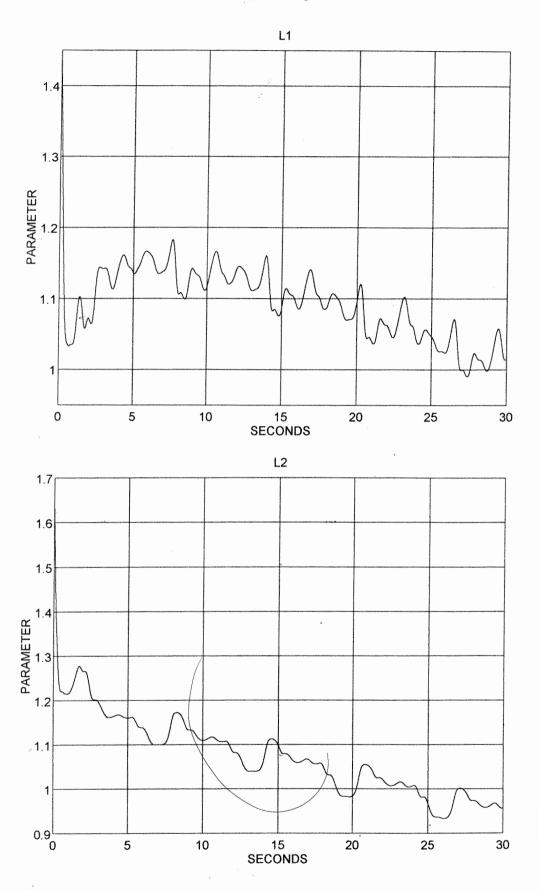


Fig. 4 b. (L1,L2) Convergence of the parameter estimation

5. CONCLUSION

In this paper, a computationally efficient adaptive control scheme is presented to achieve trajectory following for a robot manipulator. This scheme, which utilizes a direct approach, employs desired trajectories instead of actual trajectories. Usually, these quantities are known in advance and therefore the calculations can be executed off-line, which reduces the computational burden in each sampling period. The convergence properties of the proposed scheme are established through a basic theorem, using the theory of passive systems. The basic theorem utilizes a general lemma on passive theory, one which the authors believe is proposed for the first time. The basic theorem results in a simpler set of linear and decoupled design conditions in comparison with the previous works in this area. The controller parameters can be selected in a very simple and retractable manner using these design conditions. For the case of unknown parameters, a gradient estimator is proposed which preserves the passivity property of the overall system. The proposed controller and its related parameter estimation algorithm are simulated on a digital computer and the results confirm the theoretical studies.

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